P1 $X = \{1,2,3,4,5\}$

$U(p) = \sum_{i=1}^{5} p_i u(i)$, U lineal, u cóncava.
$c(p,u) \leq p \cdot \bar{x}$

$$\frac{1}{2}u(1) + \frac{1}{2}u(5) > \frac{1}{2}u(2) + \frac{1}{2}u(3) \iff \boxed{u(1)+u(5) \overset{*}{>} u(2)+u(3)}$$

Note:

$$U(C) = \frac{2}{5}u(1) + \frac{1}{5}u(4) + \frac{2}{5}u(5) > \frac{2}{5}u(1) + \frac{1}{5}u(3) + \frac{2}{5}u(5) = U(B)$$

$\Rightarrow \quad C \succ B.$

$$U(C) = \frac{2}{5}u(1) + \frac{1}{5}u(4) + \frac{2}{5}u(5) = \frac{2}{5}\left(u(1)+u(5)\right) + \frac{1}{5}u(4)$$
$$\overset{*}{>} \frac{u(1)+u(5)}{5} + \frac{u(2)+u(3)}{5} + \frac{1}{5}u(4) = U(A)$$

$\Rightarrow \quad C \succ A.$

Por aversión al riesgo, $u(\cdot)$ es cóncavo.

$\Rightarrow \quad u(n+1) - u(n)$ decrece en n.

$$U(B) - U(A) = \frac{1}{5}u(1) - \frac{1}{5}u(2) - \frac{1}{5}u(4) + \frac{1}{5}u(5)$$
$$= \frac{1}{5}\left(u(1) - u(2) + u(5) - u(4)\right)$$
$$< \frac{1}{5}\left(u(1) - u(2) + u(4) - u(3)\right)$$
$$< \frac{1}{5}\left(u(1) - u(2) + u(3) - u(2)\right)$$
$$< \frac{1}{5}\left(u(1) - u(2) + u(2) - u(1)\right) = 0 \quad //$$

$\therefore \quad B \prec A$

$$\boxed{\therefore \quad C \succ A \succ B}$$ //

P2) $p^m \in \mathbb{R}^L$, $m \in \{A, B\}$ $\quad x^m = ((\ ), \ldots, (\ )) \in \mathbb{R}^{I^m L}$

$(p^m, x^m) \in \mathbb{R}^L \times \mathbb{R}^{L I^m}$

$p^A = \alpha p^B, \quad \alpha > 0.$

EW en economía integrada.

a) $(p, x) \in \mathbb{R}^L \times \mathbb{R}^{IL}$, $\quad I = I^A + I^B$. $\quad p > 0$

1) $\forall i \in I$

$$x^i \in \underset{y \in \mathbb{R}^L}{\operatorname{argmax}}\ u_i(y) \qquad , w^i \text{ dotación de } i.$$

$$\text{s.a} \quad \underset{y \geq 0}{p \cdot y \leq p \cdot w^i}$$

2) $\displaystyle\sum_{i \in I} x^i(p) = \sum_{i \in I} w^i$

b) Sea $x = (x^A, x^B) \in \mathbb{R}^{LI}$

Pd: (p^A, x) es EW.

Nota: $p^A \cdot y \leq p^A \cdot w^i \iff \sum p_j^A y_j \leq \sum p_j^A w_j^i$

$\iff \sum \alpha p_j^B y_j \leq \sum \alpha p_j^B w_j^i$

$\iff \sum p_j^B y_j \leq \sum p_j^B w_j^i$

$\iff p^B \cdot y \leq p^B w^i \quad \forall i \in I$

Luego, (p^A, x^A) EW en A.

$$\Rightarrow \quad x^i \in \underset{y \geq 0}{\operatorname{argmax}}\ u_i(x^i)$$

$$\text{s.a} \quad p^A \cdot x^i \leq p^A \cdot w^i \quad \forall i \in I^A$$

(p^B, x^B) EW en B.

$$\Rightarrow \quad x^i \in \underset{y \geq 0}{\operatorname{argmax}}\ u_i(x^i) \qquad = \underset{y \geq 0}{\operatorname{argmax}}\ u_i(x^i)$$

$$\text{s.a} \quad p^B \cdot y \leq p^B \cdot w^i \qquad \text{s.a} \quad p^A \cdot y \leq p^A \cdot w^i$$

$\forall i \in I^B$. $\therefore\ x^i(p^B) = x^i(p^A) \quad \forall i \in I^B$

$\therefore$ a precio p^A, cada $i \in I$ maximiza su utilidad.

Por otro lado

$$\sum_{i \in I^A} x^i(p^A) = \sum_{i \in I^A} w^i, \quad \sum_{i \in I^B} x^i(p^B) = \sum_{i \in I^B} w^i = \sum_{i \in I^B} x^i(p^A)$$

$$\Rightarrow \quad \sum_{i \in I} x^i(p^A) = \sum_{i \in I} w^i \quad //$$

$\therefore\ (p^A, x)$ es EW.

P2)

c) Pd: $x = (x^a, x^b)$ es Pareto óptimo.

Como (p^*, x) es EW y u_i crecientes

$\overset{1^\circ \text{ Teo B.}}{\Longrightarrow}$ x P.O//

d)

P3) $u_1(x,y) = \ln(x) + \ln(y)$, $u_2(x,y) = \min\{2x, y\}$

$e_1 = (5,5)$, $e_2 = (10,10)$

a) Curva de Contrato:

$y^2 = 2x^2 \iff \frac{y^2}{x^2} = 2 \iff \frac{15 - y^1}{15 - x^1} = 2 \iff 15 - y^1 = 30 - 2x^1$

$\iff \boxed{2x^1 - 15 = y^1}$ //

(*) Si (x^2, y^2) en Eq $\frac{y^2}{x^2} > 2 \Rightarrow y^2 > 2x^2 \Rightarrow u_2(x^2, y^2) = 2x^2$

$\Rightarrow$ puedo quitarle $\varepsilon > 0$ del bien y para dárselo a 1, sin disminuir u_2 y aumentando u_1

$\Rightarrow (x^2, y^2)$ no es PO. Análogo si $\frac{y^2}{x^2} < 2$ //

b) EW:

$\frac{y}{x} = \frac{p_x}{p_y}$

$p_y y = p_x x$

$\Rightarrow 2 p_x x = 5(p_x + p_y)$

$\Rightarrow x = 5 \frac{(p_x + p_y)}{2 p_x}$

$\Rightarrow 2 p_y y = 5(p_x + p_y)$

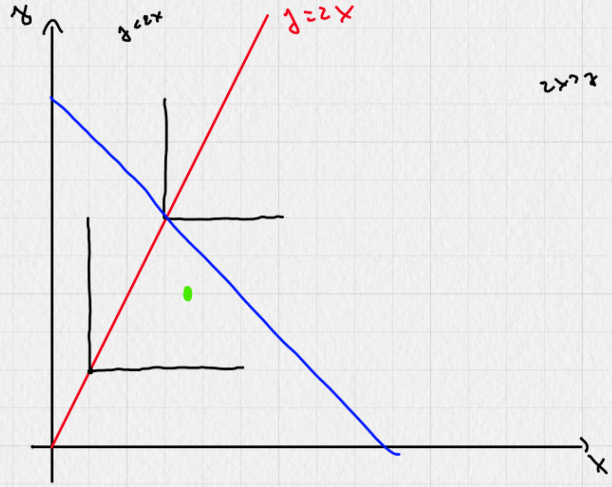
$\Rightarrow y = 5 \frac{p_x + p_y}{2 p_y}$

$2x = y$, $p_x x + p_y y = 10(p_x + p_y)$

$\Rightarrow x(p_x + 2p_y) = 10(p_x + p_y)$

$\Rightarrow x = 10 \frac{p_x + p_y}{p_x + 2p_y}$

$y = 20 \frac{p_x + p_y}{p_x + 2p_y}$



Mercados se limpian:

$\frac{5}{2} \frac{(p_x + p_y)}{p_x} + 10 \frac{(p_x + p_y)}{p_x + 2p_y} = 15$, $p_x = 1$

$\frac{5}{2}(1 + p_y) + 10 \frac{(1 + p_y)}{1 + 2p_y} = 15 \quad / 2(1 + 2p_y)$

$5(1 + 2p_y + p_y + 2p_y^2) + 20(1 + p_y) = 30(1 + 2p_y)$

$10 p_y^2 + 15 p_y + 5 + 20 + 20 p_y = 30 + 60 p_y \quad / \frac{1}{5}$

$2 p_y^2 + 7 p_y + 5 = 6 + 12 p_y$

$2 p_y^2 - 5 p_y = 1$

$p_y^2 - \frac{5}{2} p_y = \frac{1}{2}$

$\left(p_y - \frac{5}{4}\right)^2 = \frac{1}{2} + \frac{25}{16} = \frac{33}{16}$

$\left| p_y - \frac{5}{4} \right| = \sqrt{\frac{33}{16}} \Rightarrow \boxed{p_y = \sqrt{\frac{33}{16}} + \frac{5}{4} \approx 2{,}7}$

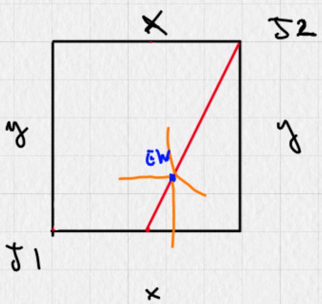
P3) Kdo: $p_x = 1$ $p_y = \sqrt{\frac{33}{16}} + \frac{5}{4} \approx 2,7$

$$x^2 = 10 \frac{(1+p_y)}{1+2p_y} = 10 \frac{3,7}{6,4} \approx 5,78$$

$$y^2 = 2x^2 \approx 11,56$$

$$x^1 = 15 - x^2 \approx 9,22$$

$$y^1 = 15 - y^2 \approx 3,44$$



$y^1 = 2x^1 - 15$