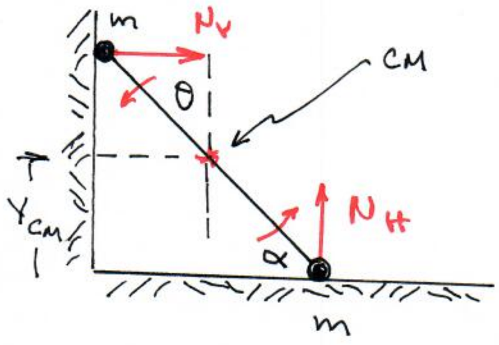
Fi 2001 SESION 23 17/06/21 P. ACEITUNO ①

EJEMPLO



$t=0$ ESTRUCTURA EN REPOSO $\alpha = \frac{\pi}{4}$

¿ $\dot{\theta}^*$ CUANDO N_V SE ANULA ?

CONSERVACIÓN DE ENERGÍA

$$K + V_G = mgL \operatorname{sen}\frac{\pi}{4}$$

$$K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM}\dot{\theta}^2$$

$$V_G = mgL \operatorname{sen}\alpha = mgL\cos\theta$$

$$x_{CM} = \frac{L}{2}\operatorname{sen}\theta \rightarrow \dot{x}_{CM} = \frac{L}{2}\cos\theta\,\dot{\theta}$$

$$y_{CM} = \frac{L}{2}\cos\theta \rightarrow \dot{y}_{CM} = -\frac{L}{2}\operatorname{sen}\theta\,\dot{\theta}$$

$$v_{CM}^2 = \left(\frac{L}{2}\cos\theta\,\dot{\theta}\right)^2 + \left(-\frac{L}{2}\operatorname{sen}\theta\,\dot{\theta}\right)^2 = \frac{L^2}{4}\dot{\theta}^2$$

$$I_{CM} = 2m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$$

$$K = \frac{1}{2} m \frac{L^2}{4}\dot{\theta}^2 + \frac{1}{2}\frac{mL^2}{2}\dot{\theta}^2$$

$$\boxed{K = \frac{3}{8} m L^2 \dot{\theta}^2}$$

$$\frac{3}{8}\cancel{m}L^{\cancel{2}}\,\dot\theta^2 + \cancel{m}g\cancel{L}\cos\theta = \cancel{m}g\cancel{L}\,\frac{1}{\sqrt2}$$

$$\boxed{\dot\theta^2 = \frac{8}{3}\frac{g}{L}\left(\frac{1}{\sqrt2}-\cos\theta\right)}$$

DE AQUÍ SE PUEDE DERIVAR $\ddot\theta$

$$2\cancel{\dot\theta}\,\ddot\theta = \frac{8}{3}\frac{g}{L}\,\operatorname{sen}\theta\,\cancel{\dot\theta}$$

$$\boxed{\ddot\theta = \frac{4}{3}\frac{g}{L}\,\operatorname{sen}\theta}$$

★ EC. MOV. CENTRO DE MASA

$$M_T\,\vec a_{cm} = 2m\vec g + N_H + N_V$$

$\hat i)$ $2m\,\ddot x_{cm} = N_V$

$\hat j)$ $2m\,\ddot y_{cm} = N_H - 2mg$

$$\ddot x_{cm} = -\frac{L}{2}\operatorname{sen}\theta\,\dot\theta^2 + \frac{L}{2}\cos\theta\,\ddot\theta$$

$$\cancel{2}m\left(-\frac{\cancel{L}}{\cancel{2}}\operatorname{sen}\theta\,\frac{8}{3}\frac{g}{\cancel{L}}\left(\frac{1}{\sqrt2}-\cos\theta\right) + \frac{\cancel{L}}{\cancel{2}}\cos\theta\cdot\frac{4}{3}\frac{g}{\cancel{L}}\operatorname{sen}\theta\right) = N_V$$

$$\boxed{N_V = \frac{mg\,\operatorname{sen}\theta}{3}\left[12\cos\theta - \frac{8}{\sqrt2}\right]}$$

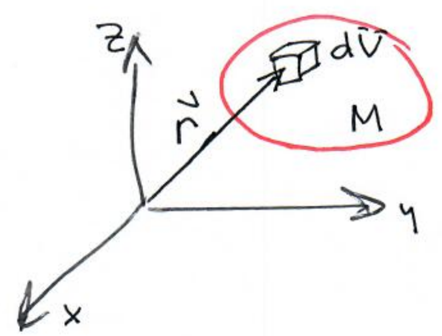
LA PARTÍCULA QUE DESLIZA POR LA PARED SE SEPARA DE ELLA CUANDO $N_V = 0$

$$12 \cos\theta^* - \frac{8}{\sqrt{2}} = 0$$

$$\cos\theta^* = \frac{4\sqrt{2}}{12} = \frac{\sqrt{2}}{3}$$

$$\theta^* = \arccos\left(\frac{\sqrt{2}}{3}\right) \sim 61.9°$$

★ MOVIMIENTO DE UN SÓLIDO



$$dV = dx\,dy\,dz$$

$$dm = D\,dV$$

DENSIDAD

VAMOS A ESTUDIAR EL MOVIMIENTO DE SÓLIDOS CON DENSIDAD D HOMOGÉNEA

EN UN S.P. $\vec{R}_{CM} = \frac{1}{M_T} \sum_i m_i \vec{r}_i$

EN UN SÓLIDO $\vec{R}_{CM} = \frac{1}{M} \iiint dm\, \vec{r}$

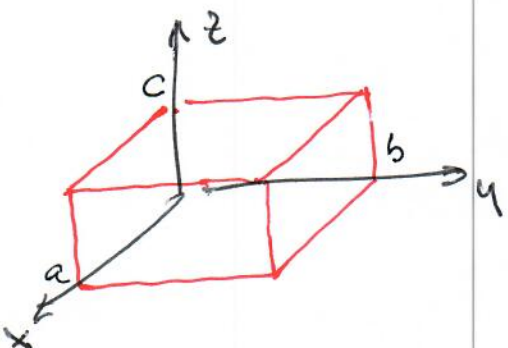
INTEGRAL DE VOLUMEN

$$\vec{R}_{CM} = \frac{1}{M}\iiint D\,\vec{r}\,dV \equiv \frac{D}{M}\iiint \vec{r}\,dx\,dy\,dz$$

$$\boxed{\vec{R}_{CM} = \frac{1}{V}\iiint \vec{r}\,dx\,dy\,dz}$$

↑ VOLUMEN DEL SÓLIDO

EJEMPLO



$$V = a\cdot b\cdot c$$

$$\vec{R}_{CM} = \frac{1}{V}\iiint (x\hat{i} + y\hat{j} + z\hat{k})\,dx\,dy\,dz$$

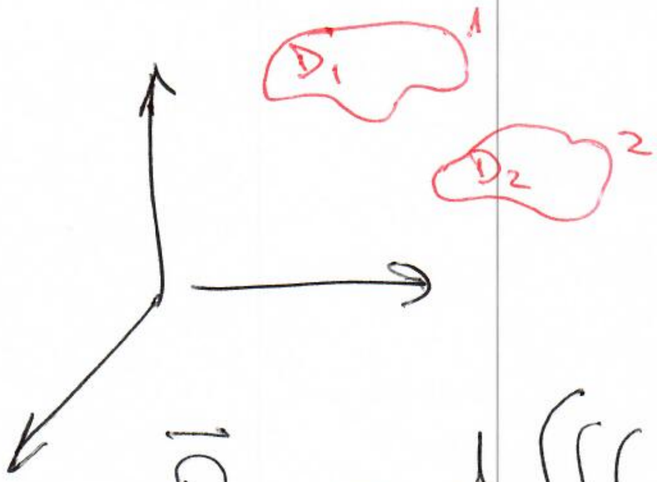
COMPONENTE DE $\vec{R}_{CM}$ SEGÚN $\hat{i}$

$$X_{CM} = \frac{1}{abc}\int_0^a\int_0^b\int_0^c x\,dx\,dy\,dz$$

$$= \frac{1}{abc}\int_0^a x\,dx \cdot \int_0^b dy \cdot \int_0^c dz$$

$$X_{CM} = \frac{b\cdot c\cdot \frac{1}{2}a^2}{abc} = \frac{a}{2}$$

C.M. DE DOS (O MAS...) SÓLIDOS



SUPONGAMOS QUE LAS DENSIDADES SON HOMOGÉNEAS PERO DIFERENTES D_1 y D_2

$$\vec{R}_{CM} = \frac{1}{M_T}\iiint dm\,\vec{r} \qquad M_T = M_1 + M_2$$

$$= \frac{1}{M_T}\left[\iiint_{V_1} D_1 \vec{r}\,d\vec{V} + \iiint_{V_2} D_2 \vec{r}\,d\vec{V}\right]$$

$$= \frac{1}{M_T}\left[\underbrace{\frac{M_1 D_1}{M_1}\iiint_{V_1}\vec{r}\,d\vec{V}}_{=\vec{R}_{CM_1}} + \underbrace{\frac{M_2 D_2}{M_2}\iiint_{V_2}\vec{r}\,d\vec{V}}_{=\vec{R}_{CM_2}}\right]$$

$$\vec{R}_{CM} = \frac{1}{M_1+M_2}\left[M_1\vec{R}_{CM_1} + M_2\vec{R}_{CM_2}\right]$$

PARA N CUERPOS

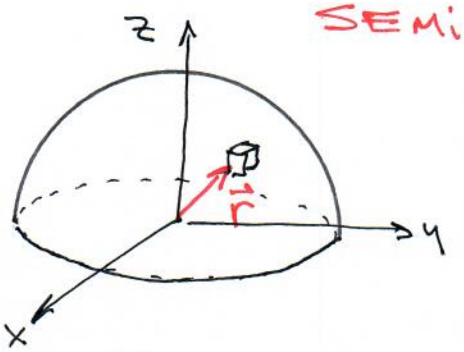
$$\boxed{\vec{R}_{CM} = \frac{1}{\sum_i M_i}\sum_i M_i\vec{R}_{CM_i}}$$

ANALOGAMENTE SE PUEDE DEMOSTRAR QUE

$$y_{cm} = \frac{b}{2} \qquad z_{cm} = \frac{c}{2}$$

$$\vec{R}_{cm} = \frac{a}{2}\hat{i} + \frac{b}{2}\hat{j} + \frac{c}{2}\hat{j}$$

SEMI-ESFERA DE RADIO R



$$\vec{R}_{cm} = \frac{1}{V}\iiint \vec{r}\, d\bar{V}$$

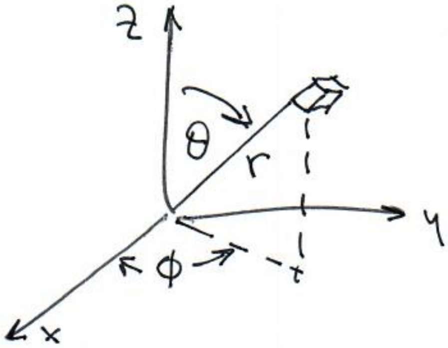
$\hat{i})$ $$X_{cm} = \frac{1}{V}\iiint x\, d\bar{V}$$

POR SIMETRÍA SE CONCLUYE QUE $X_{cm} = 0$

POR CADA ELEMENTO $d\bar{V}$ localizado en una posición x $\exists$ otro elemento $d\bar{V}$ localizado en la posición $-x$

$\hat{j})$ $$y_{cm} = \frac{1}{V}\iiint y\, d\bar{V} = 0$$ MISMA RAZÓN

$\hat{k})$ $$z_{cm} = \frac{1}{V}\iiint z\, d\bar{V}$$



COORD. ESFÉRICAS

$$d\bar{V} = (dr)(r\,d\theta)(r \operatorname{sen}\theta\, d\phi)$$

$$z = r\cos\theta$$

$$V = \frac{2}{3}\pi R^3$$

$$z_{cm} = \frac{1}{V}\iiint z\, dV = \frac{1}{V}\iiint r\cos\theta\,(dr)(r\,d\theta)(r\operatorname{sen}\theta\, d\phi)$$

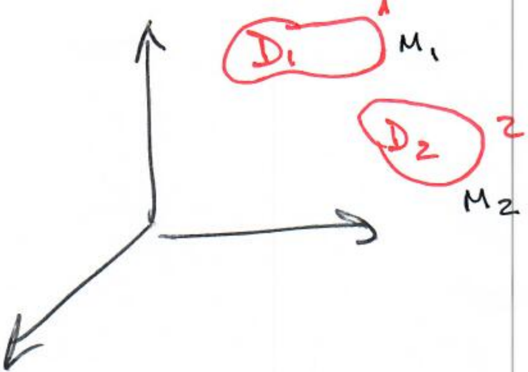
$$z_{cm} = \frac{1}{V}\iiint r^3 \operatorname{sen}\theta\cos\theta\, dr\, d\theta\, d\phi$$

$$= \frac{1}{V}\underbrace{\left[\int_0^R r^3\,dr\right]}_{\frac{1}{4}R^4}\underbrace{\left[\int_0^{\pi/2}\operatorname{sen}\theta\cos\theta\, d\theta\right]}_{\frac{1}{2}\operatorname{sen}^2\theta\Big|_0^{\pi/2}}\underbrace{\left[\int_0^{2\pi} d\phi\right]}_{2\pi}$$

$$z_{cm} = \left[\frac{2}{3}\pi R^3\right]^{-1}\frac{R^4}{4}\cdot\frac{1}{2}\cdot 2\pi$$

$$\boxed{z_{cm} = \frac{3}{8}R}$$

CENTRO DE MASA DE UNA ESTRUCTURA FORMADA POR DOS O MAS CUERPOS DE DENSIDADES HOMOGÉNEAS PERO DISTINTAS



$$\vec{R}_{CM} = \underbrace{\frac{1}{M_T}}_{M_1 + M_2} \underbrace{\iiint dm\, \vec{r}}_{\text{INTEGRACIÓN SOBRE LOS DOS VOLÚMENES}}$$

$$\vec{R}_{CM} = \frac{1}{M_T}\left[\iiint_{V_1} D_1 \vec{r}\, dV + \iiint_{V_2} D_2 \vec{r}\, dV\right]$$

$$\vec{R}_{CM} = \frac{1}{M_T}\left[M_1 \underbrace{\frac{D_1}{M_1}\iiint_{V_1} \vec{r}\, d\vec{V}}_{\vec{R}_{CM_1}} + M_2 \underbrace{\frac{D_2}{M_2}\iiint \vec{r}\, d\vec{V}}_{\vec{R}_{CM_2}}\right]$$

$$\vec{R}_{CM} = \frac{1}{M_1 + M_2}\left[M_1 \vec{R}_{CM_1} + M_2 \vec{R}_{CM_2}\right]$$

PARA N CUERPOS ...

$$\boxed{\vec{R}_{CM} = \frac{1}{\sum_i M_i} \sum_i M_i \vec{R}_{CM_i}}$$