

P11

a) $X_i = \# \text{ intentos etapa } i$

$$\rightarrow P(X_i = x) = \prod_{l=0}^{x-1} (1-p_i) \cdot p_i \quad x \in \{1, 2, 3, \dots\}$$

$$\rightarrow X_i \sim \text{geom.}(p_i)$$

b) $X = \# \text{ intentos totales para } m \text{ niveles}$
 $= \sum_{i=1}^m X_i$

$$\rightarrow E[X] = E\left[\sum X_i\right] = \sum E[X_i] \quad \text{linealidad de la esperanza}$$
$$= \sum_{i=1}^m 1/p_i$$

c) $p_i = p \quad \forall i \in \{1, 2, 3, \dots, m\}$

Si pasa el nivel i en K intentos

$$\rightarrow \begin{aligned} P(X_1 = K) &= (1-p)^{K-1} p \\ P(X_2 = K) &= (1-p)^{K-1} p \end{aligned}$$

$$\rightarrow X = \sum_{i=1}^m X_i \sim \text{BN}(m, p) \rightarrow \binom{x-1}{m-1} p^m (1-p)^{x-m}$$
$$x \in \{m, m+1, m+2, \dots\}$$

d) R monedas si $X_i < h$
paga c por cada nivel

$$\rightarrow P(X_i < h) = \sum_{k=0}^{h-1} p(1-p)^k = q \rightarrow \text{prob. de éxito a conseguir la recompensa}$$

$Z = \# \text{ de veces que consigo la recompensa}$
 $\sim \text{Bem}(n, q)$

Costo total = $n \cdot c$

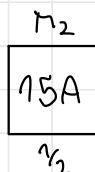
$$\rightarrow Y = R \cdot Z - mc$$

$$\begin{aligned}\rightarrow E[Y] &= E[RZ - mc] = E[RZ] - E[mc] \\ &= R \cdot E[Z] - mc \\ &= R \cdot m_{\mathcal{Z}} - mc\end{aligned}$$

$$\begin{aligned}\rightarrow \text{Var}(Y) &= \text{Var}(RZ - mc) = \text{Var}(RZ) = R^2 \text{Var}(Z) \\ &= R^2 \cdot m_{\mathcal{Z}}(1 - \rho)\end{aligned}$$

P2

a)



+6N

$$P(M_1 | D.N) = P(D.N | M_1) \frac{P(M_1)}{P(D.N)} \quad \text{Bayes}$$

$$\rightarrow P(D.N) = P(D.N | M_1) P(M_1) + P(D.N | M_2) P(M_2) \quad \text{prob. totales}$$

$$P(M_1 | D.N) = \frac{P(D.N | M_1) P(M_1)}{P(D.N | M_1) P(M_1) + P(D.N | M_2) P(M_2)}$$

$$= \frac{\frac{6}{16} \cdot \frac{1}{2}}{\frac{6}{16} \cdot \frac{1}{2} + \frac{6}{21} \cdot \frac{1}{2}} = \frac{0.1875}{0.3304} = 0.567$$

b) K mochilas com M doces azules
 ↓
 N " negros

$$P(D.A_K) = P(D.A_1)$$

$$\rightarrow P(D.A \text{ em } M_1) = \frac{M}{M+N} \quad 1 \rightarrow 2$$

$$P(D.A \text{ em } M_2) \quad 2 \rightarrow 3$$

$$= P(D.A \text{ em } M_2 | D.A \text{ em } M_1) P(D.A \text{ em } M_1) + P(D.A \text{ em } M_2 | D.N \text{ em } M_1) \cdot P(D.N \text{ em } M_1)$$

$$= \frac{M+1}{M+1+N} \cdot \frac{M}{M+N} + \frac{M}{M+N+1} \cdot \frac{N}{M+N}$$

$$= \frac{(M+1)M + MN}{(M+N)(M+N+1)} = \frac{M[M+1+N]}{(M+N)(M+N+1)} = \frac{M}{M+N}$$

→ El caso para llegar a K es análogo y como la prob. se mantiene

$$\rightarrow P(\text{DA en } M_1) = P(\text{DA en } M_K)$$

P3

a) No ordenado con reemplazo

$$m=20 \quad K=4 \quad \binom{20+4-1}{4} = \frac{23!}{19!4!} \text{ equipos distintos}$$

$$\begin{aligned} \{ABC\} &\rightarrow ABC, ACB, BAC, BCA, CAB, CBA & 6 &= \frac{3!}{1!1!1!} \Rightarrow \\ \{AAB\} &\rightarrow AAB, ABA, BAA & 3 &= \frac{3!}{2!1!} \end{aligned} \quad \begin{matrix} P(ABC) \neq \\ P(AAB) \end{matrix}$$

b) $P(\text{Al menos 1 de esos 3}) = 1 - P(\text{ninguno})$ ordenado con reemplazo

$$\rightarrow P(\text{ninguno}) = \frac{\text{Casos Fav.}}{\text{Casos tot.}} = \frac{17 \cdot 17 \cdot 17 \cdot 17}{20 \cdot 20 \cdot 20 \cdot 20}$$

$$\rightarrow P(\text{Al menos 1}) = 1 - \frac{17^4}{20^4}$$

c) 5 equipos de 4 clw. Sin repetición y ordenado

$$\begin{aligned} \rightarrow & \binom{20}{4} \cdot \binom{16}{4} \cdot \binom{12}{4} \cdot \binom{8}{4} \cdot \binom{4}{4} = \binom{20}{4,4,4,4,4} \\ &= \frac{20!}{4!^5} \end{aligned}$$