

Prop1 Considere la ecuación diferencial lineal homogénea

$$y^{(4)} + 2y''' + 11y'' + 2y' + 10y = 0$$

$$P(\lambda) = \lambda^4 + 2\lambda^3 + 11\lambda^2 + 2\lambda + 10 = 0$$

$$= \lambda^4 + 2\lambda^3 + 10\lambda^2 + \lambda^2 + 2\lambda + 10 = 0$$

$$= \lambda^2(\lambda^2 + 2\lambda + 10) + 1(\lambda^2 + 2\lambda + 10) = 0$$

$$= (\lambda^2 + 1)(\lambda^2 + 2\lambda + 10) = 0$$

$$\frac{-2 \pm \sqrt{4 - 40}}{2} = -\frac{2 \pm 6i}{2} = \boxed{-1 \pm 3i}$$

$$B_{\text{ASC}} = \{\cos(x), \sin(x), e^{-x}\cos(3x), e^{-x}\sin(3x)\}$$

$$y_H(x) = A\cos(x) + B\sin(x) + C e^{-x}\cos(3x) + D e^{-x}\sin(3x)$$

Prop2 Calcule mediante coeficientes indeterminados:

$$y'' + 2y' - 8y = 2e^{-2x} - e^{-x}, \text{ con } y(0) = 1, y'(0) = 0$$

$$\begin{aligned} (D^2 + 2D - 8)y &= 2e^{-2x} - e^{-x} \\ (D+4)(D-2)y &= \underbrace{2e^{-2x}}_{y_{p1}} - \underbrace{e^{-x}}_{y_{p2}} \end{aligned}$$

$$y_H = A e^{-4x} + B e^{2x}$$

$$y_{p1} = (A_1 e^{-2x}), \quad y_{p2} = (A_2 e^{-x})$$

$$D y_{p1} = -2A_1 e^{-2x}, \quad D y_{p2} = -A_2 e^{-x}$$

$$D^2 y_{p1} = 4A_1 e^{-2x}, \quad D^2 y_{p2} = A_2 e^{-x}$$

$$(4 - 4 - 8)A_1 = 2, \quad (1 - 2 - 8)A_2 = -1$$

$$A_1 = -\frac{1}{4}, \quad A_2 = \frac{1}{9}$$

$$\Rightarrow y_p(x) = -\frac{e^{-2x}}{4} + \frac{e^{-x}}{9}$$

$$y(x) = A e^{-4x} + B e^{2x} - \frac{e^{-2x}}{4} + \frac{e^{-x}}{9}$$

$$\frac{4}{9} - \frac{1}{9}$$

$$y(0) = A + B - \frac{1}{4} + \frac{1}{9} = 1$$

$$= A + B = \frac{1}{4} - \frac{1}{9} + \frac{36}{36} = \frac{9-4+36}{36} = \frac{41}{36}$$

$$y'(0) = -4A + 2B + \frac{1}{2} - \frac{1}{9} = 0$$

$$= -4A + 2B = -\frac{9-2}{18} = -\frac{7}{18} \quad (2)$$

$$4 \cdot (1) + (2) \Rightarrow 6B = \frac{41}{9} - \frac{7}{18}$$

$$= \frac{82-7}{18} = \frac{75}{18} = \frac{25}{6}$$

$$\Rightarrow \boxed{B = \frac{25}{36}}$$

$$A + B = \frac{41}{36} \Rightarrow A = \frac{16}{36} = \frac{8}{18} = \boxed{\frac{4}{9}}$$

$$\Rightarrow y(x) = \frac{4}{9} e^{-4x} + \frac{25}{36} e^{2x} - \frac{e^{2x}}{4} + \frac{e^{-x}}{9}$$

$$- e^{-4x}$$

$$- 2x$$

$$2$$

$$1$$

$$= \frac{1}{36} (16 - e \cdot 9 + 4e^3 + 25e^6)$$