71) $tg^2(\alpha) = 2tg^2(\beta) + 1 \Rightarrow \cos^2(\alpha) + sen^2(\beta) = sen^2(\alpha)$

$$tg^2(\beta) = \frac{tg^2(\alpha) - 1}{2}$$

$$\Leftrightarrow \frac{sen^2(\beta)}{\cos^2(\beta)} = \frac{tg^2(\alpha) - 1}{2}$$

$$\Leftrightarrow \frac{sen^2(\beta)}{1 - sen^2(\beta)} = \frac{tg^2(\alpha) - 1}{2}$$

$$\Leftrightarrow 2sen^2(\beta) = (tg^2(\alpha) - 1) - (tg^2(\alpha) - 1))\, sen^2(\beta)$$

$$\Leftrightarrow 2\, sen^2(\beta) + (tg^2(\alpha) - 1)\, sen^2(\beta) = tg^2(\alpha) - 1$$

$$\Leftrightarrow sen^2(\beta)\,(tg^2(\alpha) + 1) = tg^2(\alpha) - 1$$

$$\Leftrightarrow sen^2(\beta) = \frac{tg^2(\alpha) - 1}{tg^2(\alpha) + 1} = 1 - \frac{2}{tg^2(\alpha) + 1}$$

$$\Leftrightarrow sen^2(\beta) = 1 - \frac{2}{\frac{sen^2(\alpha) + 1}{\cos^2(\alpha)}} = 1 - 2\cos^2(\alpha)$$

$$\Rightarrow \cos^2(\alpha) + sen^2(\beta) = \cos^2(\alpha) + 1 - 2\cos^2(\alpha)$$

$$= 1 - \cos^2(\alpha) = sen^2(\alpha)$$

P2 $f(x) = \dfrac{x}{x^2-1}$

i) Dom, ceros, sign, paridad, asintotas

Dom : $\mathbb{R} \setminus \{-1, 1\}$

Ceros : $x = 0$

paridad : $f(-x) = \dfrac{-x}{(-x)^2-1} = -\dfrac{x}{x^2-1} = -f(x) \longrightarrow$ impar

asintotas verticales: $x = 1$, $x = -1$

asintota horizontal: $y = 0$

ii) PDQ: $f(x_2) - f(x_1) = \dfrac{(x_1 - x_2)(1 + x_1 x_2)}{(x_1^2-1)(x_2^2-1)}$

$$f(x_2) - f(x_1) = \frac{x_2}{x_2^2-1} - \frac{x_1}{x_1^2-1}$$

$$= \frac{x_2(x_1^2-1) - x_1(x_2^2-1)}{(x_2^2-1)(x_1^2-1)}$$

$$= \frac{x_2 x_1^2 - x_2 - x_1 x_2^2 + x_1}{(x_2^2-1)(x_1^2-1)}$$

$$= \frac{x_1 x_2 (x_1 - x_2) + (x_1 - x_2)}{(x_2^2-1)(x_1^2-1)} = \frac{(x_1 x_2 + 1)(x_1 - x_2)}{(x_2^2-1)(x_1^2-1)}$$

f es impar $\longrightarrow$ estudiemos crec. en $\mathbb{R}^+$

$x_1, x_2 \in (0,1)$, $x_1 < x_2 \Rightarrow f(x_2) - f(x_1) = \dfrac{(<0)(>0)}{(<0)(<0)} < 0$

$\Rightarrow$ es decrec.

$x_1, x_2 \in (1,\infty)$, $x_1 < x_2 \Rightarrow f(x_2) - f(x_1) = \dfrac{(<0)(>0)}{(>0)(>0)} < 0$

$\Rightarrow$ es decrec.

23) $P = (x_0, y_0)$, se mueve en $x^2 = 4y + 4$, $x_0 \neq 0$

$A = (0,2)$, $Q = (x_0, 0)$

PDQ: L_{OP} y L_{AQ} se intersectan en R tq su dist al origen es cte.

$$L_{OP}: \quad y = \frac{y_0}{x_0} x$$

$$L_{AQ}: \quad y = -\frac{2x}{x_0} + 2$$

$$L_{OP} \cap L_{AQ} = \quad \frac{y_0 x}{x_0} = -\frac{2x}{x_0} + 2 \Rightarrow x = \frac{2x_0}{y_0 + 2}$$

$$\Rightarrow y = \frac{2y_0}{y_0+2}$$

$$\therefore R = \left(\frac{2x_0}{y_0+2} , \frac{2y_0}{y_0+2} \right)$$

$$d(R, 0) = \sqrt{\frac{4x_0^2}{(y_0+2)^2} + \frac{4y_0^2}{(y_0+2)^2}}$$, $(x_0, y_0) \in$ parabola $\Rightarrow x_0^2 = 4y_0 + 4$

$$= \sqrt{\frac{4y_0^2 + 16y_0 + 16}{(y_0+2)^2}}$$

$$= \sqrt{4 \cdot \frac{(y_0+2)^2}{(y_0+2)^2}} = \sqrt{4} = 2 \ //$$

P4)

$$\frac{1 - tg(x)}{1 + tg(x)} = 1 + sen(2x) \longrightarrow cos(x) \neq 0$$

$$\Leftrightarrow \frac{1 - \frac{sen(x)}{cos(x)}}{1 + \frac{sen(x)}{cos(x)}} = 1 + sen(2x)$$

$$\Leftrightarrow \frac{\frac{cos(x) - sen(x)}{\cancel{cos(x)}}}{\frac{cos(x) + sen(x)}{\cancel{cos(x)}}} = 1 + sen(2x)$$

$$\Leftrightarrow cos(x) - sen(x) = (cos(x) + sen(x))(1 + \underbrace{sen(2x)}_{2sen(x)cos(x)})$$

$$\Leftrightarrow \cancel{cos(x)} - sen(x) = \cancel{cos(x)} + sen\,x + 2sen(x)cos^2(x) + 2\,sen^2(x)\,cos(x)$$

$$\Leftrightarrow 0 = 2sen(x)\left(1 + cos^2(x) + sen(x)\,cos(x)\right)$$

$$\Rightarrow \underbrace{sen(x) = 0}_{(i)} \quad \vee \quad \underbrace{1 + cos^2(x) + sen(x)\,cos(x)}_{(ii)}$$

i) $sen(x) = 0 \Leftrightarrow x = k\pi$, $k \in \mathbb{Z}$

ii) $1 + cos^2(x) + sen(x)\,cos(x) = 0$

$$\Leftrightarrow \underbrace{cos^2(x)}_{>0} + sen(x)\,cos(x) = -1 \quad \times$$

no hay solución.