P1

a) PDF de (V, X) en T que es cte.

$$\iint_T F_{VX}(v,x)\, dv\, dx = 1$$

$$\iint C\, dv\, dx = 1 \qquad v + x \leq 6, \quad v \leq 6 - x$$

$$\rightarrow \int_0^6 \int_0^{6-x} C\, dv\, dx = 1 \rightarrow C \int_0^6 [6 - x]\, dx$$

$$\rightarrow C\left[\int_0^6 6\, dx - \int_0^6 x\, dx\right] = C\left[6 \cdot 6 - \frac{6^2}{2}\right]$$

$$= C[36 - 18] = 18C = 1 \rightarrow C = 1/18$$

$$\therefore F_{VX}(v,x) = \begin{cases} 1/18 & \text{si } (v,x) \in T \\ 0 & \sim \end{cases}$$

b) $$F_V(v) = \int_0^{6-v} \frac{1}{18}\, dx = \frac{1}{18}[6 - v] \qquad v + x \leq 6, \quad x \leq 6 - v$$

$$\rightarrow \begin{cases} 1/18\,[6 - v] & \text{si } 0 \leq v \leq L \\ 0 & \sim \end{cases}$$

$$F_X(x) = \int_0^{6-x} \frac{1}{18}\, dv = \frac{1}{18}[6 - x]$$

$$\rightarrow \begin{cases} 1/18\,[6 - x] & \text{si } 0 \leq x \leq L \\ 0 & \sim \end{cases}$$

c) $\mathrm{Cov}(V,X) = E[VX] - E[V]E[X]$

$\rightarrow E[V] = \int_0^6 v \cdot F_V \, \partial v = \int_0^6 v \cdot \frac{1}{18}[6-v]\,\partial v$

$= \int_0^6 v\left[\frac{1}{3} - \frac{v}{18}\right]\partial v = \int_0^6 \left(\frac{v}{3} - \frac{v^2}{18}\right)\partial v$

$= \int_0^6 \frac{v}{3}\,\partial v - \int_0^6 \frac{v^2}{18}\,\partial v = \frac{1}{3}\cdot\frac{6^2}{2} - \frac{1}{18}\cdot\frac{6^3}{3}$

$= 6 - 4 = 2$

$\rightarrow E[X] = \int_0^6 x\, F_X \,\partial x = \int_0^6 x \cdot \frac{1}{18}[6-x]\,\partial x$

$= \int_0^6 x\left[\frac{1}{3} - \frac{x}{18}\right]\partial x$ $\rightarrow$ análogo al anterior

$\therefore E[X] = 2$

- $E[VX] = \int_0^6 \int_0^{6-x} vx \cdot \frac{1}{18}\,\partial v\,\partial x = \frac{1}{18}\int_0^6 \int_0^{6-x} v\cdot x \,\partial v\,\partial x$

$= \frac{1}{18}\int_0^6 x\,\partial x \cdot \frac{[6-x]^2}{2} = \frac{1}{36}\int_0^6 x[36 - 12x + x^2]\,\partial x$

$= \frac{1}{36}\left[\int_0^6 36x\,\partial x - \int_0^6 12x^2\,\partial x + \int_0^6 x^3\,\partial x\right]$

$= \frac{1}{36}\left[36\cdot\frac{6^2}{2} - 12\cdot\frac{6^3}{3} + \frac{6^4}{4}\right]$

$$= \frac{1}{36}\left[648 - 864 + 324 \right] = 3$$

- $Cov(V, X) = 3 - 2 \cdot 2 = -1$

d) $Z = \dfrac{X}{6-V}$, (V, Z)

$\rightarrow (V, Z) = g(V, X)$ / g^{-1} $g(v, x) = \left(v, \dfrac{x}{6-v}\right)$

$g^{-1}(v, z) = (v, x)$

- $g^{-1}(v, z) = (v, z(6-v))$
- $$J_g = \begin{pmatrix} \frac{\partial g_1}{\partial v} & \frac{\partial g_1}{\partial x} \\ \frac{\partial g_2}{\partial v} & \frac{\partial g_2}{\partial x} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{x}{(6-v)^2} & \frac{1}{6-v} \end{pmatrix}$$

$$\frac{\partial}{\partial v}\left(\frac{x}{6-v}\right) = x \cdot \frac{\partial}{\partial v}\left(\frac{1}{6-v}\right) = x \cdot (-1) \cdot \frac{1}{(6-v)^2} \cdot (0-1)$$
$$= \frac{(-x)}{(6-v)^2} \cdot -1 = \frac{x}{(6-v)^2}$$

$$\det(J_g) = \frac{1}{6-v} - 0 = \frac{1}{6-v}$$

$$\rightarrow F_{VZ} = F_{VX}(g^{-1}(v, z)) \cdot (6-v)$$
$$= F_{VX}(v, z(6-v)) \cdot (6-v)$$
$$= \begin{cases} \frac{1}{18} \cdot (6-v) & \text{si } (v, z(6-v)) \in T \\ 0 & \sim \end{cases}$$

$$= \begin{cases} \frac{1}{18} \cdot (6-v) & \text{si } 0 \leq v \leq 6 \,;\, 0 \leq z \leq 1 \\ 0 & \sim \end{cases}$$

- $F_V(v) = \int F_{VZ} \, dz = \int_0^1 \frac{1}{18}(6-v)\, dz$

$$= \frac{1}{18}(6-v)$$

- $F_Z(z) = \int F_{VZ} \, dv = \int_0^6 \frac{1}{18}(6-v)\, dv = \frac{1}{18}\int_0^6 (6-v)\, dv$

$$= \frac{1}{18}\left[\int_0^6 6\,dv - \int_0^6 v\,dv\right] = \frac{1}{18}\left[6 \cdot 6 - \frac{36}{2}\right]$$

$$= \frac{1}{18} \cdot 18 = 1$$

$\therefore$ V y Z son indep. pues la multiplicación de sus densidades marginales es igual a la conjunta

P2)

$$\iint F_{XY}\, dx\, dy = 1$$

$$\rightarrow \int_X^\infty \int_1^\infty \frac{C}{x^2 y^2}\, dx\, dy = \int_1^\infty \int_X^\infty \frac{C}{x^2 y^2}\, dy\, dx$$

$$= \int_1^\infty \frac{C}{x^2}\, dx \int_x^\infty \frac{dy}{y^2} = \int_1^\infty \frac{C}{x^2}\, dx \cdot \frac{y^{-1}}{-1}\Big|_x^\infty$$

$$= \int_1^\infty \frac{C}{x^2}\, dx \left[0 + \frac{1}{x}\right] = \int_1^\infty \frac{C}{x^3}\, dx$$

$$= C \cdot \left[-0 + \frac{1}{2}\right] = \frac{C}{2} = 1 \rightarrow C = 2$$

- $$F_X(x) = \int_X^\infty \frac{2}{x^2 y^2}\, dy = \frac{2}{x^2}\left[-0 + \frac{1}{x}\right] = \frac{2}{x^3}$$

- $$F_{Y|X}(y|x) = \frac{F_{XY}(x,y)}{F_X(x)} = \frac{2/x^2 y^2}{2/x^3} = \frac{x^{\cancel{3}} \cdot \cancel{2}}{\cancel{x^2} y^2 \cdot \cancel{2}}$$

$$= \frac{x}{y^2}$$

P3 X = # personas que paga la cuenta

- $X_i = \begin{cases} 1 & \text{si } i \text{ paga} \\ 0 & \sim \end{cases} \rightarrow X_i \sim \text{Bernoulli}(p)$

$i-1$	i	$i+1$
S	C	S
C	S	C

$$p = \frac{\text{casos Fav.}}{\text{casos tot.}}$$

$$= \frac{2}{2^3} = \frac{1}{4}$$

$$X = \sum X_i \rightarrow E[X] = E[X_1 + X_2 + \_ + X_n]$$

$$= E[X_1] + E[X_2] + \_ + E[X_n]$$

$$= 1/4 \cdot n$$

- $Var(X) = Var(\sum X_i) = \sum Var(X_i) + 2\sum_{i<j} Cov(X_i, X_j)$

  - $Var(X_i) = p(1-p) = 1/4 \cdot 3/4 = 3/16$
  - $\sum Var(X_i) = n \cdot 3/16$

- $Cov(X_i, X_j)$

  - i y j no son vecinos

$i-1$	i	$i+1$	$i+2$	$i+3$
S	C	S	|	|
C	S	C		

el resultado de $i+2$, no influye al resultado de i, ie, no afecta si i tiene que pagar o no

$\therefore$ i y j son indep

$\rightarrow \operatorname{Cov}(X_i, X_j) = 0$

• i y j son vecinos

$$\operatorname{Cov}(X_i, X_j) = E[X_i, X_j] - E(X_i)E(X_j)$$

• $$E[X_i X_j] = \sum_{\substack{x_i \in \{0,1\} \\ x_j \in \{0,1\}}} x_i x_j \, P(X_i = x_i, X_j = x_j)$$

Si x_i o x_j son 0, no aportan a la esperanza

$$= 1 \cdot 1 \cdot P(X_i = 1, X_j = 1)$$

$i-1$	i	j	$i+2$
S	C	S	C
C	S	C	S

$$\rightarrow P(X_i = 1, X_j = 1) = \frac{\text{casos fav.}}{\text{casos tot.}} = \frac{2}{2^4} = \frac{1}{8}$$

$$\rightarrow \operatorname{Var}(X) = \frac{3m}{16} + 2\sum\left(\frac{1}{8} - \frac{1}{4} \cdot \frac{1}{4}\right)$$

$$= \frac{3m}{16} + 2\sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{1}{16} = \frac{3m}{16} + \frac{2m}{16} = \frac{5m}{16}$$