Pauta Ejercicio 2 - P2

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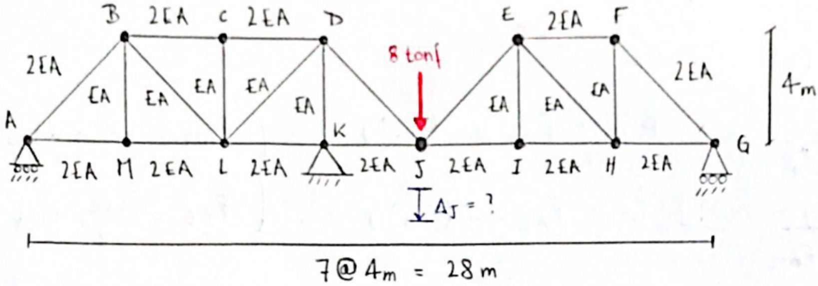
$E = 10000\ MPa$
$A = 50\ cm^2$



(*) Utilizando el principio de igualdad de energía interna con el trabajo externo, determinar el desplazamiento vertical Δ_J.

En primer lugar, calculemos el GIE del enrejado.

$$GIE = \text{Reacciones} - \text{Ecuaciones eq.} = 4 - (3 + 1)$$ → Rotula J

$$\longrightarrow GIE = 0$$ ✓✓ (Isostática)

Ahora, de las ecuaciones de equilibrio:

$\sum F_x = 0$: $H_K = 0$ tonf

$\sum F_y = 0$: $V_A + V_K + V_G = 8$

$\sum M_A^{\curvearrowleft +} = 0$: $28V_G - 8 \cdot 16 + 12 V_K = 0$

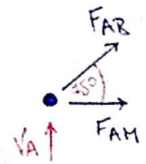
$\sum M_{\overrightarrow{R}}^{\curvearrowleft +} = 0$: $12 V_G = 0$

$$\boxed{\begin{aligned} H_K &= 0 \text{ tonf} \\ V_K &= \tfrac{32}{3} \text{ tonf} \\ V_A &= -\tfrac{8}{3} \text{ tonf} \\ V_G &= 0 \text{ tonf} \end{aligned}}$$

El lado derecho del enrejado tendrá esfuerzos nulos, ya que no tiene ni cargas ni reacciones asociadas.

Por lo tanto, veamos los esfuerzos del lado izquierdo.

Nodo A:



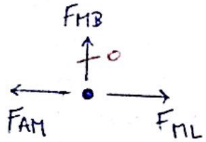
$\hat{x}]\ F_{AM} + F_{AB}\cos(45°) = 0$

$\hat{y}]\ V_A + F_{AB}\,\text{sen}(45°) = 0$

$F_{AM} = -\frac{8}{3}$ tonf ✓✓

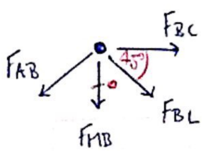
$F_{AB} = \frac{8\sqrt{2}}{3}$ tonf ✓✓

Nodo M:



$\hat{x}]\ F_{ML} = \frac{8}{3}$ tonf ✓✓

Nodo B:

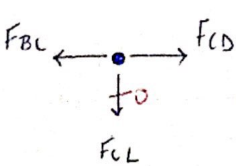


$\hat{x}]\ -F_{AB}\,\text{sen}(45°) + F_{BC}\cos(45°) + F_{BC} = 0$

$\hat{y}]\ -F_{AB}\cos(45°) - F_{BL}\,\text{sen}(45°) = 0$

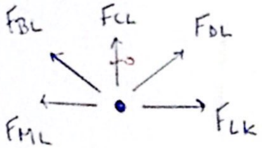
$\longrightarrow F_{BC} = \frac{16}{3}$ tonf ✓✓ $\wedge$ $F_{BL} = -\frac{8\sqrt{2}}{3}$ tonf ✓✓

Nodo C:



$\hat{x}]\ F_{CD} = \frac{16}{3}$ tonf ✓✓

Nodo L :

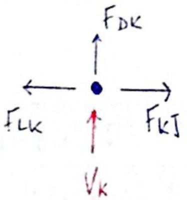


$\hat{x}\rfloor \quad -F_{ML} - F_{BL}\cos(45^\circ) + F_{DL}\cos(45^\circ) + F_{LK} = 0$

$\hat{y}\rfloor \quad F_{BL}\,\text{sen}(45^\circ) + F_{DL}\,\text{sen}(45^\circ) = 0$

$\longrightarrow \quad F_{LK} = -8\,\text{tonf}$ ✓✓ $\wedge \quad F_{BL} = \dfrac{8\sqrt{2}}{3}\,\text{tonf}$ ✓✓

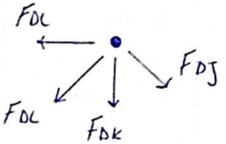
Nodo K :



$\hat{x}\rfloor \quad F_{KJ} = F_{LK}$ $\quad F_{KJ} = -8\,\text{tonf}$ ✓✓

$\hat{y}\rfloor \quad F_{DK} = -V_K$ $\quad F_{DK} = -\dfrac{32}{3}\,\text{tonf}$ ✓✓

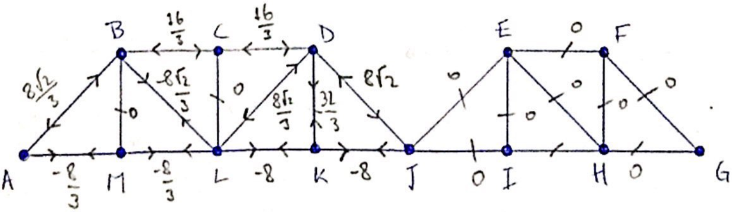
Nodo D :



$\hat{x}\rfloor \quad -F_{DC} - F_{DL}\,\text{sen}(45^\circ) + F_{DJ}\,\text{sen}(45^\circ) = 0$

$\hat{y}\rfloor \quad -F_{DL}\cos(45^\circ) - F_{DK} - F_{DJ}\cos(45^\circ) = 0$

$\longrightarrow \quad F_{DJ} = 8\sqrt{2}\,\text{tonf} \quad \wedge \quad F_{KD} = -\dfrac{32}{3}\,\text{tonf}$ ✓✓



Posteriormente, calculamos el trabajo externo:

$$W_{EXT} = \frac{P\Delta_J}{2} = \frac{8 \cdot \Delta_J}{2} \longrightarrow \boxed{W_{EXT} = \frac{8\Delta_J}{2}}$$

Luego, calculamos la energía interna del enrejado:

$$U_{INT} = \frac{1}{2}\sum_{i}^{n} \frac{N_i^2 L_i}{EA_i}$$

$$EA = 5 \cdot 10^7 [N] = 5 \cdot \frac{10^7}{9806{,}65} [tonf]$$

$$EA = 5098{,}6 \ [tonf]$$

$$2EA = 10197{,}2 \ [tonf]$$

Biela	N [tonf]	L [m]	EA [tonf]	U_i [tonf-m]
AB	$\frac{8\sqrt{2}}{3}$	$4\sqrt{2}$	2EA	0,00789
AM	$-\frac{8}{3}$	4	2EA	0,00279
BL	$-\frac{8\sqrt{2}}{3}$	$4\sqrt{2}$	EA	0,01578
ML	$-\frac{8}{3}$	4	2EA	0,00279
BC	$\frac{16}{3}$	4	2EA	0,01116
CD	$\frac{16}{3}$	4	2EA	0,01116
LD	$\frac{8\sqrt{2}}{3}$	$4\sqrt{2}$	EA	0,01578
LK	-8	4	2EA	0,02511
DJ	$8\sqrt{2}$	$4\sqrt{2}$	2EA	0,07100
KJ	-8	4	2EA	0,02511

DK	$-\frac{32}{3}$	4	EA	0,08926

$$\therefore \boxed{U_{INT} = \frac{1}{2}\sum_{i}^{n} \frac{N_i^2 L_i}{EA_i} = \frac{1}{2}\cdot 0,27783 \text{ [tonf - m]}}$$

Finalmente, se determina el desplazamiento en J con el principio de Clayperon:

$$W_{EXT} = U_{INT} \longrightarrow 8\Delta_J \cdot \cancel{\frac{1}{2}} = \cancel{\frac{1}{2}} \cdot 0,27783$$

$$\therefore \boxed{\Delta_J \approx 3,5 \text{ [cm]}}$$