P1 $\quad u(x) = -e^{-\lambda x}, \quad \lambda > 0.$

a)

$r_A(x) = -\frac{u''(x)}{u'(x)}$

$u'(x) = \lambda e^{-\lambda x}, \quad u''(x) = -\lambda^2 e^{-\lambda x}$

$\Rightarrow r_A(x) = \frac{\lambda^2 e^{-\lambda x}}{\lambda e^{-\lambda x}} = \lambda$ //

$\frac{1}{2} \cdot -e^{-\lambda 1000} + \frac{1}{2} \cdot -e^{-\lambda 0} = -e^{-\lambda 470}$

$e^{-1000\lambda} + 1 = 2e^{-470\lambda}$

b) $-e^{-1500\lambda} \frac{1}{2} + -e^{-500\lambda} \frac{1}{2} = e^{-\lambda c} \quad / \cdot -2e^{500\lambda}$

$\Rightarrow e^{-1000\lambda} + 1 = 2e^{(500-c)\lambda}$

$\Rightarrow 500 - c = -470$

$\Rightarrow 970 = c$

P2 $\succsim$ continua ssi $\forall\, p, p', p''$ loterías tq $p \succsim p' \succsim p''$ $\exists\, \alpha \in [0,1]$ con $\alpha p + (1-\alpha) p'' \sim p'$

$$p \succsim q \quad \text{ssi} \quad \sum_{x\in X}\left(p(x) - \frac{1}{|X|}\right)^2 \leq \sum_{x\in X}\left(q(x) - \frac{1}{|X|}\right)^2$$

Completa: Sea $p_1, p_2 \in \Delta(X)$.

Note: $\sum_{x\in X}\left(p_i(x) - \frac{1}{|X|}\right)^2 \in \mathbb{R}$ completo luego ($\succsim$).

$\therefore$ Siempre puedo comparar p_1 y p_2.

Transitiva: Sean $p, q, r \in \Delta(X)$

$$F : \Delta(X) \to \mathbb{R} \qquad F(p) = \sum_{x\in X}\left(p(x) - \frac{1}{|X|}\right)^2$$

Note: $p \succsim q$ y $q \succsim r \iff F(p) \leq F(q)$ y $F(q) \leq F(r)$

$(\mathbb{R}, \geq)$ es transitivo $\Rightarrow F(p) \leq F(r)$

$\Rightarrow p \succsim r$ $\therefore$ $\succsim$ transitivo.

Continua: Sean $p \succ q \succ r$

$$\Rightarrow \underbrace{\sum_{x\in X}\left(p(x) - \frac{1}{|X|}\right)^2}_{F(p)} < \underbrace{\sum_{x\in X}\left(q(x) - \frac{1}{|X|}\right)^2}_{F(q)} < \underbrace{\sum_{x\in X}\left(r(x) - \frac{1}{|X|}\right)^2}_{F(r)}$$

$$G(\alpha) \equiv \sum_{x\in X}\left(\alpha r(x) + (1-\alpha) p(x) - \frac{1}{|X|}\right)^2$$

G continua.

$$G(0) = F(p) < F(q) < F(r) = G(1)$$

$\Rightarrow \exists\, z \in (0,1)$ tq $G(z) = F(q)$

$$\therefore \exists\, z \in (0,1) \text{ tq } \sum_{x\in X}\left(z\, r(x) + (1-z) p(x) - \frac{1}{|X|}\right)^2 = \sum_{x\in X}\left(q(x) - \frac{1}{|X|}\right)^2$$

$\iff z r + (1-z) p \sim q$ //

P2| Cont.

Independiente:

Sea $X = \{x_1, x_2\}$

$p = \left(\frac{1}{2}, \frac{1}{2}\right)$ Sea $\alpha = \frac{1}{2}$

$p' = \left(\frac{3}{4}, \frac{1}{4}\right)$

$p'' = \left(\frac{1}{4}, \frac{3}{4}\right)$

Es claro: $p \succ p'$

$$F(p) = 0 < F(p') = \left(\frac{3}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{4} - \frac{1}{2}\right)^2 = \frac{2}{16} = \frac{1}{8}$$

Pero $\frac{1}{2} p + \frac{1}{2} p'' \not\succ \frac{1}{2} p' + \frac{1}{2} p''$:

$$F(\tfrac{1}{2} p + \tfrac{1}{2} p'') = F\left(\left(\tfrac{3}{8}, \tfrac{5}{8}\right)\right)$$

$$= \left(\frac{3}{8} - \frac{1}{2}\right)^2 + \left(\frac{5}{8} - \frac{1}{2}\right)^2 = \frac{2}{64} = \frac{1}{32}$$

$$F(\tfrac{1}{2} p' + \tfrac{1}{2} p'') = F\left(\left(\tfrac{1}{2}, \tfrac{1}{2}\right)\right) = 0 < \frac{1}{32}$$

//

P3

$$U_1(x,y) = x^{1/2} y^{1/2} \quad , \quad U_2(x,y) = x^{1/3} y^{2/3}$$

$$e_1 = (6,3) \quad , \quad e_2 = (3,3)$$

$$\tilde{U}_1(x,y) = \frac{1}{2}\ln(x) + \frac{1}{2}\ln(y) \quad , \quad \tilde{U}_2(x,y) = \frac{1}{3}\ln(x) + \frac{2}{3}\ln(y)$$

1)
$$\max_{x,y} \frac{1}{2}\ln(x) + \frac{1}{2}\ln(y) \quad , \quad I_1 = p_x e_{1x} + p_y e_{1y} = 6p_x + 3p_y .$$

$$\text{s.a} \quad p_x X + p_y y = I_1$$

$$\max_x \frac{1}{2}\ln(x) + \frac{1}{2}\ln\left(\frac{I_1 - p_x X}{p_y}\right)$$

CPO:
$$\frac{1}{2}\frac{1}{x} - \frac{1}{2}\frac{p_y}{I_1 - p_x X}\frac{p_x}{p_y} = 0$$

$$\frac{1}{x} = \frac{p_x}{I_1 - p_x X} \Rightarrow I_1 - p_x X = p_x X \Rightarrow \boxed{x = \frac{I_1}{2p_x}}$$

$$\Rightarrow \boxed{y = \frac{I_1 - p_x \frac{I_1}{2p_x}}{p_y} = \frac{I_1}{2p_y}}$$

2)
$$TMS_{xy} = \frac{p_x}{p_y}$$

$$\frac{\frac{\partial \tilde{U}_2}{\partial x}}{\frac{\partial \tilde{U}_2}{\partial y}} = \frac{\frac{1}{3}\frac{1}{X}}{\frac{2}{3}\frac{1}{y}} = \frac{1}{2}\frac{y}{X} = \frac{p_x}{p_y} \quad , \quad p_x X + p_y y = I_2$$

$$y = \frac{I_2 - p_x X}{p_y}$$

$$\Rightarrow \frac{1}{2}\frac{1}{X}\frac{I_2 - p_x X}{p_y} = \frac{p_x}{p_y}$$

$$\Rightarrow I_2 - p_x X = 2 X p_x \Rightarrow \boxed{X = \frac{I_2}{3p_x}}$$

$$\boxed{y = \frac{I_2 - p_x \frac{I_2}{3p_x}}{p_y} = \frac{2}{3}\frac{I_2}{p_y}} \qquad I_2 = 3p_x + 3p_y$$

$$\therefore D_1 = \left(\frac{6p_x + 3p_y}{2p_x}, \frac{6p_x + 3p_y}{2p_y}\right)$$

$$D_2 = \left(\frac{3(p_x + p_y)}{3p_x}, \frac{3(p_x + p_y)}{p_y}\frac{2}{3}\right)$$

Mercado de limpiar:

$D_1 + D_2 = e_1 + e_2$ $\qquad$ $e_1 + e_2 = (9, 6)$

Imponemos $p_y = 1$

$$D_1 = \left(\frac{6p_x + 3}{2p_x} , \frac{6p_x+3}{2} \right)$$

$$D_2 = \left(\frac{3(p_x+1)}{3p_x} , 3(p_x+1)\frac{2}{3} \right)$$

$$\Rightarrow \frac{6p_x+3}{2p_x} + \frac{3(p_x+1)}{3p_x} = 9 \qquad / \cdot 2p_x$$

$$6p_x + 3 + 2(p_x+1) = 18p_x$$

$$4 = 10p_x \Rightarrow \boxed{p_x = \frac{2}{5}}$$

$$D_1 = \left(\frac{6 \cdot 2/5 + 3}{4/5} , \frac{6 \cdot 2/5 + 3}{2} \right)$$

$$D_1 = \left(\frac{12 + 15}{4} , \frac{12+15}{10} \right)$$

$$D_1 = \left(\frac{27}{4} , \frac{27}{10} \right)$$