Exemplo Laplace Sist Lineal

$$x' + x + y' = 0$$
$$4x + y' + y = 0$$

$$x(0) = 2, \quad y(0) = 1$$

$\mathcal{L}(\ )(s)$

Denotando $X := \mathcal{L}(x)(s)$
$Y := \mathcal{L}(y)(s)$

$$\left.\begin{aligned} sX - 2 + X + sY - 1 &= 0 \\ 4X + sY - 1 + Y &= 0 \end{aligned}\right\}$$

$$\Leftrightarrow \quad (s+1)X + sY = 3$$
$$4X + (s+1)Y = 1$$

Ahora resolvemos el sistema de ecuaciones como prefieran.

$$\begin{bmatrix} \lambda+1 & \lambda \\ 4 & \lambda+1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \lambda+1 & \lambda \\ 4 & \lambda+1 \end{bmatrix}^{-1} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \frac{1}{(\lambda-1)^2} \begin{bmatrix} \lambda+1 & -\lambda \\ -4 & \lambda+1 \end{bmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{(\lambda-1)^2} \begin{bmatrix} 2\lambda+3 \\ \lambda-11 \end{bmatrix}$$

$$\Rightarrow \quad x = \frac{2\lambda}{(\lambda-1)^2} + \frac{3}{(\lambda-1)^2} = \frac{2}{\lambda-1} + \frac{5}{(\lambda-1)^2}$$

$$X = 2\,\mathcal{L}(e^{t})(s) + 5\,\mathcal{L}(t)(s-1)$$

$$= \mathcal{L}(2e^{t})(s) + 5\,\mathcal{L}(e^{t}t)(s)$$

$$= \mathcal{L}\left(e^{t}(2+5t)\right)(s)$$

$$\Rightarrow \quad x(t) = e^{t}(2+5t)$$

$$Y = \frac{s-1}{(s-1)^2} - \frac{10}{(s-1)^2} = \frac{1}{s-1} - \frac{10}{(s-1)^2}$$

$$= \mathcal{L}(e^{t})(s) - 10\,\mathcal{L}(te^{t})(s)$$

$$= \mathcal{L}(e^{t}(1-10t)(s)$$

$$\Rightarrow \quad y(t) = e^{t}(1-10t)$$