Auxiliar 10

Fuerzas → conservativas → hay potencial

Fuerzas → no conservativas → calculo de trabajo

$$\Delta K = \sum W = \underbrace{\sum W_C}_{} + \sum W_{NC} = -\Delta U + \sum W_{NC}$$

$$W_C^{a\to\vec{r}} = -(U(\vec{r}) - U(a))$$

$$U(\vec{r}) = -W_C^{\vec{r}_0\to\vec{r}} + U(\vec{r}_0)$$

$$\Rightarrow \Delta K + \Delta U = \sum W_{NC}$$

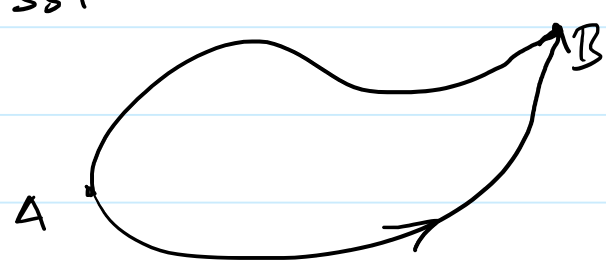
$$\Delta(K+U) = \sum W_{NC}$$

$$\Delta E = \sum W_{NC}$$

si no hay trabajos N.C. $\Rightarrow \Delta E = 0 \Leftrightarrow E(i) = E(f)$

$\vec{F}$ es conservativa ssi

$\nabla \times \vec{F} = 0$



$\oint \vec{F}\cdot d\vec{r} = 0$

$\exists\, U(\vec{r})$ tal que $-\nabla U(\vec{r}) = \vec{F}(\vec{r})$

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$$\vec{F} = f(\rho)\,\hat{\rho} + 0\,\hat{\phi} + 0\,\hat{z} \Rightarrow F_\rho = f(\rho),\ F_\phi = 0,\ F_z = 0$$

$$\nabla \times \vec{F} = \left[\frac{1}{\rho}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right]\hat{\boldsymbol{\rho}} + \left[\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho}\right]\hat{\boldsymbol{\phi}} + \frac{1}{\rho}\left[\frac{\partial(\overset{\rho\cdot 0}{\rho F_\phi})}{\partial \rho} - \frac{\partial F_\rho}{\partial \phi}\right]\hat{\boldsymbol{k}}$$

$F_z = 0$ $\quad F_\phi = 0$ $\quad f$ No depende de z

$$\nabla \times \vec{F} = \left(\cancel{\frac{1}{\rho}\frac{\partial F_z}{\partial \phi}}^{0} - \cancel{\frac{\partial F_\phi}{\partial z}}^{0}\right)\hat{\rho} + \left(\cancel{\frac{\partial f(\rho)}{\partial z}}^{0} - \cancel{\frac{\partial F_z}{\partial \rho}}^{0}\right)\hat{\phi} + \frac{1}{\rho}\left[\cancel{\frac{\partial(\rho F_\phi)}{\partial \rho}}^{0} - \cancel{\frac{\partial f(\rho)}{\partial \phi}}^{0}\right]\hat{k}$$

$\Rightarrow \vec{F}$ es una fuerza conservativa

$$U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{r}$$

$$d\vec{r} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{k}}$$

$$= -\int_{\vec{r}_0}^{\vec{r}} f(\rho)\,\hat{\rho}\cdot\left(d\rho\,\hat{\rho} + \rho\, d\phi\,\hat{\phi} + dz\,\hat{k}\right)$$

$$\boxed{U(\rho) = -\int_{\rho_0}^{\rho} f(\rho)\,d\rho}$$

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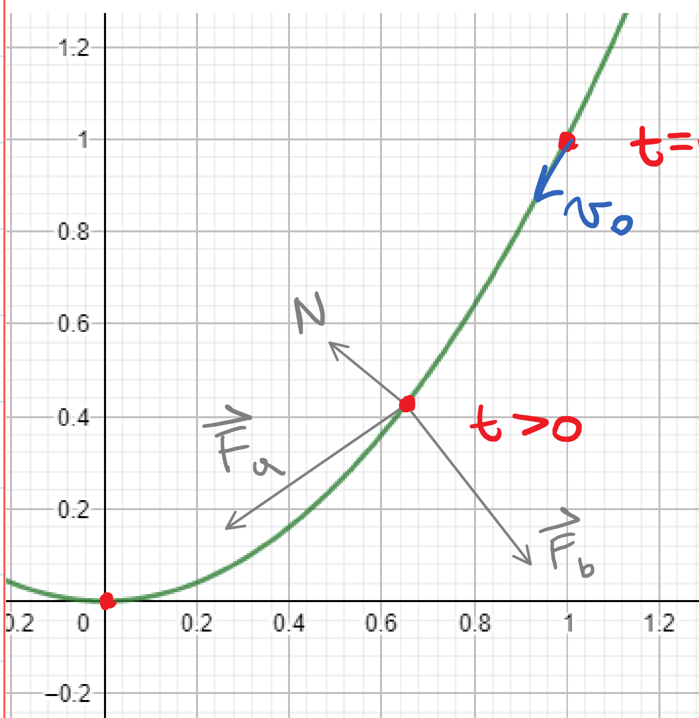

La Normal N no hace trabajo

$$\vec{F}_a = -A\rho^3\,\hat{\rho}$$

$$\vec{F}_b = B\left(y^2\hat{x} - x^2\hat{y}\right)$$

$$F_{bx} = By^2 \,,\quad F_{by} = -Bx^2 \,,\quad F_{bz} = 0$$

Por la P1 sabemos que $\vec{F}_a$ es conservativa con $f(\rho) = -A\rho^3$

$$U_a(\rho) = +\int_{\rho_0}^{\rho} +A\rho^3\, d\rho = A\left(\frac{\rho^4}{4} - \frac{\rho_0^4}{4}\right)$$

$$\rho_0 = 0 \Rightarrow \boxed{U_a(\rho) = \frac{A\rho^4}{4}}$$

$$\nabla \times \vec{F}_b = \left(\frac{\partial F_{by}}{\partial x} - \frac{\partial F_{bx}}{\partial y}\right)\hat{z}$$

$$= \left(-2Bx - 2By\right)\hat{z} \Rightarrow \text{No es conservativa}$$

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$$W_b^{(1,1)\to(0,0)} = \int_{(1,1)}^{(0,0)} \vec{F}_b \cdot d\vec{r}$$

$$= \int_{(1,1)}^{(0,0)} B\left(y^2\hat{x} - x^2\hat{y}\right)\cdot\left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right)$$

$$= B\int_{(1,1)}^{(0,0)} y^2\,dx - x^2\,dy$$

$$= x^2$$

$$y^2 = x^4$$

$$= B\int_{(1,1)}^{(0,0)} x^4\,dx - y\,dy$$

$$= B\left(\int_1^0 x^4\,dx - \int_1^0 y\,dy\right)$$

$$= B\left(-\frac{1}{5} + \frac{1}{2}\right) = \frac{3B}{10}$$

$$\boxed{W_b^{(1,1)\to(0,0)} = \frac{3B}{10}}$$

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$$U_a(\rho) = \frac{A\rho^4}{4}$$

$$\rho(1,1) = \sqrt{2}$$

$$\rho(0,0) = 0$$

$$U_a(1,1) = \frac{A(\sqrt{2})^4}{4} = A$$

$$U_a(0,0) = \frac{A \cdot 0}{4} = 0$$

$$\Delta E = \sum W_{NC}$$

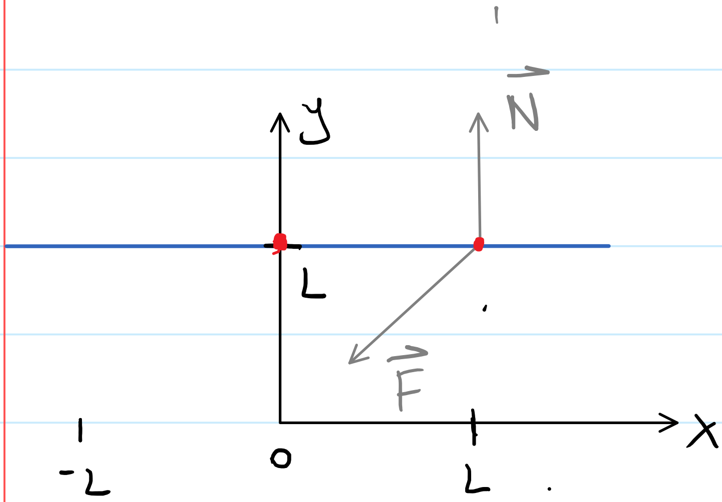
$$E(0,0) - E(1,1) = \frac{3B}{10}$$

$$K(0,0) + U(0,0) - K(1,1) - U(1,1) = \frac{3B}{10}$$

$$\frac{1}{2}mv_f^2 + 0 - \frac{1}{2}mv_0^2 - A = \frac{3B}{10}$$

$$\boxed{v_f = \sqrt{\frac{3B}{5m} + \frac{2A}{m} + v_0^2}}$$

Auxiliar 10



$\vec{N} \perp \Delta\vec{r}$

$\Rightarrow \vec{N}$ no hace trabajo

$$\vec{F} = -\alpha x\,\hat{x} - \alpha y\,\hat{y}$$

$$F_x = -\alpha x, \quad F_y = -\alpha y$$

$$W^{(L,L)\to(0,L)} = \int_{(L,L)}^{(0,L)} \vec{F}\cdot d\vec{r}$$

$$= -\alpha\left(\int_{(L,L)}^{(0,L)} (x\,\hat{x} + \boxed{y\,\hat{y}})\cdot(dx\,\hat{x} + \boxed{dy\,\hat{y}} + dz\,\hat{z})\right)$$

$$= -\alpha\left(\int_{(L,L)}^{(0,L)} x\,dx + y\,dy\right)$$

$$= -\alpha\left[\int_{L}^{0} x\,dx + \cancel{\int_{L}^{L} y\,dy}\right]$$

$$\boxed{W^{(L,L)\to(0,L)} = \frac{\alpha L^2}{2}}$$

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$$\nabla \times \vec{F} = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$$

$$= (0 - 0)\hat{z} = 0$$

$\Rightarrow \vec{F}$ es conservativa $\Rightarrow \exists\, U(\vec{r})\,,\ -\nabla U(\vec{r}) = \vec{F}$

$$-\nabla U = -\frac{\partial U}{\partial x}\hat{x} \boxed{-\frac{\partial U}{\partial y}\hat{y}} = -ax\hat{x} \boxed{-ay\hat{y}}$$

$$\Rightarrow \underbrace{\frac{\partial U}{\partial x} = ax}\ \wedge\ \underbrace{\frac{\partial U}{\partial y} = ay}$$

$$\rightarrow U = \frac{ax^2}{2} + f(y) \qquad U = \frac{ay^2}{2} + g(x)$$

$$\Rightarrow U(x,y) = \frac{ax^2}{2} + \frac{ay^2}{2}$$

$(L, L) \longrightarrow (x, L)$

No hay Fzas. N.C. $\Rightarrow \sum W_{NC} = 0$

$\Rightarrow E(L,L) = E(x,L)$

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$$E = K + U$$

$$E(L,L) = E(x,L) \qquad U(x,y) = \frac{ax^2}{2} + \frac{ay^2}{2}$$

$$K(L,L) + U(L,L) = K(x,L) + U(x,L)$$

P7 Burna

$$\frac{aL^2}{2} + \cancel{\frac{aL^2}{2}} = \frac{1}{2}m(\dot{x}^2 + \cancel{\dot{y}^2}) + \frac{ax^2}{2} + \cancel{\frac{aL^2}{2}}$$

$$aL^2 = m\dot{x}^2 + ax^2$$

$$m\dot{x}^2 = a(L^2 - x^2)$$

$$\dot{x} = \sqrt{\frac{a}{m}(L^2 - x^2)}$$

$$\dot{x}_{max} = \sqrt{\frac{aL^2}{m}} \qquad (x = 0)$$

$$U = U(x,y)$$

$$-\frac{\partial U}{\partial x}\Big|_{\bar{x}} = 0 \longrightarrow \bar{x} \Rightarrow \text{ptos de equilibrio}$$

$$\frac{\partial^2 U}{\partial x^2}\Big|_{\bar{x}} \begin{cases} > 0 \longrightarrow \text{eq estable} \\ < 0 \longrightarrow \text{eq inestable} \end{cases}$$