FI 2001 MECÁNICA SESIÓN 15 P. ACEITUNO 11/05/21

ÓRBITAS PLANETARIAS

$$\rho = \rho_0 \frac{1+e}{1+e\cos\theta} \quad (\theta = 0)$$

$$\rho_0 = \frac{h^2}{GM(1+e)} \qquad h = \frac{l_0}{m}$$

SISTEMA SOLAR

EXCENTRICIDADES (e)

TIERRA = 0.017
MARTE = 0.093
VENUS ; 0.007
JÚPITER : 0.048
PLUTÓN : 0,249
COMETA HALLEY : 0.967

ÓRBITAS SATELITALES TERRESTRES

$$\frac{GM}{R_T^2} = g \qquad \rho_0 = \frac{h^2}{g R_T (1+e)}$$

ÓRBITA GEOESTACIONARIA ECUATORIAL

EL SATÉLITE SE MUEVE CON LA MISMA VELOCIDAD ANGULAR QUE LA TIERRA

$$\omega_0 = \frac{2\pi}{T} \qquad T = 24 \times 3600$$

$$T = 86400$$

$$\cancel{m}\Big(\underbrace{\ddot{\rho}}_{=0} - \rho\,\underbrace{\dot{\theta}^2}_{\omega_0^2}\Big) = -\frac{G M_T \cancel{m}}{\rho^2} = -\frac{g R_T^2 \cancel{m}}{\rho^2}$$

$$\rho^3 = \frac{g R_T^2}{\omega_0^2}$$

$$R_T = 6360 \text{ Km} = 6.36\times 10^6 \text{ m}$$

$$\rho^3 = \frac{9.81\ 6.36^2\ 10^{12} \cdot 8.64^2\ 10^8}{4\pi^2}$$

$$\rho = 42.177 \text{ Km}$$

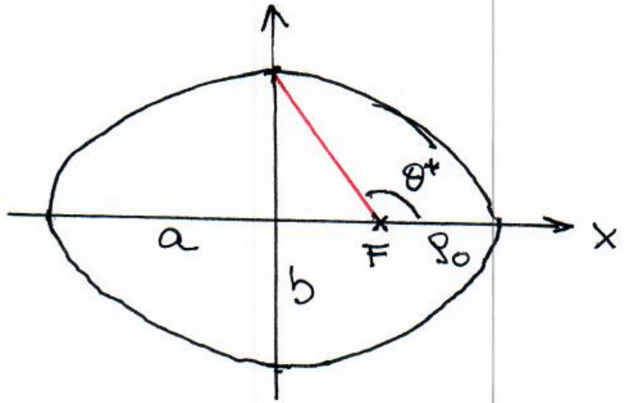
ALTURA DE LA ÓRBITA GEOSTACIONARIA SOBRE LA SUPERFICIE DE LA TIERRA

$$\boxed{z^* = 35.817 \text{ Km}}$$

ÓRBITAS ELÍPTICAS

TERCERA LEY DE KEPLER

(PERIODO ORBITAL)^2 $\propto$ (SEMI-EJE MAYOR)^3



$$a = \frac{1}{2}(\rho_0 + \rho_1)$$

$$\rho_0 = \rho(\theta = 0)$$

$$\rho_1 = \rho(\theta = \pi)$$

$$\rho = \rho_0 \frac{1+e}{1+e\cos\theta} \rightarrow \rho_1 = \rho_0 \frac{(1+e)}{(1-e)}$$

$$a = \frac{1}{2}\left[\rho_0 + \frac{\rho_0(1+e)}{(1-e)}\right] = \frac{\rho_0}{1-e} \qquad \boxed{a = \frac{\rho_0}{1-e}}$$

$$y = \rho \operatorname{sen}\theta \qquad y_{MAX} = b$$

$$y = \rho_0(1+e)\underbrace{(1+e\cos\theta)^{-1}\operatorname{sen}\theta}_{f(\theta)}$$

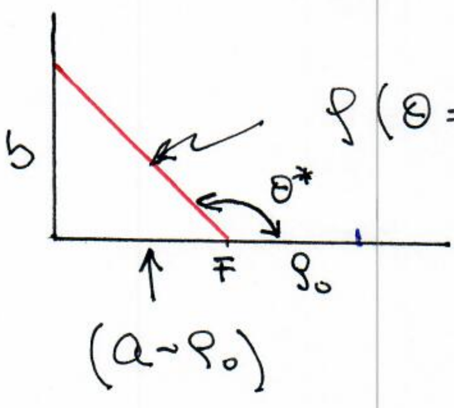
HAY QUE ENCONTRAR EL MAX. DE $f(\theta)$

$$\frac{df}{d\theta} = 0 \rightarrow -(1+e\cos\theta)^{-2}(-\overset{e}{\downarrow}\operatorname{sen}\theta)\operatorname{sen}\theta + $$
$$+(1+e\cos\theta)^{-1}\cos\theta$$

$$\frac{df}{d\theta} = \frac{e\operatorname{sen}^2\theta + (1+e\cos\theta)\cos\theta}{(1+e\cos\theta)^2} = 0$$

$$\frac{d f}{d\theta} = 0 \longrightarrow e + \cos\theta = 0$$

$$\boxed{\cos\theta^* = -e}$$



$$\rho(\theta = \theta^*) = \rho_0 \frac{(1+e)}{1+e\cos\theta^*}$$

$$= \frac{\rho_0(1+e)}{1-e^2} = \frac{\rho_0}{1-e} = a$$

$$a - \rho_0 = \frac{\rho_0}{1-e} - \rho_0 = \frac{\rho_0 e}{1-e} = ae$$

$$b^2 + (a-\rho_0)^2 = \rho^2(\theta^*)$$

$$b = \sqrt{a^2 - (ae)^2} \longrightarrow \boxed{b = a\sqrt{1-e^2}}$$

2ª LEY DE KEPLER

$$\frac{dS}{dt} = \frac{1}{2m} l_0$$

l_0 : MAGNITUD DEL MOMENTUM ANGULAR CONSTANTE

$$\boxed{S = \frac{1}{2m} l_0 T}$$

T : PERIODO ORBITAL

EN EL CASO DE UNA ÓRBITA ELÍPTICA

$$\boxed{S = \pi a b}$$

$$T = \frac{2m\pi a b}{l_0} = \frac{2\pi a b}{h} \qquad h = \frac{l_0}{m}$$

$$T^2 = \frac{(4\pi)^2}{h^2} a^2 b^2 = \frac{(4\pi)^2}{h^2} a^2 a^2(1-e^2)$$

$$a(1-e^2) = \frac{\rho_0}{1-e}(1-e^2) = \rho_0(1+e)$$

PERO $\rho_0(1+e) = \dfrac{h^2}{GM}$

$$T^2 = \frac{(4\pi)^2}{\cancel{h^2}} a^3 \frac{\cancel{h^2}}{GM}$$

$$\boxed{T^2 = \frac{(4\pi)^2}{GM} a^3}$$

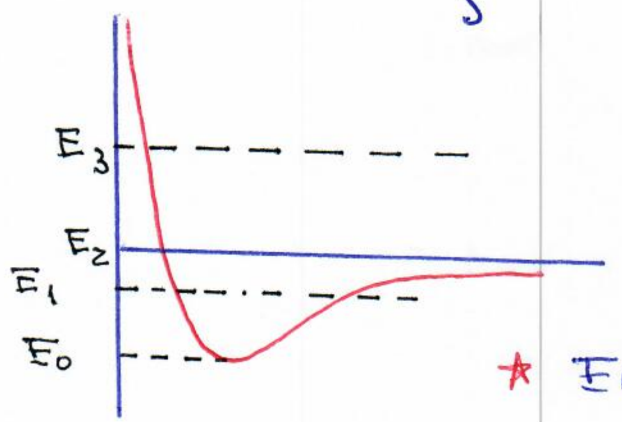
ENERGÍA MECÁNICA Y EXCENTRICIDAD

FUERZA GRAVITACIONAL $\vec{F} = -\frac{GMm}{\rho^2}\hat{\rho}$

POTENCIAL GRAVITATORIO $V = -\frac{GMm}{\rho}$

$$V^* = -\frac{GMm}{\rho} + \frac{mh^2}{2\rho^2}$$

$V = 0 \quad \rho = \infty$



EC. CONSERVACIÓN ENERGÍA

$$\boxed{\frac{1}{2}m\dot{\rho}^2 + V^* = E_0}$$

* ENERGÍA E_0 (<0) ÓRBITA CIRCULAR
* ENERGÍA E_1 (<0): ÓRBITA ELÍPTICA
* ENERGÍA E_2 $(=0)$ ÓRBITA PARABÓLICA

$$\lim_{\rho \to \infty} K_\rho = 0$$

* ENERGÍA E_3 (>0) ÓRBITA HIPERBÓLICA

$$\lim_{\rho \to \infty} K_\rho = E_3$$

$$\boxed{K_\rho = \frac{1}{2}m\dot{\rho}^2}$$

⑦

$$\frac{1}{2} m v^2 + V(\rho) = E_0$$

$$v^2 = \dot{\rho}^2 + (\rho\dot{\theta})^2 \qquad \xi = \frac{1}{\rho}$$

$$\frac{d\rho}{dt} = \frac{d\rho}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(\frac{1}{\xi}\right)\dot{\theta} \qquad \boxed{\rho^2\dot{\theta} = \frac{l_0}{m} = h}$$

$$\frac{d\rho}{dt} = -\frac{1}{\cancel{\xi^2}}\frac{d\xi}{d\theta}\cdot h\cancel{\xi^2} = -h\left(\frac{d\xi}{d\theta}\right) \qquad \boxed{\dot{\theta} = h\xi^2}$$

$$v^2 = h^2\left(\frac{d\xi}{d\theta}\right)^2 + \frac{1}{\xi^2}h^2\xi^4$$

$$v^2 = h^2\left[\left(\frac{d\xi}{d\theta}\right)^2 + \xi^2\right]$$

$$E = \frac{m}{2}h^2\left[\left(\frac{d\xi}{d\theta}\right)^2 + \xi^2\right] - GMm\xi$$

$$\frac{2E}{m} = h^2\left[\left(\frac{d\xi}{d\theta}\right)^2 + \xi^2\right] - 2GM\xi$$

PERO $$\xi = \frac{1 + e\cos\theta}{\rho_0(1+e)}$$

$$\frac{df}{d\theta} = \frac{-e \operatorname{sen}\theta}{\rho_0(1+e)}$$

$$\frac{2E}{m} = h^2\left[\left(\frac{-e\operatorname{sen}\theta}{\rho_0(1+e)}\right)^2 + \frac{(1+e\cos\theta)^2}{\rho_0^2(1+e)^2}\right] - 2GM\frac{1+e\cos\theta}{\rho_0(1+e)}$$

$$= h^2\frac{(e^2+1+2e\cos\theta)}{[\rho_0(1+e)]^2} - 2GM\frac{(1+e\cos\theta)}{\rho_0(1+e)}$$

PERO $\boxed{\rho_0(1+e) = \frac{h^2}{GM}}$

$$\frac{2E}{m} = \frac{\cancel{h^2}(GM)^2}{h^{\cancel{4}2}}(e^2+1+2e\cos\theta) - 2GM\cdot\frac{GM}{h^2}(1+e\cos\theta)$$

$$\frac{2E}{m} = \left(\frac{GM}{h}\right)^2(e^2-1)$$

$$\boxed{e = \left[1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2\right]^{1/2}}$$

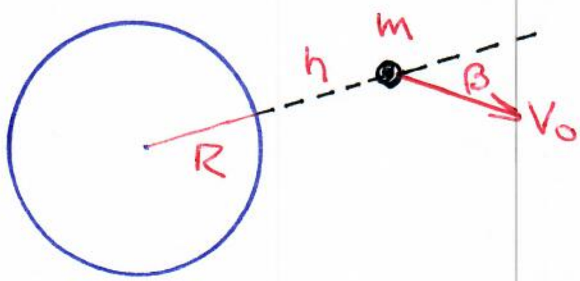
$E<0 \rightarrow e<1$ CÍRCULO + ELIPSE

$E=0 \rightarrow e=1$ PARÁBOLA

$E>0 \rightarrow e>1$ HIPÉRBOLA

EJ. D.19

⑨



¿ EC. DE TRAYECTORIA ?

$$E = \frac{1}{2} m v_0^2 - \frac{GMm}{(R+h)}$$

$$h = (R+h)\, v_0 \operatorname{sen}\beta$$

$$e = \left[1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2\right]^{1/2}$$

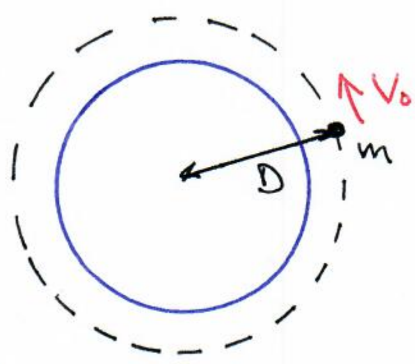
$$\rho_0 (1+e) = \frac{h^2}{GM} \longrightarrow \rho_0 = \frac{h^2}{GM}\frac{1}{(1+e)}$$

ECUACIÓN DE TRAYECTORIA

$$\rho = \rho_0 \frac{1+e}{1+e\cos\theta} = \frac{h^2}{GM}\frac{1}{1+e\cos\theta}$$

$$\rho(\theta) = \frac{h^2}{GM}\frac{1}{1+e\cos\theta}$$

EJ. D.21



SATÉLITE EN ÓRBITA CIRCULAR DE RADIO D

$$\frac{\cancel{m} v_0^2}{D} = \frac{GM\cancel{m}}{D^2} \qquad V_0 = \left(\frac{GM}{D}\right)^{1/2}$$

* IMPULSO Δv TANGENCIAL PARA QUE EL SATÉLITE ESCAPE DEL CAMPO DE ATRACCIÓN TERRESTRE

$$\frac{1}{2} m v^{*2} - \frac{GMm}{D} = 0 \rightarrow v^* = \left(\frac{2GM}{D}\right)^{1/2}$$

$$\Delta v = \left(\frac{2GM}{D}\right)^{1/2} - \left(\frac{GM}{D}\right)^{1/2} = \left(\frac{GM}{D}\right)^{1/2} (\sqrt{2} - 1)$$

* IDEM CON UN Δv RADIAL

$$\frac{1}{2} \cancel{m} \left(v_0^2 + (\Delta v)^2\right) - \frac{GM\cancel{m}}{D} = 0$$

$$(\Delta v)^2 = 2\frac{GM}{D} - v_0^2 = \frac{2GM}{D} - \frac{GM}{D}$$

$$\Delta v = \left(\frac{GM}{D}\right)^{1/2}$$