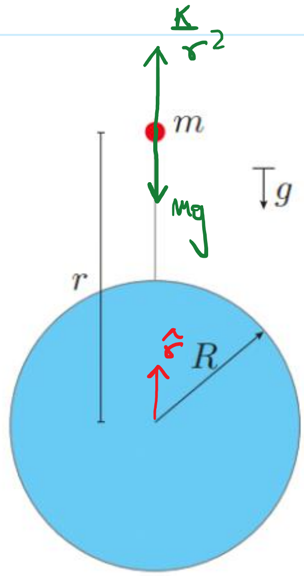
Auxiliar 13



$$\vec{F}_{neta} = \sum \vec{F} = \left(\overset{F_1}{\frac{K}{r^2}} - \overset{F_2}{mg}\right)\hat{r} = m\,\vec{a}_r$$

$$V_1(\vec{r}),\ V_2(\vec{r}) \quad , \quad -\nabla V_i = \vec{F}_i \quad , \quad i=\{1,2\}$$

$$-\nabla V_1(\vec{r}) = \frac{K}{r^2}\hat{r}$$

$$-\nabla V_2(\vec{r}) = -mg\,\hat{r}$$

$$\Rightarrow \quad -\frac{\partial V_1}{\partial r}\hat{r} = \frac{K}{r^2}\hat{r} \quad \wedge \quad -\frac{\partial V_2}{\partial r}\hat{r} = -mg\,\hat{r}$$

$$\frac{\partial V_1}{\partial r} = -\frac{K}{r^2} \quad \wedge \quad \frac{\partial V_2}{\partial r} = mg$$

$$\int_{r_0}^{r} dV_1 = \int_{r_0}^{r} -\frac{K}{r^2}\,dr \quad \wedge \quad \int_{r_0}^{r} \partial V_2 = \int_{r_0}^{r} mg\,dr$$

$$V_1(r) - V_1(r_0) = \frac{K}{r} - \frac{K}{r_0} \qquad V_2(r) - V_2(r_0) = mg(r - r_0)$$

$r_0 \to \infty$ $\qquad\qquad$ $r_0 \to 0$

$V_1(r_0) = 0$ $\qquad\qquad$ $V_2(r_0) = 0$

$$V_1(r) = \frac{K}{r} \qquad\qquad V_2(r) = mgr$$

$$V(r) = V_1(r) + V_2(r) = \frac{K}{r} + mgr$$

Auxiliar 13

$$V(r) = mgr + \frac{K}{r} \qquad \{m, g, K\}$$

pto. de equilibrio r_e

$$\left.\frac{\partial V}{\partial r}\right|_{r=r_e} = 0 \quad , \quad \frac{\partial V}{\partial r} = mg - \frac{K}{r^2}$$

$$\left.\frac{\partial V}{\partial r}\right|_{r=r_e} = mg - \frac{K}{r_e^2} = 0$$

$$r_e^2 = \frac{K}{mg} \Rightarrow \boxed{r_e = \sqrt{\frac{K}{mg}}}$$

$$\frac{\partial^2 V}{\partial r^2} = +\frac{2K}{r^3} \qquad \left.\frac{\partial^2 V}{\partial r^2}\right|_{r=r_e} = \frac{2K}{\left(\frac{K}{mg}\right)^{3/2}} > 0$$

$\Rightarrow$ r_e es estable

$\Rightarrow$ existen pequeños oscilaciones ($\omega = \frac{2\pi}{T}$

$$\omega_{po}^2 = \frac{\left.\frac{\partial^2 V}{\partial r^2}\right|_{r=r_e}}{m} = \frac{2K}{m\left(\frac{K}{mg}\right)^{3/2}} = \frac{4\pi^2}{T^2}$$

$$T^2 = \frac{2\pi^2 m \left(\frac{K}{mg}\right)^{3/2}}{K}$$

Auxiliar 13

$$V(r) = mgr + \frac{K}{r^2}$$

$$V(r_e) = mg r_e + \frac{K}{r_e^2}$$

$$\frac{\partial V}{\partial r}(r_e) = 0$$

$$\frac{\partial^2 V}{\partial r^2}(r_e) = \frac{2K}{\left(\frac{K}{mg}\right)^{3/2}}$$

$$V(r) = V(r_e) + \frac{\partial V}{\partial r}(r_e)(r-r_e) + \frac{\partial^2 V}{\partial r^2}(r_e)\frac{(r-r_e)^2}{2}$$

Auxiliar 13

$$\Delta K = \sum W \qquad \text{pq no hay}$$

$$= \sum W_c + \cancel{\sum W_{NC}}^{\,0} \qquad \text{fzas N.C.}$$

$$= -\Delta V$$

$$\cancel{\Delta K}^{\,0} = -\Delta V \Rightarrow \quad \Delta K + \Delta V = 0$$

$$\Delta(\underbrace{K+V}_{E}) = 0$$

$$\Delta E = 0$$

$$\Rightarrow \quad -\Delta V = 0 \iff \Delta V = 0$$

$$V = mgr + \frac{K}{r} \qquad r_i = r_0$$

$$r_f = R$$

$$\Delta V = V(R) - V(r_0) = 0 \Rightarrow V(R) = V(r_0)$$

$$= mgR + \frac{K}{R} - \left(mgr_0 + \frac{K}{r_0} \right) = 0$$

$$\Rightarrow \quad mgRr_0 + \frac{K}{R} r_0 - mgr_0^2 - K = 0$$

$$mgr_0^2 - r_0\left(mgR - \frac{K}{R} \right) + K = 0$$

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$$mg r_0^2 - r_0\left(mgR + \frac{K}{R}\right) + K = 0$$

$$\cancel{mgR^2} - \cancel{mgR^2} - \cancel{R \cdot \frac{K}{R}} + \cancel{K}$$

$$mg\cancel{(r_0 - R)} = \cancel{(r_0 - R)}\frac{K}{r_0 R}$$

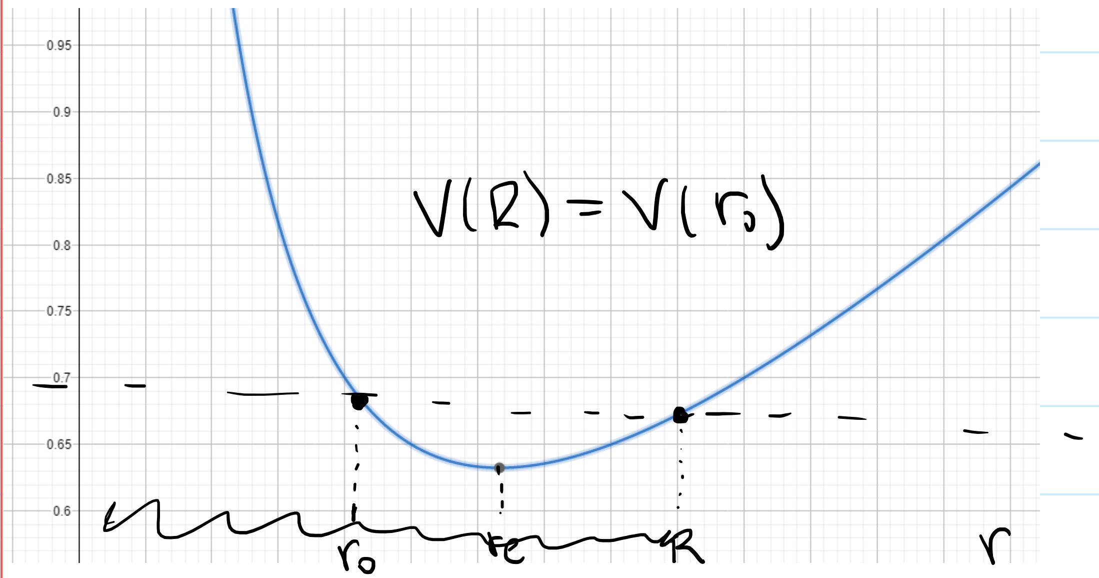
$r_0 \neq R$

$$mg = \frac{K}{r_0 R}$$

$$V(x) = \frac{1}{2} K x^2$$

$$\boxed{r_0 = \frac{K}{mgR}}$$

Auxiliar 13



$R < r_e \iff r_0 > r_e$

$R > r_0$

Auxiliar 13

L.G.U. : $\vec{F}_g = -\frac{GMm}{r^2}\hat{r}$

$$V_g(r) = -\frac{GMm}{r}$$

cumple $\quad -\nabla V_g = \vec{F}_g$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$E_1 = \frac{1}{2}mv_0^2 - \frac{GMm}{R_0}$$

$$E_2 = \frac{1}{2}m(\alpha v_0)^2 - \frac{GMm}{R_0}$$

$$E_3 = 0$$

$$\frac{1}{2}\cancel{m}(\alpha v_0)^2 - \frac{GM\cancel{m}}{R_0} = 0$$

$$\boxed{\alpha^2 = \frac{2GM}{v_0^2 R_0}} = \frac{2\cancel{GM}}{\cancel{GM}} = 2$$

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$$+\frac{GM\cancel{m}}{R_0^2} = +\cancel{m}\,\frac{v_0^2}{R_0}$$

$$\boxed{GM = v_0^2 R_0}$$

$$\Rightarrow \quad \alpha \leq \sqrt{2}$$

Auxiliar 13

$$u = \frac{1}{r(\theta)}$$

$$-mh^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) = F(u^{-1})$$

$$h = \frac{L}{m} = \frac{\cancel{m}\cdot \alpha v_0 \cdot R_0}{\cancel{m}} = \alpha v_0 R_0$$

$$h^2 = \alpha^2 v_0^2 R_0^2 \leftarrow$$

$$F(r) = -\frac{GMm}{r^2} \Leftrightarrow F(u^{-1}) = -GMmu^2$$

$$\cancel{+m}\alpha^2 v_0^2 R_0^2 \cancel{u^2}\left(\frac{d^2u}{d\theta^2} + u\right) = +GM\cancel{m}\cancel{u^2}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{\alpha^2 v_0^2 R_0^2} = \frac{\cancel{GM}}{\alpha^2 R_0 (\cancel{v_0^2 R_0})} \cancel{GM}$$

$$\frac{d^2u}{d\theta^2} + 1u = \frac{1}{\alpha^2 R_0}$$

$$u = A\cos(\theta + \delta) + \frac{1}{\alpha^2 R_0}$$

ctes de integracion

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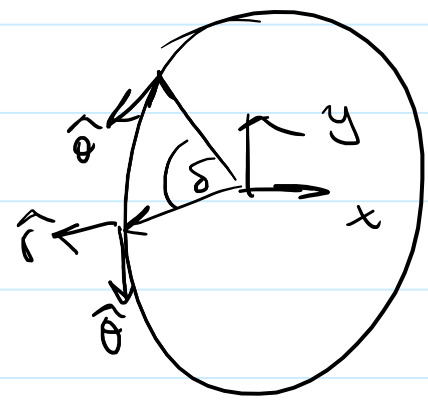
$$u(\theta) = A\cos(\theta+\delta) + \frac{1}{\alpha^2 R_0}$$

$\delta = 0$



$$\frac{1}{r(\theta)} = A\cos(\theta) + \frac{1}{\alpha^2 R_0}$$

$$r(\theta) = \frac{1}{\frac{1}{\alpha^2 R_0} + A\cos(\theta)}$$

$$r(\theta=0) = R_0$$

$$\frac{1}{\frac{1}{\alpha^2 R_0} + A} = R_0 \Rightarrow A = \frac{\alpha^2 - 1}{\alpha^2 R_0} > 0$$

$$= A > 0 \qquad \alpha > 1$$

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$$r(\theta) = \frac{1}{\frac{1}{\alpha^2 R_0} + A\cos(\theta)} \quad , \quad A = \frac{\alpha^2 - 1}{\alpha^2 R_0} > 0$$

$$R_+ = r(\theta^*)$$

$$\frac{1}{\alpha^2 R_0} + A\cos(\theta^*)$$ sea lo mas pequeño posible

$$\theta^* = \pi \Rightarrow \cos(\theta^* = \pi) = -1$$

$$r(\pi) = R_+ = \frac{1}{\frac{1}{\alpha^2 R_0} - \left(\frac{\alpha^2 - 1}{\alpha^2 R_0}\right)}$$

$$R_+ = \frac{1}{\frac{1 - \alpha^2 + 1}{\alpha^2 R_0}}$$

$$\boxed{\frac{R_+}{R_0} = \frac{\alpha^2}{2 - \alpha^2}}$$

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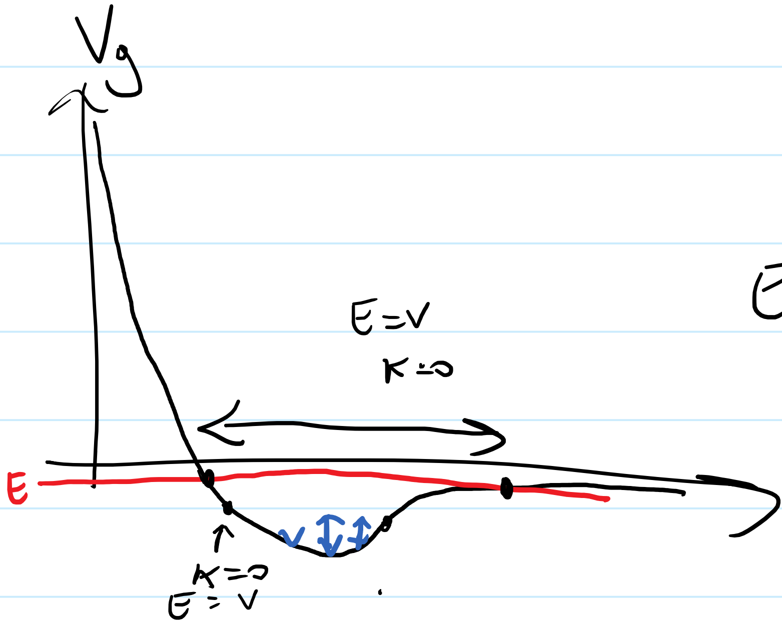

$r \to \infty$

$E = 0$ la energía de escape

r V_g

$E = K + V$