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ADVANCED LABORATORY FOR
GEOSTATISTICAL SUPERCOMPUTING

Lab 4: Comparative statistics

Statistical and Geostatistical Data Analysis.

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Comparative Test

Student, Fisher y ANOVA

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Comparative test:

- Student test: Compares two means
- Fisher test: Compares two variances
- ANOVA: Compares multiple means

Student test:

- Data distribution may not be exactly Gaussian but the tests are ROBUST.

There are 4 types:

- Compares the mean against a specific value
- Compares 2 means with equal variance
- Compares 2 means with different variances
- Compares 2 means for paired data

Student test

Compares the mean against a specific value

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0$$

Degrees of freedom: $n - 1$

Student test

Compares means of populations
with the same variance

$$S = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

$$\boldsymbol{H_0 : \mu = \mu_1 = \mu_2}$$
$$\boldsymbol{H_1 : \mu_1 \neq \mu_2}$$

$$T = \frac{\bar{\boldsymbol{X}}_1 - \bar{\boldsymbol{X}}_2}{\boldsymbol{S}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Degrees of freedom $n_1 + n_2 - 2$

Student test

Compares means of populations with different variances

$$\boldsymbol{H_0: \mu = \mu_1 = \mu_2}$$
$$\boldsymbol{H_1: \mu_1 \neq \mu_2}$$

$$S = \sqrt{\frac{(n_1 \; - 1)S_1^2 + (n_2 \; - 1)S_2^2}{n_1 + n_2 - 2}}$$

$$T = \frac{\bar{\boldsymbol{X}}_{\mathbf{1}} - \bar{\boldsymbol{X}}_{\mathbf{2}}}{\boldsymbol{S}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Degrees of freedom:

$$\frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^{\mathbf{2}}}{\left(\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}\right)}$$

Student test

Compares means for paired data

Se define $\boldsymbol{D = X_1 - X_2}$

$$\boldsymbol{H_0: \mu = \mu_0}$$
$$\boldsymbol{H_1: \mu \neq \mu_0}$$

$$\boldsymbol{T = \frac{\bar{D}}{S/\sqrt{n}}}$$

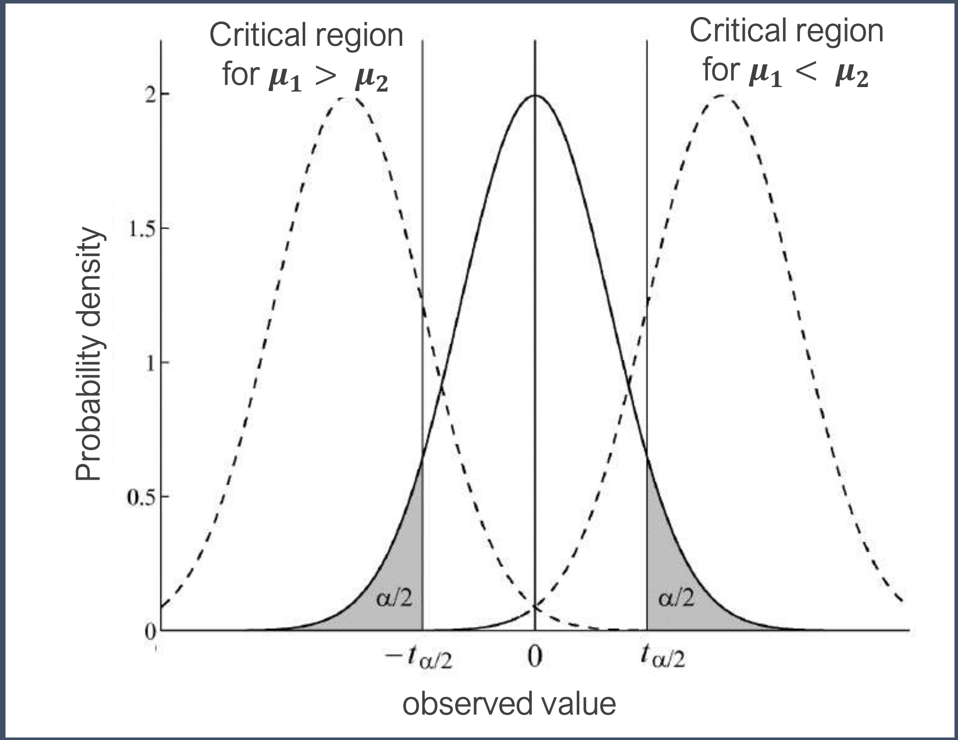
Degrees of freedom: $\boldsymbol{n - 1}$

Student test

Depending on the alternative hypothesis chosen, a one-way test or two-way test will be used.

Alternative hypothesis : $\boldsymbol{H_1}: \boldsymbol{\mu_1} > \boldsymbol{\mu_2}$
(One-way test)

Alternative hypothesis : $\boldsymbol{H_1}: \boldsymbol{\mu_1} \neq \boldsymbol{\mu_2}$
(Two-way test)



Fisher test

Compares two variances

Consider independent Gaussian variables with $S_1^2 \geq S_2^2$ variances.

$$\boldsymbol{H_0}: \sigma_1^2 = \sigma_2^2 = \sigma^2$$
$$\boldsymbol{H_1}: \sigma_1^2 \neq \sigma_2^2$$

$$F = \frac{S_1^2}{S_2^2}$$

Degrees of freedom $n_1 - 1\ y\ n_2 - 1$

Selecting the sample size

$$n \geq 2\left(\frac{S z_{\alpha}}{\delta}\right)^{2}$$

With

- z_{α} : Gaussian threshold (one-way or two-way, depending on the alternative hypothesis) for a significance level $1-\alpha$.
- S: expected standard deviation.
- δ : minimum difference that should be detected by the test.

Once the value of n has been calculated, one can replace z_{α} by the Student threshold with $2n-2$ degrees of freedom for the same significance level $1-\alpha$, then recalculate the value of n.

Problems

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Problem 1:

- As a chief metallurgist, you receive the proposal of a seller who claims that sulfide minerals recovery in the flotation process could increase from 89% to 95%, thanks to a new reagent. The proposal seems interesting enough to make an evaluation. To this end, the plant metallurgist performs an experiment with the new reagent for 6 consecutive days and compares the results with those of the 20 days previous to the experiment. (File *"Reactivo planta de flotación.xls"*).

Average recovery:

- It is asked to test:

1. Whether or not the average recovery with the new reagent is equal to 95%.
2. Whether or not the recovery variances are the same with the two reagents.
3. Whether or not the average recoveries are the same with the two reagents.

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Problem 2:

- Using one hundred duplicated data, sent to two different laboratories, it is of interest to know whether or not there is a significant difference between the laboratories. (File*"Duplicados.xls"*).

Problem 3 (proposed):

- One has a set of diamond drill hole and blast hole data, with measurements of the total copper grade (in %). (File *"Leyes de cobre.xls"*)
- It is of interest to design a test in order to decide whether or not the two types of measurements have the same quality.