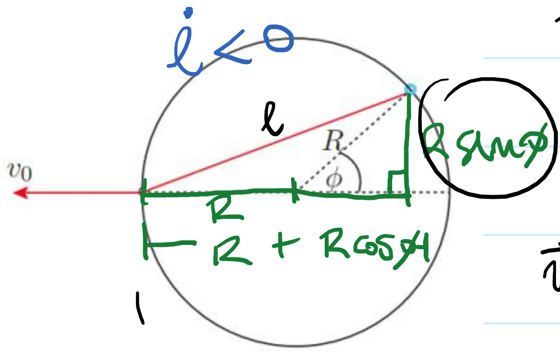
Auxiliar 2



$\vec{r} = \rho\hat{\rho} + z\hat{z}$; $\rho = R$, $z = 0$

$= R\hat{\rho}$ | $\dot{\rho} = 0$, $\dot{z} = 0$

$\vec{v} = R\dot{\phi}\hat{\phi}$ | $\dfrac{d\hat{\rho}}{dt} = \dot{\phi}\hat{\phi}$

$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k}$

$\ell^2 = R^2 + R^2 - 2R \cdot R\cos(\pi - \phi)$ | $\cos(\pi - \phi) = -\cos\phi$

$\ell^2 = (R\,\text{sen}\,\phi)^2 + (R + R\cos\phi)^2$

$= \underline{R^2\,\text{sen}^2\phi} + R^2 + \underline{R^2\cos^2\phi} + 2R^2\cos\phi$

$= 2R^2 + 2R^2\cos\phi$ | $\dot{\ell} = -v_0$

$\ell^2 = 2R^2(1 + \cos\phi)$ / $\dfrac{d}{dt}$ | $\ell = \sqrt{2}R\sqrt{1+\cos\phi}$

$\cancel{2}\ell\dot{\ell} = \cancel{2}R^2 \cdot -\text{sen}\,\phi \cdot \dot{\phi}$

$\sqrt{2}\cancel{R}\sqrt{1+\cos\phi}\,(+v_0) = R^{\cancel{2}} \cdot +\text{sen}\,\phi\,\dot{\phi}$

$$\dot{\phi} = \frac{\sqrt{2}v_0\sqrt{1+\cos\phi}}{R\,\text{sen}\,\phi}$$

$$\boxed{\vec{v} = \frac{\sqrt{2}v_0\sqrt{1+\cos\phi}}{\text{sen}\,\phi}\,\hat{\phi}}$$

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$$\rho = R \rightarrow \dot{\rho} = \ddot{\rho} = 0$$

$$\dot{\phi} = \frac{\sqrt{2}v_0\sqrt{1+\cos\phi}}{R\,\mathrm{sen}\,\phi} \qquad \dot{\phi}\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}v_0}{R}$$

$$\dot{\phi} = \frac{\sqrt{2}v_0}{R}\left[(1+\cos\phi)^{1/2}\,\mathrm{sen}^{-1}\phi\right] \quad / \quad \frac{d}{dt}$$

$$\ddot{\phi} = \frac{\sqrt{2}v_0}{R}\Big[\frac{1}{2}(1+\cos\phi)^{-1/2}\cdot -\cancel{\mathrm{sen}\,\phi}\,\dot{\phi}\,\cancel{\mathrm{sen}^{-1}\phi}$$

$$+(1+\cos\phi)^{1/2}(-1)\,\mathrm{sen}^{-2}\phi\cdot\cos\phi\,\dot{\phi}\Big]$$

$$\ddot{\phi} = \frac{\sqrt{2}v_0}{R}\left[-\frac{1}{2}\frac{1}{\sqrt{1+\cos\phi}} - \frac{\sqrt{1+\cos\phi}\cdot\cos\phi}{\mathrm{sen}^2\phi}\right]\dot{\phi}$$

$$\ddot{\phi}\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}v_0}{R}\left[-\frac{1}{2} - 0\right]\dot{\phi}\left(\frac{\pi}{2}\right)$$

$$= \frac{\cancel{\sqrt{2}}v_0}{R}\cdot -\frac{1}{\cancel{2}}\,\frac{\cancel{\sqrt{2}}v_0}{R} = -\frac{v_0^2}{R^2}$$

$$\vec{a} = (\cancel{\ddot{\rho}}^{\,0} - \rho\dot{\phi}^2)\hat{\boldsymbol{\rho}} + (\cancel{2\dot{\rho}\dot{\phi}}^{\,0} + \boxed{\rho\ddot{\phi}})\hat{\boldsymbol{\phi}} + \cancel{\ddot{z}\boldsymbol{k}}^{\,0}$$

$$\vec{a}\left(\frac{\pi}{2}\right) = -R\frac{2v_0^2}{R^2}\hat{\rho} - R\frac{v_0^2}{R^2}\hat{\phi}$$

$$\boxed{\vec{a}\left(\frac{\pi}{2}\right) = -\frac{2v_0^2}{R}\hat{\rho} - \frac{v_0^2}{R}\hat{\phi}} \qquad \|\vec{a}\| = \frac{v_0^2}{R}\sqrt{2^2+1^2} = \frac{\sqrt{5}v_0^2}{R}$$

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$$\frac{d}{dt} f(\phi(t)) = \frac{df}{d\phi} \cdot \frac{d\phi}{dt} \quad \dot{\phi}$$

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$$y(x) = y_0 - \frac{1}{\ell}(x-x_0)^2 \quad , \quad \|\vec{v}\| = v_0 \quad , \quad \vec{v} = \dot{x}\,\hat{x} + \dot{y}\,\hat{y}$$

$$\frac{dy}{dt} = \dot{y} = -\frac{1}{\ell}\,2(x-x_0)\cdot 1\cdot \dot{x}$$

$$\dot{y} = -\frac{2\dot{x}(x-x_0)}{\ell}$$

$$\vec{v} = \dot{x}\,\hat{x} - \frac{2\dot{x}(x-x_0)}{\ell}\,\hat{y}$$

$$\vec{v} = \dot{x}\left[1\,\hat{x} - \frac{2(x-x_0)}{\ell}\,\hat{y}\right]$$

$$\|\vec{v}\| = \dot{x}\sqrt{1^2 + \frac{4(x-x_0)^2}{\ell^2}} = v_0$$

$$\boxed{\dot{x} = \frac{v_0}{\sqrt{1 + \frac{4(x-x_0)^2}{\ell^2}}}}$$

$$\boxed{\dot{y} = -\frac{2\dot{x}(x-x_0)}{\ell} = -\frac{2(x-x_0)}{\ell}\,\frac{v_0}{\sqrt{1+\frac{4}{\ell^2}(x-x_0)^2}}}$$

Auxiliar 2

$$\vec{a} = a_t \hat{t} + a_n \hat{n}$$

$$= \left(\frac{d}{dt}\|\vec{v}\|\right)\hat{t} + \frac{\|\vec{v}\|^2}{R_c}\hat{n}$$

$$\|\vec{v}\| = v_0 \longrightarrow \frac{d}{dt}\|\vec{v}\| = \frac{d}{dt} v_0 = 0$$

$$\vec{a} = \frac{\cancelto{v_0^2}{\|\vec{v}\|^2}}{R_c}\hat{n} \, , \qquad a_n = \frac{v_0^2}{R_c}$$

$$R_c = \frac{(1+y')^{3/2}}{|y''|}$$

$$y(x) = y_0 - \frac{1}{\ell}(x-x_0)^2$$

$$y'(x) = -\frac{2}{\ell}(x-x_0)$$

$$y''(x) = -\frac{2}{\ell}$$

$$R_c = \frac{\left(1 - \frac{2}{\ell}(x-x_0)\right)^{3/2}\ell}{2}$$

$$\boxed{a_n = \frac{2v_0^2}{\left(1 - \frac{2}{\ell}(x-x_0)\right)^{3/2} \cdot \ell}}$$

Auxiliar 2

$$\vec{a} = \ddot{x}\,\hat{x} + \ddot{y}\,\hat{y}$$

$$\dot{x} = v_0 \left(1 + \frac{4}{\ell^2}(x - x_0)^2\right)^{-1/2} \quad \Big/ \frac{d}{dt}$$

$$\ddot{x} = v_0 \left(-\frac{1}{\cancel{2}}\right) \left(1 + \frac{4}{\ell^2}(x - x_0)^2\right)^{-3/2} \cdot \frac{\cancel{8}\,4}{\ell^2}(x - x_0)\,\dot{x}$$

$$= -\frac{4 v_0^2 (x - x_0)}{\left(1 + \frac{4}{\ell^2}(x - x_0)^2\right)^{3/2} \cdot \ell^2} \left(1 + \frac{4}{\ell^2}(x - x_0)^2\right)^{-1/2}$$

$$\boxed{\ddot{x} = -\frac{4 v_0^2 (x - x_0)}{\left[1 + \frac{4}{\ell^2}(x - x_0)^2\right]^2 \cdot \ell^2}}$$

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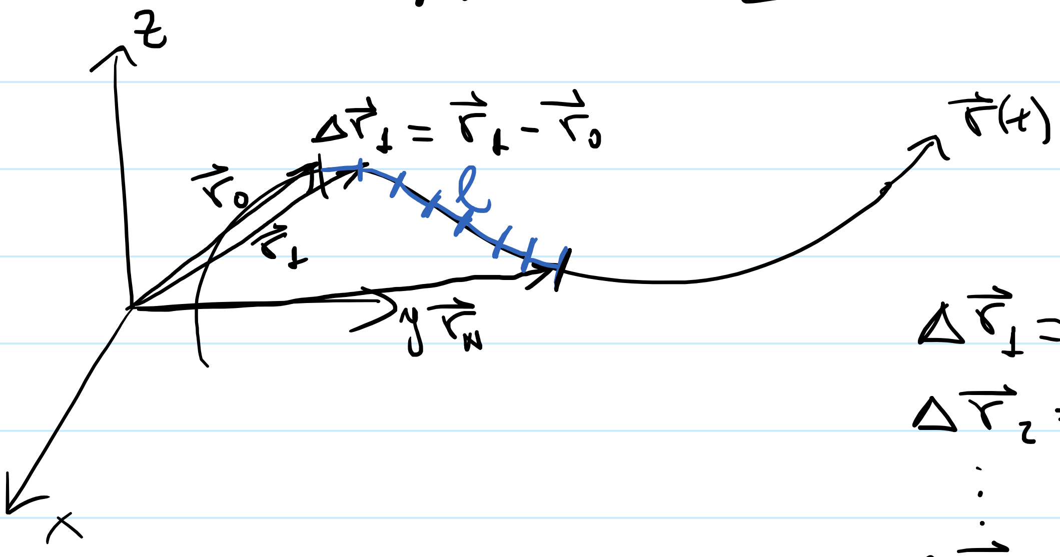

$$\Delta\vec{r}_1 = \vec{r}_1 - \vec{r}_0$$
$$\Delta\vec{r}_2 = \vec{r}_2 - \vec{r}_1$$
$$\vdots$$
$$\Delta\vec{r}_i = \vec{r}_i - \vec{r}_{i-1}$$

$$\ell \sim \|\Delta\vec{r}_1\| + \|\Delta\vec{r}_2\| + \cdots + \|\Delta\vec{r}_N\|$$
$$\sim \sum_{i=1}^{N} \|\Delta\vec{r}_i\|$$

$$\ell = \lim_{N\to\infty} \sum_{i=1}^{N} \|\Delta\vec{r}_i\| = \boxed{\int \|d\vec{r}\| = \ell}$$

$$\ell = \int \frac{\|d\vec{r}\|}{\|dt\|} \cdot dt = \int \left\|\frac{d\vec{r}}{dt}\right\| dt = \int v\, dt = \int \|\vec{v}\|\, dt$$

$$\ell = \int \|d\vec{r}\| \frac{dt}{dt} \cdot \frac{dx}{dx} = \int \left\|\frac{d\vec{r}}{dt}\right\| \cdot \frac{1}{\frac{dx}{dt}} \cdot dx$$

$$= \int \|\vec{v}\| \cdot \frac{1}{\dot{x}}\, dx = v_0 \int \frac{1}{\dot{x}}\, dx$$

Auxiliar 2

$$l = v_0 \int \frac{1}{\dot{x}} dx = \cancel{v_0} \int \frac{1}{\cancel{v_0}} \sqrt{1 + \frac{4}{\ell^2}(x-x_0)^2}\, dx$$

$$= \int \sqrt{1 + \frac{4}{\ell^2}(x-x_0)^2}\, dx$$

$$u^2 = \frac{4}{\ell^2}(x-x_0)^2 \rightarrow$$ cambio de variable

$$\int \sqrt{1+u^2}\, du$$