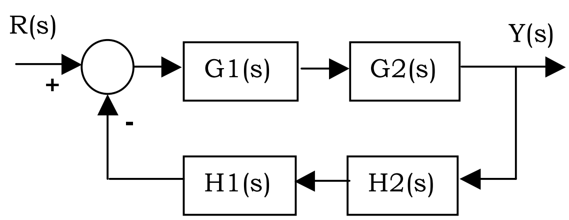
EE302 Controls – Mason's Gain Rule for Block Diagrams
DePiero

Mason's Gain Rule is a technique for finding an overall transfer function. It is helpful when trying to simplify complex systems. The purpose of using Mason's is the same as that of Block Reduction. However, Mason's is guaranteed to yield a concise result via a direct procedure, where as the process of Block Reduction can meander. Mason's method was particularly helpful before the advent of modern computers, and tools such as MatLab which can also be used to find the overall transfer function of a complex system (and then perform subsequent analysis).

The Dorf text presents Mason's Gain Rule applied to signal flow diagrams. This presentation is for block diagrams only. A derivation of Mason's Gain Rule is beyond the scope of the course. As such, just the procedure will be presented here. The Dorf text does present a specific example and draws parallel between Mason's and the solution of a set of linear equations.



Mason's procedure refers to portions of a block diagram in terms of 'paths' and 'loops'. Loops begin and end at the same point, and are described in terms of the concatenation of all the transfer function blocks encountered. This is the 'loop gain' and also includes any negations associated with a summer. Hence there is one loop in the above system, L1 = -G1G2H1H2. Blocks or summers contained within a loop should be included exactly once. ('Figure eight' patterns are two loops, not one).

A path runs from the input R(s) to the output Y(s) and is without any loops. Paths are also described by the concatenation of blocks. In the above system there is one path, described as G1G2.

Another criterion used in the evaluation of Mason's has to do with the notion of 'touching' loops and paths. Touching loops contain a common branch (or summer or block) element. A loop touches a path if they share a common block or summer.

Mason's Gain Rule states $T = \dfrac{\sum_k P_k \Delta_k}{\Delta}$ where,

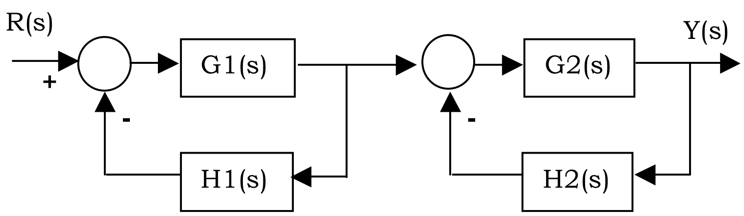
Δ = 1 – (sum of all different loop gains)
+ (sum of products of all pairs of loop gains, for non-touching loops)
– (sum of products of all triples of loop gains, for non-touching loops)
+ …

P_k = kth path from input to output.

Δ_k = The quantity Δ, but with all loops touching the kth path, P_k, removed.

This is best illustrated via examples…

Example 1a. Find T(s) = Y(s)/R(s), both algebraically and via Mason's Gain Rule



Working algebraically,

$$T = \frac{G1}{1 + G1\,H1}\,\frac{G2}{1 + G2\,H2} = \frac{G1G2}{1 + G1\,H1 + G2H2 + G1G2H1H2}$$

Via Mason's first note there is one path from R(s) to Y(s):

P1 = G1 G2

There are two loops:

L1 = -G1 H1
L2 = -G2 H2

Loops L1 and L2 are not touching. Finding Δ,

Δ = 1 – (L1 + L2) + L1 L2 = 1 + G1H1 + G2H2 + G1G2H1H2

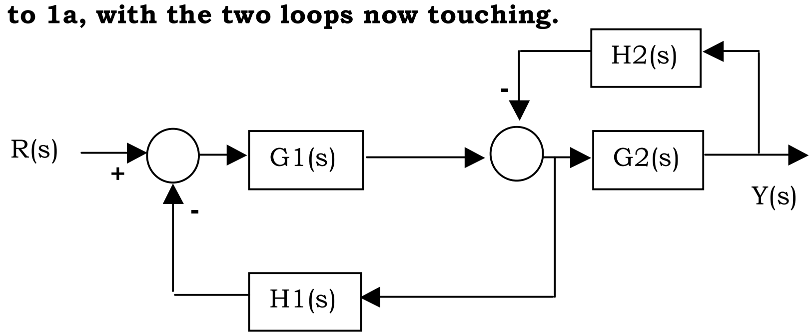
To find Δ_1, loops touching path P1 are removed from Δ. Hence $\Delta_1 = 1$.

So, Mason's technique yields:

$$T = \frac{P1\,\Delta_1}{1 - (L1 + L2) + (L1L2)} = \frac{G1G2}{1 - (-G1\,H1 - G2H2) + G1H1G2H2}$$

Which is the same result as with the algebraic method.

1b) Similar to 1a, with the two loops now touching.

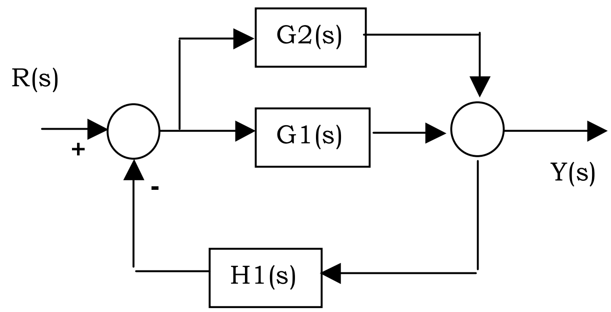


Working algebraically, first define a signal D that flows *out* of the central node. The signal D flows along the –H1 path *and* along the G2 path. This can be reduced to:

$$T = \frac{G1G2}{1 + G1\,H1 + G2H2}$$

Which is the same as the result from Mason's with the L1L2 term dropped due to the touching loops.

Example 2. Find T(s) = Y(s)/R(s), Algebraically and via Mason's Gain Rule



There are two paths from R(s) to Y(s):

$$P1 = G1 \quad \text{and} \quad P2 = G2$$

There are two loops:

$$L1 = -G1\,H1$$
$$L2 = -G2\,H1$$

Loops L1 and L2 are touching. Finding Δ,

$$\Delta = 1 - (L1 + L2) = 1 + G1H1 + G2H1$$

To find Δ_1, loops touching path P1 are removed from Δ. Δ_2 is found similarly. Hence

$$\Delta_1 = 1 \text{ and } \Delta_2 = 1$$

So Mason's technique yields:

$$T = \frac{P1\,\Delta_1 + P2\,\Delta_2}{1-(L1+L2)} = \frac{G1+G2}{1+G1H1+G2H1}$$

Working algebraically, define G = G1 + G2, then

$$T = \frac{G}{1+GH} = \frac{G1+G2}{1+(G1+G2)H1}$$