FI 2001 MECÁNICA SESIÓN 13 04/05/21 PACEITUNO (1)

FUERZAS CENTRALES (CONTINUACIÓN)

CONSIDERACIONES DE ENERGÍA

$$\frac{1}{2} m v^2 + V = E_0 \quad \text{(CONSTANTE)}$$

$$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} \quad \text{(MOV. EN PLANO)}$$

$$v^2 = \dot{\rho}^2 + \rho^2\dot{\theta}^2$$

$$\frac{1}{2} m\dot{\rho}^2 + \frac{1}{2} m \rho^2 \dot{\theta}^2 + V = E_0$$

PERO... $\rho^2\dot{\theta} = \frac{l_0}{m}$ (MOMENTUM ANGULAR POR UNIDAD DE MASA)

$$\dot{\theta}^2 = \frac{l_0^2}{m^2 \rho^4}$$

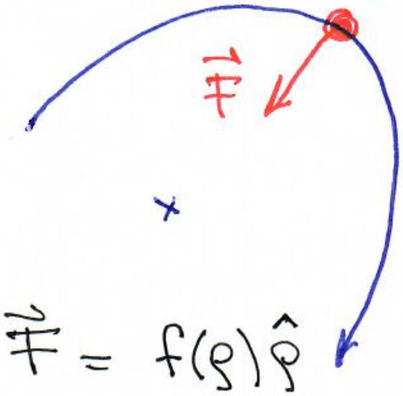
$$\boxed{\frac{1}{2} m\dot{\rho}^2 + \underbrace{V + \frac{1}{2}\frac{l_0^2}{m\rho^2}}_{V^*} = E_0}$$

V^* (POTENCIAL EFECTIVO)

$$\frac{1}{2} m\dot{\rho}^2 + V^*(\rho) = E_0$$

RECORDAR QUE $m\ddot{\rho} = f(\rho) + \frac{l_0^2}{m\rho^3} = f^*(\rho)$

MOV. BIDIMENSIONAL $\longrightarrow$ MOV. UNIDIMENSIONAL



$\Longrightarrow$

$$\vec{F} = f(\rho)\,\hat{\rho} \qquad\qquad \vec{F}^* = f^*(\rho)\,\hat{\rho}$$

$$m\vec{a} = \vec{F} \longleftrightarrow V(\rho) \qquad\qquad m\ddot{\rho}\,\hat{\rho} = f^*(\rho)\,\hat{\rho} \;\updownarrow\; V^*(\rho)$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\frac{1}{2} m v^2 + V(\rho) = E_0 \qquad\qquad \frac{1}{2} m \dot{\rho}^2 + V^*(\rho) = E_0$$

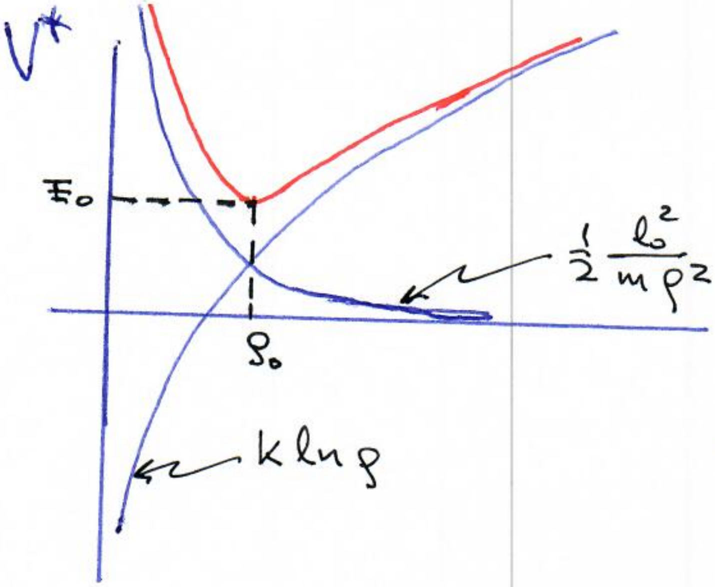
LA FORMA DE $V^*(\rho)$ PERMITE EXPLORAR EL TIPO DE MOVIMIENTO POSIBLE ALREDEDOR DEL CENTRO DE ATRACCIÓN O REPULSIÓN

EJEMPLO $\quad \vec{F} = -\frac{k}{\rho}\,\hat{\rho}$

$$\vec{F}\cdot d\vec{r} = -\frac{k}{\rho}\,d\rho = -dV \rightarrow \boxed{V = k\ln\rho}$$

$$\boxed{V^* = k\ln\rho + \frac{1}{2}\frac{l_0^2}{m\rho^2}} \longleftarrow$$

DEPENDE DE LA CONDICIÓN INICIAL



DADO l_0 (C.I.)

E_0 ES LA ENERGÍA EN UNA ÓRBITA CIRCULAR

$$E_0 = \frac{1}{2} m \dot{\rho}^2 + V^*(\rho_0)$$

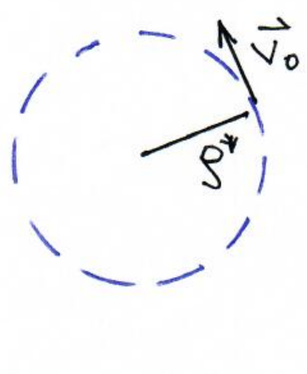
$\dot{\rho}^2 = 0$ (ÓRBITA CIRCULAR)

$V^*(\rho)$ ES MÍNIMO E ρ_0

$$\frac{dV^*}{d\rho} = \frac{k}{\rho} - \frac{l_0^2}{m\rho^3} = 0 \Longrightarrow \rho^{*2} = \frac{l_0^2}{mk}$$

COMO LA ÓRBITA ES CIRCULAR

$$l_0 = m \rho^* v_0 \qquad \cancel{\rho^{*2}} = \frac{m^{\cancel{2}} \cancel{\rho^{*2}} v_0^2}{\cancel{m} k}$$



$$v_0 = \sqrt{\frac{k}{m}}$$

NO DEPENDE DE ρ^*

SI SE LANZA UNA PARTÍCULA CON VELOCIDAD $v_0 \hat{\theta}$ A CUALQUIER DISTANCIA ρ^* DEL ORIGEN RESULTA UN MOVIMIENTO CIRCULAR

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QUE SUCEDE SI SE PERTURBA LA ÓRBITA CIRCULAR CON UNA PEQUEÑA PERTURBACIÓN RADIAL $\Delta\dot{\rho}$ LO CUAL <u>NO</u> CAMBIA l_0

$$m\ddot{\rho} = -\frac{dV^*}{d\rho}$$

$$V^*(\rho) = V^*(\rho^*) + \underbrace{\frac{dV^*}{d\rho}\Big|_{\rho^*}}_{=0}(\rho-\rho^*) + \frac{1}{2}\frac{d^2V}{d\rho^2}\Big|_{\rho^*}(\rho-\rho^*)^2 + \dots$$

$$\frac{dV^*}{d\rho} = \frac{d^2V}{d\rho^2}\Big|_{\rho^*}(\rho-\rho^*)$$

EC. MOV. $\quad m\ddot{\rho} + \frac{d^2V^*}{d\rho^2}\Big|_{\rho^*}(\rho-\rho^*) = 0$

$$\rho' = \rho - \rho^* \longrightarrow \ddot{\rho}' + \frac{1}{m}\frac{d^2V^*}{d\rho^2}\Big|_{\rho^*}\rho' = 0$$

Si $\frac{d^2V}{d\rho^2}\Big|_{\rho^*} > 0$ LA DISTANCIA RADIAL ρ OSCILA SEGÚN UN M.A.S ALREDEDOR DE ρ^*

EN ESTE CASO

$$\frac{dV^*}{d\rho} = \frac{k}{\rho} - \frac{l_0^2}{m\rho^3}$$

$$\frac{d^2V}{d\rho^2} = -\frac{k}{\rho^2} + \frac{3l_0^2}{m\rho^4}$$

$$l_0 = m\rho^* v_0$$

$$v_0 = \sqrt{\frac{k}{m}}$$

$$\frac{d^2V}{d\rho^2} = -\frac{k}{\rho^{*2}} + \frac{3m^2\rho^{*2}\left(\sqrt{\frac{k}{m}}\right)^2}{m\cdot\rho^{*4}} = \frac{2k}{\rho^{*2}}$$

$$\ddot{\rho}' + \frac{1}{m}\frac{d^2V^*}{d\rho^2}\bigg|_{\rho^*}\rho' = 0$$

$$\ddot{\rho}' + \omega_0^2\rho' = 0 \qquad \omega_0^2 = \frac{2k}{m\rho^{*2}}$$

$$\omega_0 = \frac{2\pi}{T} \rightarrow T = 2\pi\frac{\sqrt{m\rho^{*2}}}{\sqrt{2k}}$$

$$T = 2\pi\rho^*\sqrt{\frac{m}{2k}}$$

PREGUNTA: ¿ES CERRADA LA TRAYECTORIA?

ECUACIÓN DE TRAYECTORIA

$$m(\ddot{\rho} - \rho\dot{\theta}^2) = f(\rho) \qquad m\rho^2\dot{\theta} = l_0$$

$$\xi = \frac{1}{\rho} \longrightarrow \dot{\theta} = \frac{l_0}{m}\xi^2$$

$$\frac{d\rho}{dt} = \frac{d\rho}{d\theta}\dot{\theta} = \frac{d\rho}{d\theta}\cdot\frac{l_0}{m}\xi^2$$

$$\downarrow$$

$$\frac{d}{d\theta}\left(\frac{1}{\xi}\right) = -\frac{1}{\xi^2}\frac{d\xi}{d\theta}$$

$$\frac{d\rho}{dt} = -\frac{l_0}{m}\frac{d\xi}{d\theta}$$

$$\frac{d^2\rho}{dt^2} = -\frac{l_0}{m}\frac{d^2\xi}{d\theta^2}\dot{\theta} = -\left(\frac{l_0}{m}\right)^2\frac{d^2\xi}{d\theta^2}\xi^2$$

$$m\left[-\left(\frac{l_0}{m}\right)^2\frac{d^2\xi}{d\theta^2}\xi^2 - \frac{1}{\xi}\left(\frac{l_0}{m}\right)^2\xi^4\right] = f\left(\frac{1}{\xi}\right)$$

$$\boxed{-m\left(\frac{l_0}{m}\right)^2\xi^2\left[\frac{d^2\xi}{d\theta^2} + \xi\right] = f\left(\frac{1}{\xi}\right)}$$

EC. DE BINET