P1 $Z \sim U[2,4]$ 8 horas
$Y \mid Z \sim \exp(Z^{-1/2})$

$\rightarrow$ Desigualdad de Chebyshev : $P(|X-\mu| \geq K) \leq \frac{\sigma^2}{K^2}$

Adaptada al problema

$$\rightarrow P(|Y - E[Y]| \geq K) \leq \frac{Var(Y)}{K^2}$$

$X \sim \exp(\lambda) \rightarrow E(X) = 1/\lambda$

$$\bullet\ E[Y] = E[E[Y \mid Z]]$$
$$= E[Z^{1/2}]$$
$$= \int_2^4 z^{1/2} \cdot F_Z\, \partial z = \int_2^4 z^{1/2} \cdot \frac{1}{4-2}\, \partial z$$
$$= \frac{1}{2}\int_2^4 z^{1/2}\, \partial z = \frac{1}{2} \cdot \frac{z^{3/2}}{3/2}\Big|_2^4$$
$$= \frac{1}{2} \cdot \frac{2}{3} z^{3/2}\Big|_2^4 = \frac{1}{3}\left(4^{3/2} - 2^{3/2}\right)$$
$$= \frac{1}{3}\left(2^3 - \sqrt{8}\right) = \frac{1}{3}(8 - 2\sqrt{2}) \approx 1{,}72$$

$$\bullet\ Var(Y) = E(Var(Y \mid Z)) + Var(E(Y \mid Z))$$

$X \sim \exp(\lambda)$
$\rightarrow Var(X) = 1/\lambda^2$

$$= E(Z) + Var(Z^{1/2})$$
$$= E(Z) + E(Z) - E(Z^{1/2})^2$$
$$= \cancel{2} \cdot \frac{4+2}{\cancel{2}} - E[Y]^2$$

$$= 6 - E(Y)^2$$

- $P(Y \geq 8) = P(Y - E(Y) \geq 8 - E(Y))$

$$\leq P(|Y - E(Y)| \geq 8 - E(Y))$$ Desigualdad

$$\leq \frac{Var(Y)}{(8 - E(Y))^2} = \frac{3.04}{39.44} = 0.077$$

$\rightarrow P(Y \geq 8) \leq 0.077 \leq 0.08$

$\therefore$ La afirmación era verdadera

P2

1. $X \sim Bin(m, p) \quad ; \quad X_K = m - X \quad (K = 1, 2, 3, \dots)$
$p = 1/2$

- Convergencia en distribución

$$\rightarrow \lim_{K \to \infty^+} P(X_K = x) = \lim_{K \to \infty^+} P(m - X = x)$$

$$\rightarrow \lim_{K \to \infty} P(X = m - x) = \lim_{K \to \infty^+} \binom{m}{m-x} p^{m-x} (1-p)^{m-(m-x)}$$

$$\rightarrow \lim_{K \to \infty} \frac{m!}{(m-x)!\,(m-(m-x))!} \left(\frac{1}{2}\right)^{m-x} \left(\frac{1}{2}\right)^{x}$$

$$\rightarrow \lim_{K \to \infty} \frac{m!}{(m-x)!\, x!} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{m-x}$$

$$\rightarrow \cancel{\lim_{K \to \infty}} \binom{m}{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{m-x} = P(X = x)$$

- Convergencia en probabilidad

$$\rightarrow \lim_{K \to \infty} P(|X_K - X| \geq \varepsilon) = 0 \quad \forall \varepsilon > 0$$

$$\rightarrow \lim_{K \to \infty} P(|m - X - X| \geq \varepsilon)$$

$$\rightarrow \cancel{\lim_{K \to \infty}} P(|m - 2X| \geq \varepsilon)$$

$$\rightarrow 1 - P(|m-2X| < \varepsilon)$$

$$\rightarrow 1 - (P(m-2X < \varepsilon) - P(-m+2X < \varepsilon))$$

$$\rightarrow 1 - (P(m-\varepsilon < 2X) - P(2X < \varepsilon + m))$$

$$\rightarrow 1 - (P(m-\varepsilon < 2X < \varepsilon+m))$$

$$\rightarrow 1 - (P(\frac{m-\varepsilon}{2} < X < \frac{\varepsilon+m}{2})) \quad \varepsilon = 2$$

$$\rightarrow 1 - (P(\frac{m}{2} - 1 < X < \frac{m}{2} + 1))$$

$$\rightarrow 1 - (P(m/2 \leq X \leq m/2))$$

$$\rightarrow 1 - P(X = m/2)$$

- $P(X = m/2) = \binom{m}{m/2} \cdot \frac{1}{2}^{m/2} \cdot \frac{1}{2}^{m/2} = \binom{m}{m/2} \cdot \frac{1}{2}^{m}$ m es par

  0 m es impar

$$\rightarrow 1 - P(X = m/2) \not\to 0$$

$\rightarrow$ X_m no converge en probabilidad

b) $X_1, \ldots, X_{60} \sim bin(30, 0.4)$; $m = 60$

$$P(\sum_{i=1}^{60} X_i \geq 680) \rightarrow 1 - P(\sum_{i=1}^{60} X_i \leq 680)$$

$$\rightarrow 1 - P(\underbrace{\frac{\sum X_i - m \cdot \mu}{\sqrt{m} \cdot \sigma}}_{Z_m} \leq \frac{680 - m\mu}{\sqrt{m} \cdot \sigma})$$

X_i son iid con $\mu < \infty$, $\sigma^2 < \infty$ -> Z_n converge en dist. normal estandar y podemos usar TCL

$\mu = E(X_i) = 30 \cdot 0{,}4 = 12$
$\sigma^2 = Var(X_i) = 30 \cdot 0{,}4 \cdot 0.6 = 7.2$

$\rightarrow 1 - P\left(Z_n \leq \dfrac{680 - 60 \cdot 12}{\sqrt{60} \cdot \sqrt{7.2}} \right)$

$\rightarrow 1 - P(Z_n \leq -1.9245)$

$\rightarrow 1 - 0.0268 = 0.9732$

o

$1 - 0.0271 = 0.9729$