Sea $\varphi(u,v) = (u-v, u+v, uv)$ y sea

D = Disco unitario del plano (u,v)

$$\varphi(D) = \left(\varphi(u,v) : u^2+v^2 \le 1 \right)$$

$$= \left(\varphi(u,v) : \begin{array}{l} v \in [-1,1] \\ u \in \left[-\sqrt{1-v^2}, \sqrt{1-v^2}\right] \end{array} \right)$$

$$= \left((u-v, u+v, uv) : v\in[-1,1],\ u\in[-\sqrt{1-v^2}, \sqrt{1-v^2}] \right)$$

$$\frac{\partial \varphi}{\partial u} = (1, 1, v) \quad 1, \ 1$$

$$\frac{\partial \varphi}{\partial v} = (-1, 1, u) \quad -1, \ 1$$

$$\Rightarrow \frac{\partial \varphi}{\partial u} \times \frac{\partial \varphi}{\partial v} = (u-v, -u-v, 1+1)$$

$$= (u-v, -u-v, 2)$$

$$\left\| \frac{\partial \varphi}{\partial u} \times \frac{\partial \varphi}{\partial v} \right\|^2 = u^2 + v^2 + 4$$

$$\left\| \frac{\partial \varphi}{\partial u} \times \frac{\partial \varphi}{\partial v} \right\| = \sqrt{u^2 + v^2 + 4}$$

Por comodidad integraremos en polares, es decir,

$$\text{Area}(\varphi(D)) = \int_{-1}^{1} \int_{-\sqrt{1-v^2}}^{\sqrt{1-u^2}} \sqrt{u^2 + v^2 + 4}\, du\, dv$$

$$= \int_0^1 \int_0^{2\pi} \sqrt{r^2 + 4}\; r\, d\theta\, dr$$

$$= 2\pi \int_0^1 \sqrt{r^2+4}\; r\, dr \qquad r^2 = z \quad 2r\,d = dz$$

$$= \pi \int_0^1 \sqrt{z+4}\; dz$$

$$= \frac{\pi}{3} 2 (z+4)^{\frac{3}{2}} \Big|_0^1 = \frac{2\pi}{3}\left(5^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$$