# Tema 6 – Reactores
Reactors

CI7115 – Biotecnología Ambiental
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# Conceptos de termodinámica en biotecnología ambiental

**Conservation of Energy - the First Law**

- energy balance on a bioreactor (digesters, landfills, compost)
- heat of metabolism
- energy recovery

**Free Energy - the Second Law**

- feasible reactions
- thermodynamic preferences
- relationship between free energy and yield

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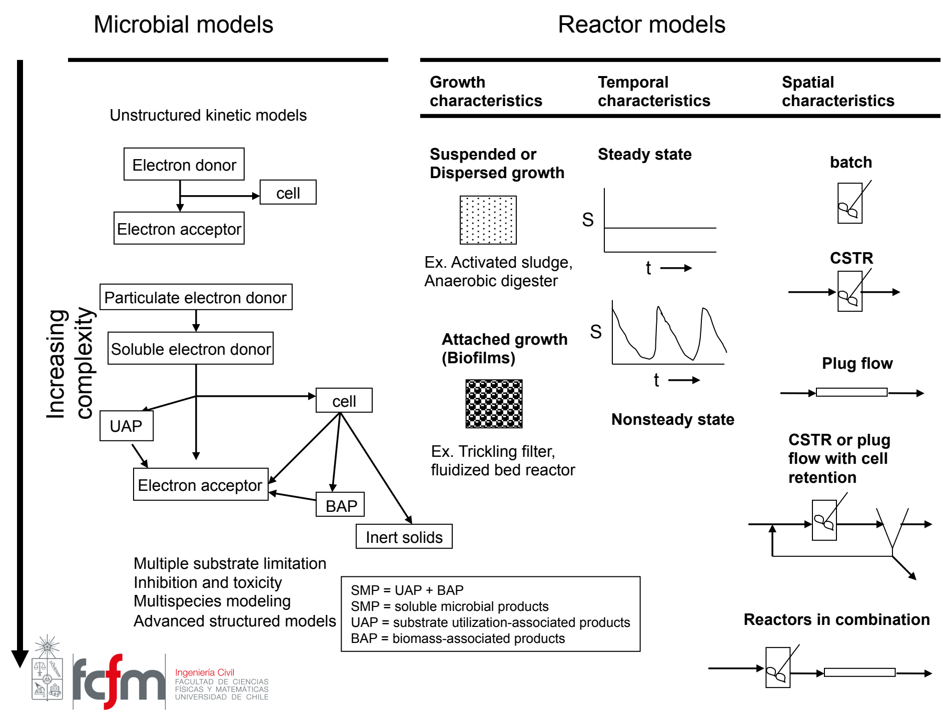
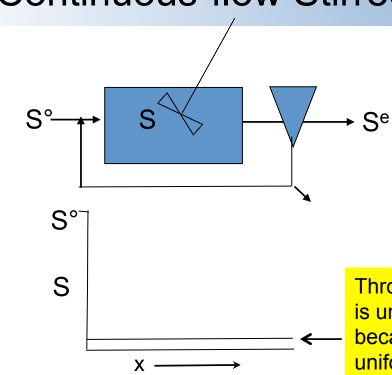
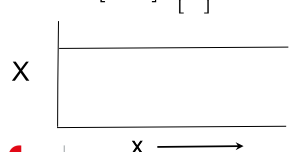



# Continuous-flow Stirred Tank Reactor (CSTR)




Throughout reactor, q is uniform and low because S is low and uniform everywhere.

$$q = \hat{q}\left[\frac{S}{K+S}\right] = \left[\frac{\hat{q}}{K}\right]S = K_1 S \text{ (for low S)}$$




Advantages:
- Resistant to toxics and shock loads because of rapid dilution
- Good for high pH waste where $CO_2$ provides neutralization
- Good for low pH volatile fatty acid wastes because they are quickly diluted and degraded
- Has spatially uniform oxygen demand.

Disadvantages:
- Less efficient
- Poor settling biomass (filamentous branching) due to uniform low substrate level

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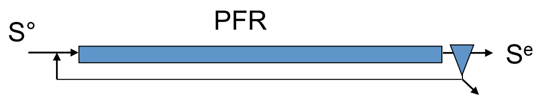
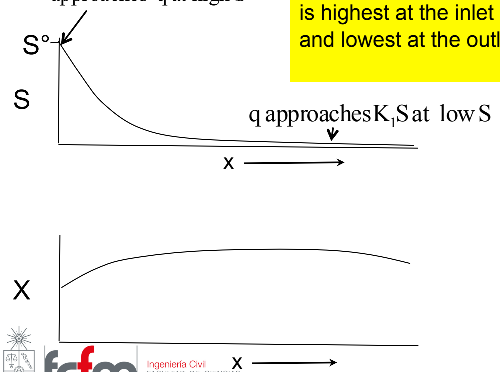
# Plug Flow Reactors (PFR)




$q = \hat{q}[(S^\circ/(K + S^\circ)]$

approaches $\hat{q}$ at high S°

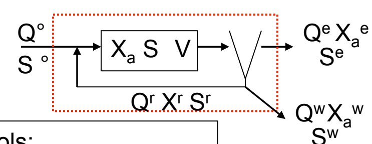
q varies throughout the reactor: it is maximum at the inlet and lowest at the outlet because S is highest at the inlet and lowest at the outlet.




Advantages:
- More efficient
- Variable substrate levels select for better settling biomass

Disadvantages:
- Can be more sensitive to toxics and shock loads because there is less dilution at the reactor entrance
- Oxygen demand is not uniform

# Dispersed Growth - CSTR ± recycle




Symbols:

Q - flowrate ($m^3/d$)
X- suspended solids concentration (i.e., biomass, mg/L = $g/m^3$
S - substrate concentration, mg/L = $g/m^3$
V - reactor volume, $m^3$

Superscripts - location
e = effluent
° = influent
r = recycle
w = waste

Subscripts - type of suspended solids X
a = active organisms (organic)
i = inert biomass (organic)
in = inorganic suspended solids

Assumptions:
1. Completely mixed (CSTR)
2. Soluble substrate
3. $X_a$ in settling tank = 0 (no storage of solids outside of the reactor)
4. Q°, S° do not vary with time.
5. V is constant.

*For a batch system (no flow), a mass balance on substrate gives*

In: 0
Out: 0
Source: 0
Sink: $qX_aV_L\Delta t$

$\Delta M = -qX_aV_L\Delta t$ (constant liquid volume)

For a nonvolatile substrate: $\Delta M/V_L = \Delta S$

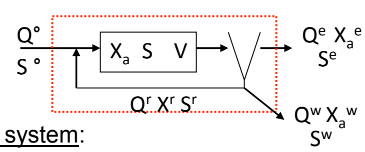
$$\frac{-dS}{dt} = qX_a \qquad \frac{-dS}{dt} = \frac{\hat{q}SX_a}{K+S}$$

A mass balance on biomass gives

$$\frac{dX_a}{dt} = Y\left(\frac{-dS}{dt}\right) \qquad X_a = X_{ao} + Y(S_o - S)$$

$$t = -\frac{1}{\hat{q}}\left(\left[\frac{-K}{X_{ao}+YS_o} - \frac{1}{Y}\right]\ln[X_{ao}+YS_o-YS] + \left[\frac{K}{X_{ao}+YS_o}\right]\ln\left[\frac{SX_{ao}}{S_o}\right] + \frac{1}{Y}[\ln X_{ao}]\right)$$

*For a CSTR (completely mixed with flow)*




Mass balance on Xa over ΔT for entire system:

in: 0

out: $(X_a^eQ^e + X_a^wQ^w)\Delta t$

source: $\left(\frac{Y\hat{q}SX_a}{K+S}\right)V\Delta t$ growth ($\mu = Yq$)

sink: $bX_aV\Delta t$ decay

Accumulation = in - out + source – sink

$$V\Delta X_a = 0 - (Q^eX_a^e + Q^wX_a^w)\Delta t + \left(\frac{Y\hat{q}SX_a}{K+S}\right)V\Delta t - bX_aV\Delta t$$

Now, Divide by $V\Delta t$

$$\frac{dX_a}{dt} = \frac{-\left(Q^e X_a^e + Q^w X_a^w\right)}{V} + \left(\frac{Y \hat{q} S X_a}{K + S}\right) - bX_a$$

Define the **solids retention time $\theta_x$** (also called mean cell residence time, solids residence time, sludge age) where:

$$\theta_x = \frac{VX_a}{\left(Q^e X_a^e + Q^w X_a^w\right)} = \frac{\text{mass of active bacteria}}{\text{rate at which active bacteria are removed from system}}$$

Note that $\theta_x$ has units of days. It represents the average time solids remain in the system.

At steady state: $\frac{dX_a}{dt} = 0$

Note that $\theta_x$ is equal to $1/\mu$. What does that mean?

So the mass balance becomes: $\frac{\left(Q^e X_a^e + Q^w X_a^w\right)}{VX_a} = \left(\frac{Y \hat{q} S}{K + S}\right) - b$

or $\frac{1}{\theta_x} = \left(\frac{Y \hat{q} S}{K + S}\right) - b$

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Solving for S:

$$S = \frac{K\left(1 + b\theta_x\right)}{\theta_x\left(Y\hat{q} - b\right) - 1}$$

For soluble substrate, with no substrate utilization in the settling tank, $S^e = S$.

Efficiency of substrate utilization: $E = 100\left(\frac{S^o - S}{S^o}\right)$

For $S \to \infty$, $\frac{1}{\theta_x} = \frac{1}{\left[\theta_x^{min}\right]_{lim}} = Y\hat{q} - b$

For $S = S^o$, $\frac{1}{\theta_x} = \frac{1}{\theta_x^{min}} = \frac{Y\hat{q}S^o}{K + S^o} - b$

If $\theta_x < \theta_x^{min}$, organism washout occurs

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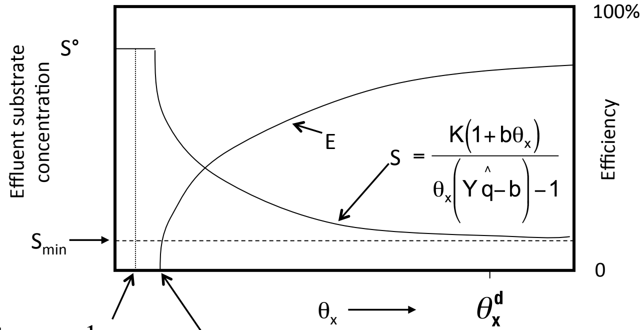
Requirements for biological treatment:

1. Efficiency
2. Reliability




$$\left[\theta_x^{min}\right]_{lim} = \frac{1}{Y\hat{q}-b}$$

$$\theta_x^{min} = \left[\frac{Y\hat{q}S^o}{K+S^o} - b\right]^{-1}$$

Below which washout occurs



## Safety Factor

Design for reliability - safety factor concept:

$$\theta_x^d = S.F.\left(\theta_x^{min}\right)_{lim}$$

| Loading classification | Implied safety factor (S.F.) |
|---|---|
| Conventional loading | 20-80 |
| High rate loadings | 3-10 |
| Low rate loadings | >100 |

Factors affecting choice of safety factor:

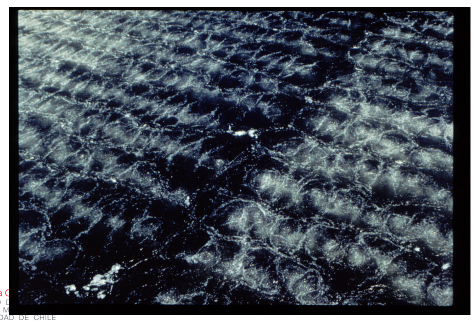
1. Expected temperature variations.
2. Expected wastewater variations (flowrate, organic conc. & composition).
3. Possible presence of inhibitors.
4. Level of operator skill.
5. Efficiency and reliability required.
6. Confidence in design coefficients.

# CSTRs with Cell Retention

The above equations are used to model dispersed growth CSTR systems with cell retention.

Applications include:

- Activated sludge (aerobic heterotrophs±nitrifiers)
- Nitrification dispersed growth
- Membrane bioreactors



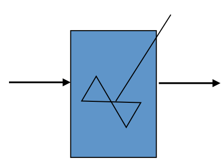
# CSTRs without Cell Retention: θx = θ = V/Q°.

Many systems have no retention of microorganisms. For such systems, $\theta_x = \theta = V/Q°$. The hydraulic residence time equals the solids residence time.

Applications of dispersed growth theory with no cell retention include:

- Chemostats - a useful research tool
- Lagoons

- Digesters



Dilution rate D= 1/θ

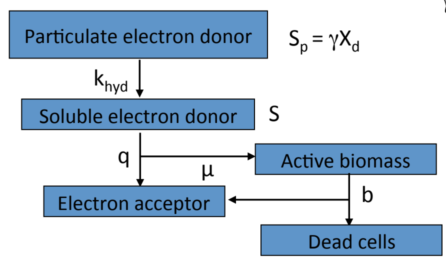
# El efecto de los sólidos suspendidos

**Why do we need a suspended solids balance?**

- Biosolids production (amount for further treatment or disposal, nutrient requirements)
- Operational control (effluent quality)
- Reactor design (sizing reactors, choosing the right size aeration equipment)

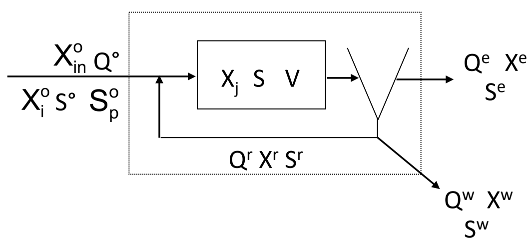
**We begin with a microbial model**:

$\gamma$ = COD/wt ratio




$$\frac{-dS_p}{dt} = k_{hyd} S_p$$

# Now we add a reactor model:

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S - substrate concentration, mg/L = g/$m^3$
V - reactor volume, $m^3$

Superscripts - location
e = effluent
° = influent
r = recycle
w = waste

Subscripts - type of suspended solids X or substrate S
a = active organisms
i = inert organic materials
d = degradable
v = S organics (volatile suspended solids)
in = inorganic
p = particulate

Total suspended solids (TSS) = Mixed Liquor Suspended Solids (MLSS)
= $\mathbf{X = X_v + X_{in}}$ = *volatile* + *inorganic*

Volatile suspended solids (VSS) = Mixed Liquor Volatile Suspended Solids (MLVSS) = $\mathbf{X_v = X_a + X_i + X_d}$ = *active* + *inert* + *biodegradable*

where:
$X_a$ = active organisms <u>produced from the consumption of soluble substrate S</u>
$X_d$ = biodegradable suspended solids = $S_p/\gamma$ so that $\mathbf{S_p = \gamma X_d}$ where
$\gamma$= COD/WT ratio of $X_d$. *(g COD/L = g COD/g VSS x g VSS/L)*

<u>Assumptions</u>:
(1) $X_{\text{settling tank}} = 0$
(2) Steady-state operation:

$$\frac{dX_a}{dt} = \frac{dQ^o}{dt} = \frac{dX^o}{dt} = \frac{dS^o}{dt} = \frac{dS_p^o}{dt} = \frac{dQ^e}{dt} = \frac{dX^e}{dt} = \frac{dS^e}{dt} = \frac{dS}{dt} = 0$$

(3) $S^e = S$

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**Mass balance**: change in storage = in - out + source -sink
Consider general suspended solids $X_j$:

$$V\Delta X_j = \underbrace{Q^o X_j^o \Delta t}_{\text{in}} - \underbrace{\left(Q^w X_j^w + Q^e X_j^e\right)\Delta t}_{\text{out}} + (\text{mass production rate})\Delta t - (\text{mass consumption rate})\Delta t$$

$$V\frac{dX}{dt} = \underbrace{Q^o X_j^o}_{\text{in}} - \underbrace{\left(Q^w X_j^w + Q^e X_j^e\right)}_{\text{out}} + \text{mass production rate} - \text{mass consumption rate}$$

<u>At steady state</u>:

$$\underbrace{Q^w X_j^w + Q^e X_j^e}_{\text{out}} = \underbrace{Q^o X_j^o}_{\text{in}} + \text{mass production rate} - \text{mass consumption rate}$$

but $\boxed{\theta_x = \dfrac{VX_j}{Q^e X_j^e + Q^w X_j^w}}$ *(previously determined)*

so $$\frac{VX_j}{\theta_x} = Q^w X_j^w + Q^e X_j^e = Q^o X_j^o + \text{mass production rate} - \text{mass consumption rate}$$

and $\boxed{\dfrac{VX_j}{\theta_x} \text{ is the mass removal rate of } X_j}$ *(i.e., outflow from the system)*

*Mass / Time*

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Assumption:

All of the different types of suspended solids have the same solids retention time.

$$(\theta_x)_{S_P} = (\theta_x)_{X_a} = (\theta_x)_{X_i} = (\theta_x)_{X_{in}} = \theta_x$$

Consider each of the different types of suspended solids in the reactor at steady state:

**Xin** - *inorganic suspended solids:*

*In Out*

$$Q^o X_{in}^o = \frac{X_{in} V}{\theta_x} \quad \text{but} \quad \frac{V}{Q^o} = \theta$$

$$\boxed{X_{in} = \frac{\theta_x}{\theta} X_{in}^o}$$

$\theta_x/\theta$ is a kind of concentrating factor

For Xin, there is no production and no removal within the reactor (conservative)

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**Sp or $X_d$** - *particulate biodegradable COD* ($S_p = \gamma X_d$ )

*(g COD/L = g COD/g VSS x g VSS/L)*

Assume a first order hydrolysis: $\frac{-dS_p}{dt} = k_{hyd} S_p$

*in source sink out*

$$Q^o S_p^o + 0 - k_{hyd} S_p V = \frac{S_p V}{\theta_x}$$

$$S_p V \left( \frac{1}{\theta_x} + k_{hyd} \right) = Q^o S_p^o$$

$$\frac{S_p (1 + k_{hyd} \theta_x)}{\theta_x} = \frac{S_p^o}{\theta} \quad so \quad \boxed{S_p = \frac{\theta_x}{\theta} \frac{S_p^o}{(1 + k_{hyd} \theta_x)}}$$

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Dividing both sides by $\gamma$:

$$X_d = \frac{\theta_x}{\theta}\frac{X_d^o}{(1+k_{hyd}\theta_x)}$$

We also note that a mass balance on soluble substrate gives:

In out sink source

$$O = Q^\circ(S^\circ - S) - qX_aV + k_{hyd}S_pV$$

$$O = S^\circ - S - qX_a\theta + k_{hyd}S_p\theta$$

Substituting in the expression for $S_p$, we obtain:

$$S_p = \frac{\theta_x}{\theta}\frac{S_p^o}{(1+k_{hyd}\theta_x)}$$

$$0 = S^o - S - qX_a\theta + k_{hyd}\theta_x\left(\frac{S_p^o}{1+k_{hyd}\theta_x}\right)$$

In the absence of particulate substrate ($S_p$=0), we have: $0 = S^\circ - S - qX_a\theta$

Comparing the previous two equations, we see that hydrolysis of particulate substrate effectively increases S°. We can define an effective influent substrate concentration:

$$S_{eff}^o = S^o + \frac{k_{hyd}\theta_x}{1+k_{hyd}\theta_x}S_p^o$$

already soluble — will be soluble

$S_{eff}$ is how we account for contribution from particulates

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This observation is helpful in preparing a mass balance on active biomass:

**$X_a$** - *active organisms from soluble substrate S*:

In source sink out

$$0 + Y(S_{eff}^o - S^e)Q^o - bX_aV = \frac{X_aV}{\theta_x}$$

$$X_a = \frac{\theta_x}{\theta}\left[\frac{Y(S_{eff}^o - S^e)}{1+b\theta_x}\right]$$

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$X_i$ = *inert matter in the influent + dead cells from decay of $X_a$:*

Let $f_d$ **= biodegradable fraction of the active cells (~0.8)**.

Then $1-f_d$ **= inert fraction of the active cells (~0.2).**

In source sink out

$$Q^o X_i^o + (1-f_d) b X_a V - 0 = \frac{X_i V}{\theta_x}$$

$$X_i = \frac{\theta_x}{\theta}\left[X_i^o + (1-f_d) b\theta_x \frac{Y(S_{eff}^o - S^e)}{1+b\theta_x}\right]$$

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$$X_i = \frac{\theta_x}{\theta}\left[X_i^o + (1-f_d) b\theta_x \frac{Y(S_{eff}^o - S^e)}{1+b\theta_x}\right]$$

# $X_v$ - *volatile suspended solids:*

$$X_a = \frac{\theta_x}{\theta}\left[\frac{Y(S_{eff}^o - S^e)}{1+b\theta_x}\right]$$

$X_v = X_d + X_a + X_i$ *Adding it all up!*

$$X_d = \frac{\theta_x}{\theta}\frac{X_d^o}{(1+k_{hyd}\theta_x)}$$

$$X_v = \frac{\theta_x}{\theta}\left[X_i^o + \frac{X_d^o}{1+k_{hyd}\theta_x} + \frac{Y(S_{eff}^o - S^e)(1+(1-f_d)b\theta_x)}{1+b\theta_x}\right]$$

For $f_d = 0.8$,

$$X_v = \frac{\theta_x}{\theta}\left[X_i^o + \frac{X_d^o}{1+k_{hyd}\theta_x} + \frac{Y(S_{eff}^o - S^e)(1+0.2b\theta_x)}{1+b\theta_x}\right]$$

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Solids resulting from biological activity:

$$X_{syn} = X_a + X_{i,syn} = \frac{\theta_x}{\theta}\left[\frac{Y\left(S^o_{eff} - S^e\right)\left(1 + 0.2b\theta_x\right)}{1 + b\theta_x}\right]$$

We see that:

$$Y_{eff} = \frac{Y\left(1 + 0.2b\theta_x\right)}{1 + b\theta_x}$$

Because Y is related to $f_s$:

$$f_s = f_s^o\left(\frac{1 + 0.2b\theta_x}{1 + b\theta_x}\right)$$

For the case with no recycle:
$Q^w=0$, $Q^e=Q^o$, $X^e=X$

$$\theta_x = \frac{VX_j}{Q^w X_j^w + Q^e X_j^e} = \frac{VX_j^e}{Q^o X_j^e} = \theta$$

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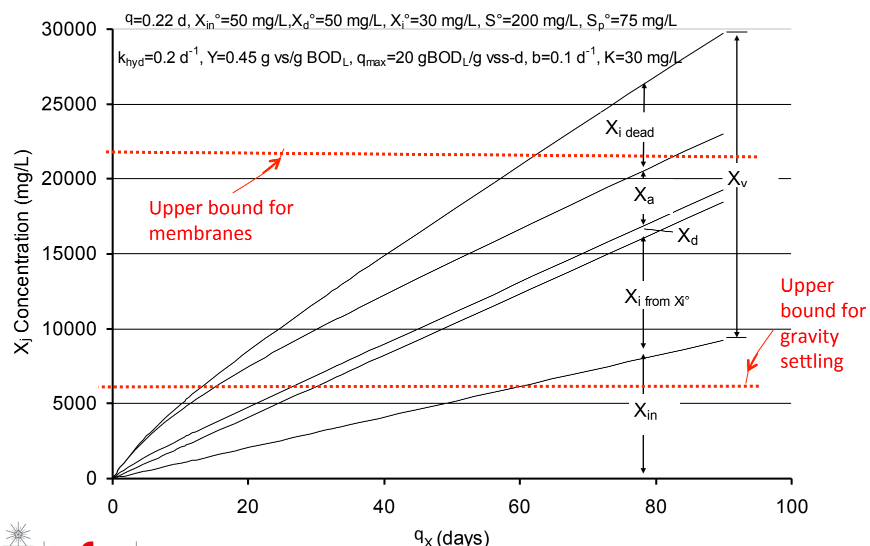
## Example: change in $X_j$ with $\theta_x$ for a fixed $\theta$

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## Steady state approach for CSTR design

1. Determine waste characteristics and effluent requirements: $Q°$, $S°_{TOTAL}$, $S°$, $S_p°$, $X_i°$, $X_{in}°$, $S^e_{max}$

   where: $S°_{TOTAL}$ = $BOD_L$ of the raw waste
   $S°$ = $BOD_L$ of the filtered waste
   $S°_{TOTAL} - S° = S_p°$
   $X_d°$ is computed from the COD/WT ratio ($\gamma$) for volatile suspended solids. ($X_d° = S_p°/\gamma$)

2. Select coefficients and design factors.

   $Y$, $\hat{q}$, $b$, $K$, S.F., $X_v$ (if cells are retained), $k_{hyd}$, SMP kinetic parameters
   Soluble microbial products

3. Calculate $\left[\theta_x^{min}\right]_{lim}$.

$$\left[\theta_x^{min}\right]_{lim} = \frac{1}{Y\hat{q} - b}$$

Apply safety factor

4. Select $\theta_x^d = S.F. \bullet \left[\theta_x^{min}\right]_{lim}$.

5. Solve:

$$S^e = \frac{K\left(1 + b\theta_x^d\right)}{\theta_x^d\left(Y\hat{q} - b\right) - 1}$$

If $S^e > S^e_{max}$, increase S.F.

6. Solve for $S^o_{eff}$.

$$S^o_{eff} = S^o + \frac{k_{hyd}\theta_x}{1 + k_{hyd}\theta_x} S^o_p$$

7. Use the expression:

$$X_v = \frac{\theta_x}{\theta}\left[X^o_i + \frac{X^o_d}{1 + k_{hyd}\theta_x} + \frac{Y\left(S^o_{eff} - S^e\right)\left(1 + 0.2b\theta_x\right)}{1 + b\theta_x}\right]$$

If the reactor has no cell retention:

$$\theta = \theta_x$$

Solve for $X_v$.

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7…Continued

If the reactor has cell retention (recycle or membrane):
Solve for $\theta$ using the design value for $X_v$.

Selection of Design $X_v$:

| | |
|---|---|
| Gravity settling | |
| Typical settling | 1,500- 3,000 mg/L |
| Poor settling (bulking) | 500-1,500 mg/L |
| Excellent settling (dense) | 3,000-5,000 mg/L |
| Membrane | 10,000-20,0000 mg/L |

8. Solve $V = Q^o\theta$

Get reactor volume required

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9. Solve for waste sludge production:

$X_v$ production rate = $\frac{X_v V}{\theta_x^d}$

Remember that $X_{syn} = X_a + X_i = \frac{\theta_x^d}{\theta}\left[\frac{Y(1+0.2b\theta_x^d)(S_{eff}^o - S^e)}{1+b\theta_x^d}\right] = \frac{\theta_x^d}{\theta}\left[\overline{X_{syn}}\right]$

where $\overline{X_{syn}} = \frac{Y(1+0.2b\theta_x^d)(S_{eff}^o - S)}{1+b\theta_x^d}$

So the cell production rate is $\frac{(X_{syn})V}{\theta_x^d} = \left[\frac{V}{\theta_x^d}\right]\left[\frac{\theta_x^d}{\theta}\right](\overline{X_{syn}}) = Q^o(\overline{X_{syn}})$

**Calculation shortcut**: it is possible to generalize the above result, as follows:

The production rate (= removal rate) for **ANY** type of suspended solid is equal to $Q^o\overline{X}_j$ where: $\overline{X}_j$ = computed term inside the parentheses of the equation in step 9.

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10. Stoichiometry and materials balance:

Substrate removal rate = $Q^o(S_{TOTAL}^o - S^e)$

N requirement = 0.12 x Cell production rate (from $C_5H_7O_2N$)

P requirement = 0.02 x Cell production rate.

$O_2$ utilization rate = $Q^o(S^o_{eff}-S)-1.42$(cell production rate)
= $Q^o(S^o_{eff}-S)f_e$
= $\Sigma$ $O_2$ equivalent input– $\Sigma O_2$ equivalent output (biomass)

Note: the value of $S_p^e$ depends upon the removal efficiency of the separation device. $S_p^e = \gamma\,(X_d/X_v)X_v^e$, where $X_v^e$ is obtained from a knowledge of clarifier performance and $X_d = (\theta_x^d/\theta)[X_d^o/(1+k_{hyd}\theta_x^d)]$.

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**Table 5.2** Summary of applicable equations for a CSTR with settling and recycle of microorganisms (operating at steady state, treating a soluble substrate, and with no input of active biomass)

Hydraulic Detention Time ($\theta$):

$$\theta = \frac{V}{Q^0} \qquad \textbf{[3.20]}$$

Solids Retention Time, SRT ($\theta_x$):

$$\theta_x = \frac{X_a V}{X_a^e Q^e + X_a^w Q^w} \qquad \textbf{[5.35]}$$

SRT at which microorganism washout results ($\theta_x^{min}$), and the limit thereto:

$$\theta_x^{min} = \frac{K + S^0}{S^0(Y\hat{q} - b) - Kb} \qquad S \to S^0 \qquad \textbf{[3.26]}$$

$$\left[\theta_x^{min}\right]_{lim} = \frac{1}{Y\hat{q} - b} \qquad S \to \infty \qquad \textbf{[3.27]}$$

Reactor or Effluent Substrate Concentration ($S = S^e$):

$$S = K \frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1} \qquad \textbf{[5.39]}$$

Reactor Minimum Substrate Concentration ($S_{min}$):

$$S_{min} = K \frac{b}{Y\hat{q} - b} \qquad \theta_x \to \infty \qquad \textbf{[3.28]}$$



Reactor Minimum Substrate Concentration ($S_{min}$):

$$S_{min} = K \frac{b}{Y\hat{q} - b} \qquad \theta_x \to \infty \qquad \textbf{[3.28]}$$

Reactor Active Microorganism Concentration ($X_a$):

$$X_a = \theta_x \frac{X(-r_{ut})}{1 + b\theta_x} \qquad \textbf{[5.40]}$$

*$r_{ut} = q X_a$*

*$= -q_{max} S Xa / (K+S)$*

$$X_a = \frac{\theta_x}{\theta} \frac{Y(S^0 - S)}{1 + b\theta_x} \qquad \textbf{[5.43]}$$

Reactor Inert Microorganism Concentration ($X_i$):

$$X_i = \frac{\theta_x}{\theta}\left[X_i^0 + X_a(1 - f_d)b\theta\right] \qquad \textbf{[5.46]}$$

Reactor volatile suspended solids concentration ($X_v$):

$$X_v = X_i + X_a$$

*No particulate substrate, biodegradable VSS, $X_d = 0$*

$$X_v = \frac{\theta_x}{\theta}\left[X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\theta_x)}{1 + b\theta_x}\right] \qquad \textbf{[5.47]}$$

Active Biological Sludge Production Rate ($r_{abp}$):

$$r_{abp} = \frac{X_a V}{\theta_x} \qquad \textbf{[5.45]}$$

Total Biological Solids Production Rate ($r_{tbp}$):

$$r_{tbp} = \frac{X_v V}{\theta_x} \qquad \textbf{[5.48]}$$

## Suspended (dispersed) vs. attached (biofilm) growth

- Suspended (dispersed) growth
  - e.g., Activated sludge, anaerobic digestion
- Attached (biofilm) growth
  - e.g., Trickling filter, fluidized bed reactor



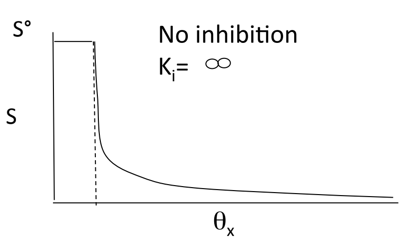
## Reactor design for inhibition

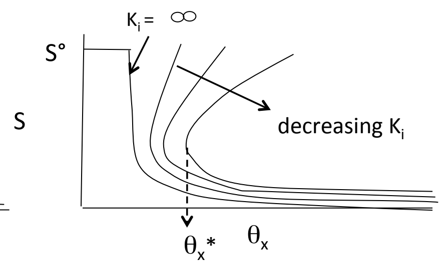
For a CSTR, $1/\mu = q_x$

$$\frac{1}{\theta_x} = \mu = Yq - b$$

**When there is inhibition, the expression used for *q* depends on the type of inhibition.**

The Andrews Equation (Haldane kinetics) and its consequence for CSTR treatment (Gaudy et al., 1988)



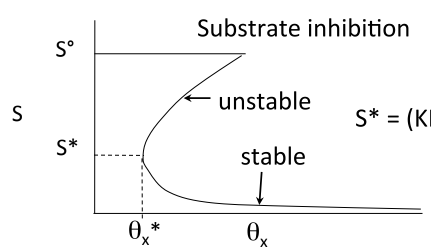




$$\mu = \frac{1}{\theta_x} = Yq - b = \frac{Y\hat{q}S}{K + S + \frac{S^2}{K_i}} - b$$

$$\mu^* = \frac{1}{\theta_x^*} = \frac{Y\hat{q}}{1 + 2\left(\frac{K}{K_i}\right)^{1/2}} - b$$

Unstable conditions occur when $\theta_x < \theta_x^* = 1/\mu^*$

S should not be allowed to exceed S*




$S^* = (KK_i)^{1/2}$

<u>Design recommendations</u>:

(1) Pick $\theta_x > \theta_x^*$

(2) Pick $\theta > \theta_x^*$ to prevent transient loadings from causing S > S*