Tarea 1 - Pauta

IN3401 - Estadstica para la Economa y la Gestin
Departamento de Ingeniera Civil Industrial, Universidad de Chile
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I. Teora de la Distribucin

S^2

$$E(S^2) = E(\frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2) = \sigma^2 - \frac{\sigma^2}{N}$$

$\bar{X} + a$

$$E(\bar{X} - a) = E(\frac{1}{N}\sum_{i=1}^{N}(X_i) - a) = E(\frac{1}{N}\sum_{i=1}^{N}(X_i)) - E(a)$$

$$= \frac{1}{N}\sum_{i=1}^{N}E(X_i) - E(a) = \frac{1}{N}\sum_{i=1}^{N}\mu - E(a)$$

$$= \frac{1}{N} * N\mu - E(a) = \mu - a$$

$M_i i = 1, ..., N \mu\sigma^2$
$S_*^2 = \frac{1}{N}\sum_{i=1}^{N}(M_i - \bar{M})^2 \bar{M} = \frac{1}{N}\sum_{i=1}^{N} M_i$
S_*^2

$$S_*^2 = \frac{1}{N}\sum_{i=1}^{N}(M_i - \bar{M})^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}(M_i^2 - 2M_i\bar{M} + \bar{M}^2)$$

$$= \frac{1}{N}[\sum_{i=1}^{N}M_i^2 - 2\sum_{i=1}^{N}M_i\bar{M} + \sum_{i=1}^{N}\bar{M}^2)]$$

$$= \frac{1}{N}\sum_{i=1}^{N}M_i^2 - 2\bar{M}\frac{1}{N}\sum_{i=1}^{N}M_i + \frac{1}{N} * N\bar{M}^2$$

$\frac{1}{N}\sum_{i=1}^{N} M_i = \bar{M}$

$$S_*^2 = \frac{1}{N}\sum_{i=1}^{N}(M_i^2 - \bar{M}^2)$$

$$E(S_*^2) = E(\frac{1}{N}\sum_{i=1}^{N}M_i^2 - \bar{M}^2) = \frac{1}{N}\sum_{i=1}^{N}E(M_i^2) - E(\bar{M}^2)$$

$Var(X) = E(X^2) - [E(X)]^2$
$E(M_i^2) = Var(M_i) + [E(M_i)]^2 = \sigma^2 + \mu^2 \Rightarrow \frac{1}{N}\sum_{i=1}^{N}E(M_i^2) = \frac{1}{N}\sum_{i=1}^{N}\sigma^2 + \mu^2 = \sigma^2 + \mu^2$
$E(\bar{M}^2) = Var(\bar{M}) + [E(\bar{M})]^2 = \frac{\sigma^2}{N} + \mu^2 \Rightarrow E(\bar{M}^2) = \frac{\sigma^2}{M} + \mu^2$

$$\frac{1}{N}\sum_{i=1}^{N}E(M_i^2) - E(\bar{M}^2) = \sigma^2 + \mu^2 - \frac{\sigma^2}{M} - \mu^2$$

$$= \sigma^2 - \frac{\sigma^2}{N}$$

$-\frac{\sigma^2}{N}$

$S_*^2 = \frac{N-1}{N}\sigma^2 \frac{N}{N-1}\sigma^2$

$N(np; np(1-p)) = N(100/9, 800/81) P(2.5 < X < 7.5)$

$$P(2.5 - 100 < X - 100 < 7.5 - 100)$$

$$P(-2.7397 < Z < -1.1487)$$

$$P(Z < -1.1487) - P(Z < -2.7397)$$

$$1 - P(Z < 1.1487) - 1 + P(Z < 2.7397))$$

$$= 1 - 0.8749 - 1 + 0.9968$$

$$= 0.1219$$

$$E(X_i) = \frac{1}{9}(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 5$$

$$V(X_i) = E(X_i^2) - E(X_i)^2$$

$$E(X_i^2) = \frac{1}{9}(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2) = \frac{95}{3}$$

$$V(X_i) = \frac{95}{3} - 25 = \frac{20}{3}$$

$N(500, 2000/3)$

$$P(\sum_{i=1}^{100} X_i < 300)$$

$$P(\frac{\sum_{i=1}^{100} X_i - 500}{\sqrt{2000/3}} < \frac{300 - 500}{\sqrt{2000/3}})$$

$$P(Z < \frac{-200}{25.82})$$

$$P(Z < \frac{-200}{25.82})$$

$$P(Z < -7.746)$$

$$= 1 - P(Z < 7.746)$$

$$= 0.000001$$

$0 < X < 7$

$$f(x) = \frac{1}{b - a}$$

$$f(x) = \frac{1}{7}$$

$x \in (0, 7)$

$$F(x) = \int_0^x \frac{1}{7} dx = \frac{x}{7}$$

$$E(x) = \int_0^7 xdx = \frac{1}{7} * \frac{x^2}{2} = \frac{x^2}{14}|_0^7 = \frac{49}{14} = \frac{7}{2}$$

$$Var(x) = E(x^2) - E(x)^2$$

$$E(x^2) = \int_0^7 \frac{1}{7}x^2dx = \frac{1}{7} * \frac{x^3}{3}|_0^7 = \frac{343}{21} = \frac{49}{3}$$

$$Var(X) = E(X^2) - E(X)^2 = \frac{49}{3} - \frac{49}{4} = \frac{49}{12}$$

$$E(X) = \frac{a+b}{2} = \frac{7}{2}$$

$$Var(X) = \frac{(b-a)^2}{12} = \frac{49}{12}$$

$X|X > 4$

$$f(X|X > 4) = \frac{f(x)}{P(X > 4)} = \frac{\frac{1}{7}}{\frac{3}{7}} = \frac{1}{3}$$

$$E(X|X > 4) = \frac{4+7}{2} = 5.5$$

$$Var(X|X > 4) = \frac{(7-4)^2}{12} = \frac{9}{12} = 0.75$$

$yxxyyf(y|x) = \frac{e^{-\beta x}(\beta x)^y}{y!}y = 0, 1, ..., x \leq 0\beta > 0f(x) = \theta e^{-\theta x}\theta > 0xy\beta xx$

$f(x, y)$

$f(x, y) = f(y|x)f(x)$

$$f(x, y) = \frac{e^{-\beta x}(\beta x)^y}{y!} * \theta e^{-\theta x} = \frac{\theta e^{-(\theta+\beta)x}(\beta x)^y}{y!}$$

$y\frac{1}{\delta(1-\delta)^y}\delta\frac{\beta+\theta}{\theta}$

$$f(y) = \int_0^\infty f(x, y)dx = \frac{\theta\beta^y}{y!}\int_0 \infty e^{-(\beta+\theta)x}x^y$$

(Γ)

$$\Gamma(z) = \int_0^\infty e^{-ax}x^tdx = \frac{z!}{a^{z+1}}$$

$$f(y) = \frac{\theta\beta^y}{y!}\frac{y!}{\beta + \theta^{y+1}} = \frac{\theta\beta^y}{(\beta+\theta)(\beta+\theta^y)} = (\frac{\theta}{\beta+\theta})(\frac{\beta}{\beta+\theta})^y$$

$$f(y) = \delta(1-\delta)^y$$

$\frac{1}{E[x]} = \theta(\frac{1}{Var[x]})^{1/2} = \theta$

$$E(X) = \int_0^\infty xf(x)dx = \int_0^\infty x\theta e^{-\theta x}dx = \theta\int_0^\infty xe^{-\theta x}dx = \theta\frac{1!}{\theta^{1+1}} = \theta\frac{1}{\theta^2} = \frac{1}{\theta}$$

$$E(X^2) = \int_0^\infty x^2 f(x)dx = \theta\frac{2!}{\theta^{2+1}} = \theta\frac{2}{\theta^3} = \frac{2}{\theta^2}$$

$E[y|x] = \beta x E[y], Var[y], Cov[x, y]\beta\theta$

$$E(Y) = E_x(E(Y|X)) = E_x(\beta X) = \beta E(X) = \beta\frac{1}{\theta} = \frac{\beta}{\theta}$$

$$Var(Y) = E(Var(Y|X)) + Var(E(Y|X)) = E(\beta X) + Var(\beta X) = \beta E(X) + \beta^2 Var(X) = \beta\frac{1}{\theta} + \beta^2\frac{1}{\theta^2}$$

$$= \frac{\beta}{\theta} + \frac{\beta^2}{\theta^2}$$

$$Cov(X, Y) = Cov(X, E(Y|X)) = Cov(X, \beta X) = \beta Var(X) = \beta\frac{1}{\theta^2} = \frac{\beta}{\theta^2}$$

yx

$E[y|x]$

$(X^tX)^{-1}(X^tY)$

$$\beta = \frac{Cov(X, Y)}{Var(X)} = \frac{6}{2} = 3$$

$$\bar{Y} - \beta\bar{X} = 2 - 3 * 1 = -1$$

yx

$$\rho^2_{X,Y} = \frac{Cov^2(X, Y)}{Var(X)Var(Y)} = \frac{6^2}{5 * 2} = \frac{36}{10} = 3.6$$

$yE[y|x]$

$$\rho^2_{Y,E[Y|X]} = \frac{Cov^2(Y, E[Y|X])}{Var(Y)Var(E[Y|X])} = \frac{Cov^2(Y, \beta_0 + \beta_1 X)}{Var(Y)Var(\beta_0 + \beta_1 X)}$$

$$= \frac{\beta_1^2 Cov^2(Y, X)}{\beta_1^2 Var(Y)Var(X)}$$

$$= \frac{Cov^2(Y, X)}{Var(Y)Var(X)}$$

$$= \frac{6^2}{5 * 2} = \frac{36}{10} = 3.6$$

$(w, z)E[w] = 0E[z] = 0Var[w] = 1Var[z] = 1Cov[w, z] = \epsilon(w_i, z_i)\epsilon$

ϵ

$$\hat{S_{w,z}} = \frac{1}{n-1}\sum_{i=1}^{n}(w_i - \bar{w})(z_i - \bar{z})$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(w_iz_i - \bar{w}z_i - w_i\bar{z} + \bar{w}\bar{z})$$

$$= \frac{n}{n-1}(\frac{1}{n}\sum_{i=1}^{n}w_iz_i - \frac{1}{n}\sum_{i=1}^{n}\bar{w}z_i - \frac{1}{n}\sum_{i=1}^{n}w_i\bar{z} + \frac{1}{n}\sum_{i=1}^{n}\bar{w}\bar{z})$$

$$= \frac{n}{n-1}(\frac{1}{n}\sum_{i=1}^{n}(w_iz_i - \bar{w}z_i - w_i\bar{z} + \bar{w}\bar{z})$$

$$= \frac{n}{n-1}(\frac{1}{n}\sum_{i=1}^{n}(w_iz_i - \bar{w}\bar{z})$$

$$\hat{S_{w,z}} = \frac{n}{n-1}(\frac{1}{n}\sum_{i=1}^{n}(w_iz_i - \bar{w}\bar{z}))$$

$$n \to \infty \Rightarrow 1 * (E(WZ) - E(W)E(Z))$$

$$= Cov(W,Z)$$

$E[w|z] = \alpha + \beta z \beta = \frac{Cov[z,w]}{Var[z]} = \epsilon \alpha = E[w] - \beta E[z] = 0 w = \epsilon z + \rho w z \epsilon$

β

$$\beta = \frac{Cov[z,w]}{Var[z]}$$

$$\beta = \frac{Cov[z,\epsilon z + \rho]}{Var[z]}$$

$$\beta = \frac{Cov[z,\epsilon z]}{Var[z]}$$

$$\beta = \epsilon\frac{Cov[z,z]}{Var[z]}$$

$$\beta = \epsilon\frac{Var[z]}{Var[z]}$$

$$\beta = \epsilon$$

$z = \frac{1}{\epsilon}w - \frac{1}{\epsilon}\rho = \delta w + \delta\rho z w \delta\epsilon\delta$

$z = \frac{1}{\epsilon}w - \frac{1}{\epsilon}\rho z = \gamma w + \gamma\rho$

$$Cov(w,z) = \gamma Cov(w,w) + \gamma Cov(w,\rho)$$

$$Cov(w,z) = \gamma(Cov(w,w) + Cov(w,\rho))$$

$$\gamma = \frac{Cov(w,z)}{Var(w) + Cov(w,\rho)} Locualesdistintoa \frac{Cov(w,z)}{Var(w)}$$

$W(\eta, 9m^2)P(W > \eta + m)$

$$P(W > \eta + m) = P(Z > \frac{\eta + m - \eta}{3m})$$

$$= P(Z > 1/3) = 1 - P(Z < 1/3) = 1 - 0.6293 = 0.3707$$

W^2
$ZN(0,1)Z^2$

$$P(Z^2 \leq z) = P(|Z| \leq \sqrt{z})$$

$$= P(-\sqrt{z} \leq Z \leq \sqrt{z})$$

$$= P(Z \leq \sqrt{z}) - P(Z \leq -\sqrt{z})$$

$$F_z(\sqrt{z}) - F_z(-\sqrt{z})$$

$$= \frac{\partial F_z(\sqrt{z})}{\partial z} - \frac{\partial F_z(-\sqrt{z})}{\partial z}$$

$$= \frac{1}{2\sqrt{z}}(\frac{e^{\frac{-(\sqrt{z})^2}{2}}}{\sqrt{2\pi}} + \frac{e^{\frac{-(-\sqrt{z})^2}{2}}}{\sqrt{2\pi}})$$

$$= \frac{e^{\frac{-z}{2}}}{\sqrt{2\pi z}}$$

$$f(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{k-1}}{\Gamma(k)}$$

$\Gamma(1/2) = \sqrt{\pi} k = 1/2 \lambda = 1/2$
χ^2_k

$$\chi^2_k = \frac{1}{2^{\frac{k}{2}}\Gamma(k/2)} z^{\frac{k}{2}-1} e^{\frac{-z}{2}}$$

$k = 1$

$$\chi^2_1 = \frac{1}{2^{\frac{1}{2}}\Gamma(1/2)} z^{\frac{-1}{2}} e^{\frac{-z}{2}}$$

$$= \frac{e^{\frac{-z}{2}}}{\sqrt{2\pi z}}$$

II. Test de Hiptesis

$\bar{X}_{45}(8, \lambda)\lambda(8, \lambda)E(X) = \frac{8}{\lambda} Var(X) = \frac{8}{2}(\bar{X}_{45}) - \frac{8}{\lambda})/\sqrt{\frac{8}{45^2}}$

$P(-1.64 < Z < 1.64) = 0.9$

$(\bar{X}_{45} - \frac{8}{\lambda})/\sqrt{\frac{8}{45\lambda^2}} N(0,1)$

$\bar{X}_{45} - \frac{8}{\lambda})/\sqrt{\frac{8}{45\lambda^2}} < 1.64)$
$= P(-1.64 < 2.37\lambda\bar{X}_{45} - 19 < 1.64)$
$= P(17.36 < 2.37\lambda\bar{X}_{45} < 20.64)$
$= P(\frac{17.36}{2.37\bar{X}_{45}} < \lambda < \frac{20.64}{2.37\bar{X}_{45}})$

$\lambda(\frac{17.36}{2.37\bar{X}_{45}}, \frac{20.64}{2.37\bar{X}_{45}})$

$N(2,\sigma^2)\sigma^2 X_i \sim N(\mu,\sigma^2)\sum_{i=1}^n (X_i-\mu)^2/\sigma^2 n$

$\sum_{i=1}^n (X_i-\mu)^2/\sigma^2 = (\frac{(n-1)S^2}{\sigma^2}) \sim \chi^2_{(n-1)}$

$(\frac{(n-1)S^2}{\sigma^2}) \sim \chi^2_{(n-1)}$
$P((\frac{(n-1)S^2}{\sigma^2}) < b) = 0.995 P(a < (\frac{(n-1)S^2}{\sigma^2}) < b) = 0.99$

$0.99 = P((\frac{(n-1)S^2}{b}) < \sigma^2 < (\frac{(n-1)S^2}{a}))$

$S^2 = \sum_{i=1}^n (X_i-\mu)^2/(n-1) = 0.887$

$\alpha = 0.01 b = \chi^2_{\alpha/2,(n-1)} = \chi^2_{0.01,9} = 21.665 a = \chi^2_{1-\alpha/2,(n-1)} = \chi^2_{0.99,9} = 2.088$

$$[\frac{(n-1)S^2}{b}, \frac{(n-1)S^2}{a}] = [0.3685; 3.8233]$$

$$\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$$

$n = 800$

$$1-\alpha = 0.9 \Rightarrow \alpha = 0.1 \Rightarrow Z_{0.1} = 1.28$$

$$\widehat{p} = \frac{casos\ favorables}{casos\ totales} = \frac{568}{800} = 0,71$$

$$IC: 0,71 \pm 1,28\sqrt{\frac{0,71(0,29)}{800}}$$

$$IC: [0,6895;\ 0,7305]$$

$H_0: \widehat{p} = 0,51$
$H_1: \widehat{p} \neq 0,51$

H_0

III. Ejercicios Empricos

$xx + 1x - 1$

bulldoser

$$F = \frac{\frac{SSB}{I-1}}{\frac{SWW}{I(J-1)}}$$

$\lambda = 0.5\mu_n = \frac{1}{0.5} = 2\sigma_n^2 = \frac{1}{0.5^2} = 4$

$\bar{y_r}$

$\bar{y_r} \mu_n \sigma_n^2 - ln(1 - ALEATORIO()) * \mu_n$

$\mu_n = 2 \sigma_n^2 = 4$