P1 $X \sim \exp(\lambda)$ $\quad Z = X + Y$

$Y \sim \exp(\lambda)$

a) $M_Z(t) = M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$

- $M_X(s) = E[e^{sx}] = \int_{R_X} e^{sx} \cdot \lambda e^{-\lambda x}\, dx = \int_0^\infty \lambda e^{x(s-\lambda)} dx$

$u = (s-\lambda)x$

$du = (s-\lambda)\,dx$

$$= \int_0^\infty \lambda e^u \frac{du}{s-\lambda} = \frac{\lambda}{s-\lambda} \int_0^\infty e^u\, du = \frac{\lambda}{s-\lambda} e^{(s-\lambda)x} \Big|_0^\infty$$

si $\lambda > s \rightarrow \lim_{x \to \infty} e^{x(s-\lambda)} = 0$

$$= \frac{\lambda}{s-\lambda}[0-1]$$

$$= \frac{\lambda}{\lambda - s} \rightarrow M_Z(t) = \left(\frac{\lambda}{\lambda - t}\right)^2$$

b) $Z = X + Y$

$$\rightarrow F_Z(z) = \int_{-\infty}^{\infty} F_X(z-y) F_Y(y)\, dy = \int_0^z \lambda e^{-\lambda(z-y)} \lambda e^{-\lambda y} dy$$

$$\rightarrow \lambda^2 \int_0^z e^{-\lambda(z - \cancel{y} + \cancel{y})} dy = \lambda^2 \int_0^z e^{-\lambda z}\, dy$$

$$\rightarrow \lambda^2 e^{-\lambda z} \cdot z \rightarrow \frac{\lambda^2 z e^{-\lambda z}}{\Gamma(2)} \qquad \frac{\lambda^\alpha z^{\alpha-1} e^{-\lambda z}}{\Gamma(\alpha)}$$

$\alpha = 2$

$\Gamma(n) = (n-1)!$ $\qquad \sim \text{Gamma}(2, \lambda)$

$\Gamma(2) = 1!$

c) $$E[Z] = \frac{\partial M_Z(t)}{\partial t}\Big|_{t=0} = \left(\lambda^2 (\lambda - t)^{-2}\right)'$$

$$= \lambda^2 (-2)(\lambda - t)^{-3}(-1)\Big|_{t=0}$$

$$\rightarrow \lambda^2 \cdot 2\,(\lambda-t)^{-3}\big|_{t=0} \rightarrow \frac{2\lambda^2}{\lambda^3} = \frac{2}{\lambda}$$

P2

- $E[X \mid Y=y] = \sum_{x\in R_X} x \cdot p_{X|Y}(x|y)$

  - $p_{X|Y}(x|y) = \frac{p_{XY}(x,y)}{p_Y(y)}$ $\rightarrow$ se tiene por enunciado

$$\rightarrow p_Y(y) = \sum_{x\in R_X} p_{XY}(x,y) = \sum_{x=1}^{y} \frac{2}{N(N+1)} = \frac{2}{N(N+1)}\, y$$

$$\rightarrow p_{X|Y}(x|y) = \frac{\frac{\cancel{2}}{\cancel{N(N+1)}}}{\frac{\cancel{2}y}{\cancel{N(N+1)}}} = \frac{1}{y}$$

$$E[X \mid Y=y] = \sum_{x\le y} x \cdot \frac{1}{y} = \sum_{x=1}^{y} x \cdot \frac{1}{y} = \frac{1}{\cancel{y}} \cdot \frac{\cancel{y}(y+1)}{2}$$

$$= \frac{y+1}{2}$$

$$\Rightarrow E[X \mid Y] = \frac{Y+1}{2}$$

- $E[Y \mid X=x] = \sum_{y\in R_Y} y \cdot p_{Y|X}(y|x)$

$N \quad (N-1)+1$

$1 \quad (N-x)+1$

$$\rightarrow p_{Y|X} = \frac{p_{XY}}{p_X} \quad \rightarrow p_X(x) = \sum_{y\in R_Y} p_{XY} = \sum_{y=x}^{N} \frac{2}{N(N+1)}$$

$$= \frac{2}{N(N+1)} \cdot (N-x+1)$$

$$\to p_{Y|X} = \frac{\frac{\cancel{2}}{\cancel{N}(\cancel{N+1})}}{\frac{\cancel{2}(N-x+1)}{\cancel{N}(\cancel{N+1})}} = \frac{1}{N-x+1}$$

$$E[Y|X=x] = \sum_{y=x}^{N} y\cdot\frac{1}{N-x+1} = \frac{1}{\cancel{N-x+1}}\cdot\frac{1}{2}\cancel{(N-x+1)}(N+x)$$

$$\to E[Y|X] = \frac{N+X}{2}$$

1ero Forma:

$$\circ\ \underbrace{E[E[Y|X]]}_{E[Y]} = E\left[\frac{N+X}{2}\right] = E\left[\frac{N}{2}\right] + E\left[\frac{X}{2}\right]$$

$$= \frac{N}{2} + \frac{1}{2}E[X] = \frac{N}{2} + \frac{1}{2}E[E[X|Y]] = \frac{N}{2} + \frac{1}{2}E\left[\frac{Y+1}{2}\right]$$

$$= \frac{N}{2} + \frac{1}{4}(1+E[Y])$$

$$\to E[Y] = \frac{N}{2} + \frac{1}{4} + \frac{E[Y]}{4}$$

$$\to \frac{3E[Y]}{\cancel{4}} = \frac{2N+1}{\cancel{4}} \to E[Y] = \frac{2N+1}{3}$$

2da:

$$E[E[Y|X]] = E[Y] = \sum_{y=1}^{N} y\cdot p_Y(y) = \sum_{y=1}^{N} y\cdot\frac{2}{N(N+1)}\cdot y$$

$$= \frac{2}{N(N+1)}\sum_{y=1}^{N} y^2 = \frac{2}{\cancel{N}(\cancel{N+1})}\cdot\frac{\cancel{N}(\cancel{N+1})(2N+1)}{6} = \frac{(2N+1)}{3}$$

P3 prob. p llover en un día

Y = # de días que trabaja entre días de lluvia
X = # de turistas que van al parque en esos Y días

$$(X \mid Y) \sim \text{Poisson}(\mu Y)$$

a) $Y \sim \text{geom}(p) \rightarrow p_Y(y) = (1-p)^{y-1} \cdot p$

Fracaso : no llueve
exito : llueva

$\rightarrow E[Y] = \dfrac{1-p}{p}$ cuenta Fracasos (nuestro caso)

$E[Y] = \dfrac{1}{p}$ contar exitos

$\text{Var}[Y] = \dfrac{1-p}{p^2}$

b) $E[X] = E[E[X \mid Y]] = E[\mu Y]$

$$= \mu \cdot \frac{1-p}{p}$$

$$\text{Var}(X) = E(\text{Var}(X \mid Y)) + \text{Var}(E(X \mid Y))$$

$$= E(\mu Y) + \text{Var}(\mu Y)$$

$$= \mu \frac{(1-p)}{p} + \mu^2 \left(\frac{1-p}{p^2} \right)$$