Auxiliar 1

CINEMATICA

$\vec{r}(t)$, $\vec{v}(t)$, $\vec{a}(t)$

$$\left.\begin{aligned}\vec{v} &= \frac{d\vec{r}(t)}{dt}\\ \vec{a} &= \frac{d\vec{v}(t)}{dt}\end{aligned}\right\} \quad \vec{a}(t) = \frac{d^2\vec{r}(t)}{dt^2}$$

D

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2}$$

$$\boxed{\begin{aligned} x(t_0) &= x_0 \\ v(t_0) &= v_0 \end{aligned}}$$

$$\alpha(t) = -\frac{g}{\ell}\,\mathrm{sen}(\theta(t))$$

$$\downarrow$$

$$\frac{d\dot{\theta}}{dt} = \frac{d^2\theta}{dt^2}$$

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$$a(t) = v(t) \cdot t$$

$$\downarrow$$

$$\frac{dv(t)}{dt} = v(t) \cdot t \iff \frac{dv(t)}{v(t)} = t \cdot dt$$

$$\int_{v(t_0)}^{v(t)} \frac{dv(t)}{v(t)} = \int_{t_0=0}^{t} t\,dt$$

$$v(t_0) = v_0$$

$$\ln(v(t))\Big|_{v_0}^{v(t)} = \frac{t^2}{2}\Big|_0^t$$

$$\ln(v(t)) - \ln(v_0) = \frac{t^2}{2}$$

$$\ln\left(\frac{v(t)}{v_0}\right) = \frac{t^2}{2}$$

$$\frac{v(t)}{v_0} = e^{t^2/2} \Rightarrow v(t) = v_0\, e^{t^2/2}$$

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$$\frac{dx(t)}{dt} = v_0\, e^{t^2/2}$$

$$dx(t) = v_0\, e^{t^2/2} \cdot dt$$

$$\int_{\substack{x(t_0)\\ \parallel \\ x_0}}^{x(t)} dx(t) = v_0 \int_{t_0=0}^{t} e^{t^2/2}\, dt$$

$$x(t) - x_0 = v_0 \int_0^t e^{t^2/2}\, dt$$

$$\boxed{x(t) = x_0 + v_0 \int_0^t e^{t^2/2}\, dt}$$

$$[dv] = \frac{L}{T}$$

$$[dt] = T$$

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$$a(t) = -\beta x^2(t)$$

$$\frac{dv(t)}{dt} = -\beta x^2(t)$$

Truquito de Mec.

$$\frac{dv(t)}{dt} \cdot \frac{dx(t)}{dx(t)} = \frac{dv(t)}{dx} \cdot \frac{dx(t)}{dt} = \boxed{\frac{dv(x)}{dx} \cdot v(x) = a}$$

$$\frac{dv(x)}{dx} \cdot v(x) = -\beta x^2$$

$$\int_{v(x_0) = v_0}^{v(x)} v \cdot dv = -\beta \int_{x_0}^{x} x^2\, dx$$

$$\left.\frac{v^2}{2}\right|_{v_0}^{v(x)} = -\beta \left.\frac{x^3}{3}\right|_{x_0}^{x}$$

$$\left(v^2(x) - v_0^2\right) = -\frac{2\beta}{3}\left(x^3 - x_0^3\right)$$

$$\boxed{v(x) = \sqrt{v_0^2 - \frac{2\beta}{3}\left(x^3 - x_0^3\right)}}$$

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$$a(t) = -2\gamma\, v(t) - \omega_0^2\, x(t)$$

$$\frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} = -2\gamma \frac{dx(t)}{dt} - \omega_0^2\, x(t)$$

$$\frac{d^2x(t)}{dt^2} + 2\gamma \frac{dx(t)}{dt} + \omega_0^2\, x(t) = 0$$

Ansatz: $x(t) = e^{\lambda t} \longrightarrow \dfrac{dx(t)}{dt} = \lambda e^{\lambda t} \rightarrow \dfrac{d^2x(t)}{dt^2} = \lambda^2 e^{\lambda t}$

$$\lambda^2 \cancel{e^{\lambda t}} + 2\gamma\lambda \cancel{e^{\lambda t}} + \omega_0^2 \cancel{e^{\lambda t}} = 0$$

$$\lambda^2 + 2\gamma\lambda + \omega_0^2 = 0 \Rightarrow \lambda = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2}$$

$$\lambda = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2} \quad \begin{cases} \lambda_+ = -\gamma + \sqrt{\gamma^2 - \omega_0^2} \\ \lambda_- = -\gamma - \sqrt{\gamma^2 - \omega_0^2} \end{cases}$$

$$x(t) = A_+ e^{\lambda_+ t} + A_- e^{\lambda_- t}$$

$$\boxed{x(t) = A_+ e^{\left(-\gamma + \sqrt{\gamma^2 - \omega_0^2}\right)t} + A_- e^{\left(-\gamma - \sqrt{\gamma^2 - \omega_0^2}\right)t}}$$

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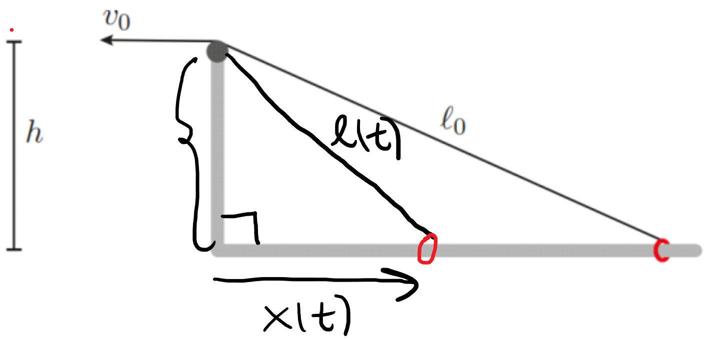

$$\frac{d\ell(t)}{dt} = \dot{\ell}(t) = -v_0$$

$$\frac{d\ell}{dt} = -v_0$$

$$\int_{\ell(0) = \ell_0}^{\ell(t)} d\ell = -\int_0^t v_0 \cdot dt$$

$$\ell(t) - \ell_0 = -v_0 t$$

$$\boxed{\ell(t) = \ell_0 - v_0 t}$$

Pitágoras: $h^2 + x^2(t) = \ell^2(t)$

$$x^2(t) = \ell^2(t) - h^2$$

$$x^2(t) = (\ell_0 - v_0 t)^2 - h^2$$

$$\boxed{x(t) = \sqrt{(\ell_0 - v_0 t)^2 - h^2}}$$

$$\frac{dx}{dt} \overset{!}{<} 0 \longrightarrow +$$