P1 Calculemos PMF p_Y

$$\rightarrow p_Y(y) = \binom{4}{y}\left(\frac{1}{6}\right)^y\left(\frac{5}{6}\right)^{4-y} \qquad y = 0, \ldots, 4$$

$\rightarrow p_{X|Y}(x|y)$, esto es la cant. de 1s en los $4-y$ tiros restantes, como sabemos, aquí no salió ningún 2, entonces la prob. de sacar un 1 es $1/5$

$$\rightarrow p_{X|Y}(x|y) = \binom{4-y}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{4-y-x}$$

∴ PMF conjunta es

$$\rightarrow p_{X,Y}(x,y) = p_Y(y)\cdot p_{X|Y}(x|y)$$
$$= \binom{4}{y}\left(\frac{1}{6}\right)^y\left(\frac{5}{6}\right)^{4-y}\cdot\binom{4-y}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{4-y-x}$$

P2 a) pares son $(1,1), (1,3), (2,1), (2,3), (4,1), (4,3)$

$$\rightarrow (1+1)c + (1+9)c + (4+1)c + (4+9)c + (16+1)c + (16+9)c$$
$$= 2c + 10c + 5c + 13c + 17c + 25c$$
$$= 72c = 1$$
$$c = \frac{1}{72}$$

b) $P(Y < x) \rightarrow (2,1), (4,1), (4,3)$

$$P(\cdot) = \frac{5}{72} + \frac{17}{72} + \frac{25}{72} = \frac{47}{72}$$

c) $P(Y > x) \rightarrow (1,3), (2,3)$

$$P(\cdot) = \frac{10}{72} + \frac{13}{72} = \frac{23}{72}$$

d) $$P(X=Y) = \frac{2}{72}$$

e) $$P(Y=3) = \frac{10}{72} + \frac{13}{72} + \frac{25}{72} = \frac{48}{72}$$

F) $$p_X(x) = \begin{cases} 2/72 + 10/72 & x=1 \\ 18/72 & x=2 \\ 42/72 & x=4 \end{cases}$$

$$p_Y(y) = \begin{cases} 24/72 & y=1 \\ 48/72 & y=3 \end{cases}$$

P3 N = # clientes totales $\sim$ Poisson(λ)
cada cliente compra con prob p (indep. cada cliente)
X = # que compra
Y = # que no compra $\qquad N = X+Y$

a) $\text{bin}(m,p) \Rightarrow P(Z=k) = \binom{m}{k} p^k (1-p)^{m-k} \qquad m \geq k$

$X \mid N=m \sim \text{bin}(m,p)$

$$\rightarrow P_X(X=k) = \sum_{m=0}^{\infty} P(X=k \mid N=m) \cdot P(N=m)$$

$$= \sum_{m=0}^{k-1} \cancel{P(X=k \mid N=m)}^{\,0} P(N=m) + \sum_{m=k}^{\infty} P(X=k \mid N=m) P(N=m)$$

$$= \sum_{m=k}^{\infty} \binom{m}{k} p^k (1-p)^{m-k} \cdot \frac{\lambda^m e^{-\lambda}}{m!}$$

$$= \sum_{m=k}^{\infty} \frac{\cancel{m!}}{(m-k)!\,k!}\, p^k (1-p)^{m-k} \cdot \frac{\lambda^m e^{-\lambda}}{\cancel{m!}}$$

$(1-p)^m (1-p)^{-k}$

$$= \frac{p^k (1-p)^{-k} e^{-\lambda}}{k!} \sum_{m=k}^{\infty} \frac{(1-p)^m \lambda^m}{(m-k)!}$$

$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

$$= \frac{p^k (1-p)^{-k} e^{-\lambda}}{k!} \sum_{m=0}^{\infty} \frac{(1-p)^{k+m} \lambda^{k+m}}{(m+\cancel{k}-\cancel{k})!}$$

$$= \frac{p^k \cancel{(1-p)^{-k}} e^{-\lambda}}{k!} \cdot \cancel{(1-p)^k} \lambda^k \sum_{m=0}^{\infty} \frac{(1-p)^m \lambda^m}{m!}$$

$((1-p)\lambda)^m$

$$= \frac{p^k e^{-\lambda} \lambda^k e^{(1-p)\lambda}}{k!} = \frac{p^k \cancel{e^{-\lambda}} \lambda^k \cancel{e^{\lambda}} e^{-\lambda p}}{k!}$$

$$= \frac{(p\lambda)^k e^{-p\lambda}}{k!} = p_X \quad \to \quad X \sim \text{poisson}(\lambda p)$$

$$Y \sim \text{poisson}(\lambda(1-p))$$

$$p_Y(k) = \frac{((1-p)\lambda)^k e^{-\lambda(1-p)}}{k!}$$

b) $P(X=k, Y=j) = \sum_{m=0}^{\infty} \overbrace{P(X=k, Y=j \mid N=m)\, P(N=m)}^{*}$

$m = k+j$

$$\to \sum_{m=0}^{k+j-1} \cancelto{0}{(*)} + P(X=k, Y=j \mid N=k+j)\, P(N=k+j) + \sum_{m=k+j+1}^{\infty} \cancelto{0}{(*)}$$

$N = k+j,\ X = k \Rightarrow Y = j$

$$= P(X=k \mid N=k+j)\, P(N=k+j)$$

$$= \binom{k+j}{k} p^k (1-p)^j \cdot \frac{\lambda^{k+j} e^{-\lambda}}{(k+j)!}$$

$$= \frac{\cancel{(k+j)!}}{j!\,k!}\, p^k (1-p)^j \frac{\lambda^k \lambda^j e^{-\lambda}}{\cancel{(k+j)!}}$$

$$= \frac{(p\lambda)^k}{k!} \cdot \frac{((1-p)\lambda)^j}{j!} e^{-\lambda} = \frac{(p\lambda)^k}{k!} \cdot \frac{((1-p)\lambda)^j}{j!} e^{-\lambda[p+(1-p)]}$$

$$= \frac{(p\lambda)^k}{k!} \cdot e^{-\lambda p} \cdot \frac{((1-p)\lambda)^j}{j!} \cdot e^{-\lambda(1-p)}$$

$$= p_X(k) \cdot p_Y(j)$$

c) $p_{XY}(k, j) = p_X(k) \cdot p_Y(j) \rightarrow$ Son independientes