# Formulario de Mecánica

Autor: Edgardo Rosas Cárcamo.

## 1 Coordenadas Cartesianas: $(x, y, z)$

### 1.1 Cinemática

| | |
|---|---|
| Posición: | $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ |
| Velocidad: | $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$ |
| Aceleración: | $\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$ |

### 1.2 Diferenciales

| | |
|---|---|
| Línea: | $d\vec{l} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$ |
| Superficie: | $d\vec{S} = dydz\hat{\imath} + dxdz\hat{\jmath} + dxdy\hat{k}$ |
| Volumen | $dV = dxdydz$ |

### 1.3 Operadores

| | | |
|---|---|---|
| Gradiente: | $\nabla\psi$ | $= \frac{\partial\psi}{\partial x}\hat{\imath} + \frac{\partial\psi}{\partial y}\hat{\jmath} + \frac{\partial\psi}{\partial z}\hat{k}$ |
| Divergencia: | $\nabla\cdot\vec{F}$ | $= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$ |
| Rotor: | $\nabla\times\vec{F}$ | $= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$ |
| Laplaciano: | $\nabla^2\psi$ | $= \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$ |

# Formulario de Mecánica

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## 1 Coordenadas Cilíndricas: $(\rho, \phi, z)$

### 1.1 Cinemática

| | |
|---|---|
| Posición: | $\vec{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\boldsymbol{k}}$ |
| Velocidad: | $\vec{v} = \dot{\rho}\hat{\boldsymbol{\rho}} + \rho\dot{\phi}\hat{\boldsymbol{\phi}} + \dot{z}\hat{\boldsymbol{k}}$ |
| Aceleración: | $\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\boldsymbol{\rho}} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\boldsymbol{\phi}} + \ddot{z}\hat{\boldsymbol{k}}$ |
| Aceleración: | $\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{d}{dt}\left(\rho^2\dot{\phi}\right)\hat{\boldsymbol{\phi}} + \ddot{z}\hat{\boldsymbol{k}}$ |

### 1.2 Diferenciales

| | |
|---|---|
| Línea: | $d\vec{l} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{k}}$ |
| Superficie: | $d\vec{S} = \rho d\phi dz\hat{\boldsymbol{\rho}} + d\rho dz\hat{\boldsymbol{\phi}} + \rho d\rho d\phi\hat{\boldsymbol{k}}$ |
| Volumen: | $dV = \rho d\rho d\phi dz$ |

### 1.3 Operadores

| | |
|---|---|
| Gradiente: | $\nabla\psi = \frac{\partial\psi}{\partial\rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial\psi}{\partial z}\hat{\boldsymbol{k}}$ |
| Divergencia: | $\nabla\cdot\vec{F} = \frac{1}{\rho}\frac{\partial(\rho F_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$ |
| Rotor: | $\nabla\times\vec{F} = \left[\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right]\hat{\boldsymbol{\rho}} + \left[\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right]\hat{\boldsymbol{\phi}} + \frac{1}{\rho}\left[\frac{\partial(\rho F_\phi)}{\partial\rho} - \frac{\partial F_\rho}{\partial\phi}\right]\hat{\boldsymbol{k}}$ |
| Laplaciano: | $\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}$ |

# Formulario de Mecánica

Autor: Edgardo Rosas Cárcamo.

## 1 Coordenadas Esféricas: $(r, \theta, \phi)$

### 1.1 Cinemática

| | |
|---|---|
| Posición: | $\vec{r} = r\hat{\boldsymbol{r}}$ |
| Velocidad: | $\vec{v} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} + r\dot{\phi}\sin\theta\hat{\boldsymbol{\phi}}$ |
| Aceleración: | $\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\boldsymbol{\theta}} + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\phi}\dot{\theta}\cos\theta)\hat{\boldsymbol{\phi}}$ |
| Aceleración: | $\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\dot{\phi}\sin^2\theta\right)\hat{\boldsymbol{\phi}}$ |

### 1.2 Diferenciales

| | |
|---|---|
| Linea | $d\vec{l} = dr\hat{\boldsymbol{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$ |
| Superficie | $d\vec{S} = r^2\sin\theta d\theta d\phi\hat{\boldsymbol{r}} + r\sin\theta dr d\phi\hat{\boldsymbol{\theta}} + r dr d\theta\hat{\boldsymbol{\phi}}$ |
| Volumen | $dV = r^2\sin\theta dr d\theta d\phi$ |

### 1.3 Operadores

| | |
|---|---|
| Gradiente: | $\nabla\psi = \frac{\partial\psi}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\hat{\boldsymbol{\phi}}$ |
| Divergencia: | $\nabla\cdot\vec{F} = \frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta F_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$ |
| Rotor: | $\nabla\times\vec{F} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta F_\phi)}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right]\hat{\boldsymbol{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial(rF_\phi)}{\partial r}\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial(rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$ |
| Laplaciano: | $\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$ |

# Formulario de Mecánica

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## 1 Transformación de Coordenadas

### 1.1 Hacia las coordenadas cartesianas: $(x, y, z)$

| | | | |
|---|---|---|---|
| Cilíndricas $(\rho, \phi, z)$ | $x = \rho\cos\phi$ | $y = \rho\sin\phi$ | $z = z$ |
| Esféricas $(r, \theta, \phi)$ | $x = r\sin\theta\cos\phi$ | $y = r\sin\theta\sin\phi$ | $z = r\cos\phi$ |

### 1.2 Hacia las coordenadas cilíndricas: $(\rho, \phi, z)$

| | | | |
|---|---|---|---|
| Cartesianas $(x, y, z)$ | $\rho = \sqrt{x^2 + y^2}$ | $\phi = \arctan(y/x)$ | $z = z$ |
| Esféricas $(r, \theta, \phi)$ | $\rho = r\sin\theta$ | $\phi = \phi$ | $z = r\cos\theta$ |

### 1.3 Hacia las coordenadas esféricas: $(r, \theta, \phi)$

| | | | |
|---|---|---|---|
| Cartesianas $(x, y, z)$ | $r = \sqrt{x^2 + y^2 + z^2}$ | $\theta = \arctan\left(\sqrt{x^2 + y^2}/z\right)$ | $\phi = \arctan(y/x)$ |
| Cilíndricas $(\rho, \phi, z)$ | $r = \sqrt{\rho^2 + z^2}$ | $\theta = \arctan(\rho/z)$ | $\phi = \phi$ |

## 2 Transformación de vectores unitarios

### 2.1 Coordenadas cilíndricas

| | | |
|---|---|---|
| $\hat{\boldsymbol{\rho}} = \cos\phi\hat{\boldsymbol{\imath}} + \sin\phi\hat{\boldsymbol{\jmath}}$ | $\hat{\boldsymbol{\phi}} = -\sin\phi\hat{\boldsymbol{\imath}} + \cos\phi\hat{\boldsymbol{\jmath}}$ | $\hat{\boldsymbol{k}} = \hat{\boldsymbol{k}}$ |

### 2.2 Coordenadas Esféricas

| | | |
|---|---|---|
| $\hat{\boldsymbol{r}} = \sin\theta\cos\phi\hat{\boldsymbol{\imath}} + \sin\theta\sin\phi\hat{\boldsymbol{\jmath}} + \cos\theta\hat{\boldsymbol{k}}$ | $\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\hat{\boldsymbol{\imath}} + \cos\theta\sin\phi\hat{\boldsymbol{\jmath}} - \sin\theta\hat{\boldsymbol{k}}$ | $\hat{\boldsymbol{\phi}} = -\sin\phi\hat{\boldsymbol{\imath}} + \cos\phi\hat{\boldsymbol{\jmath}}$ |