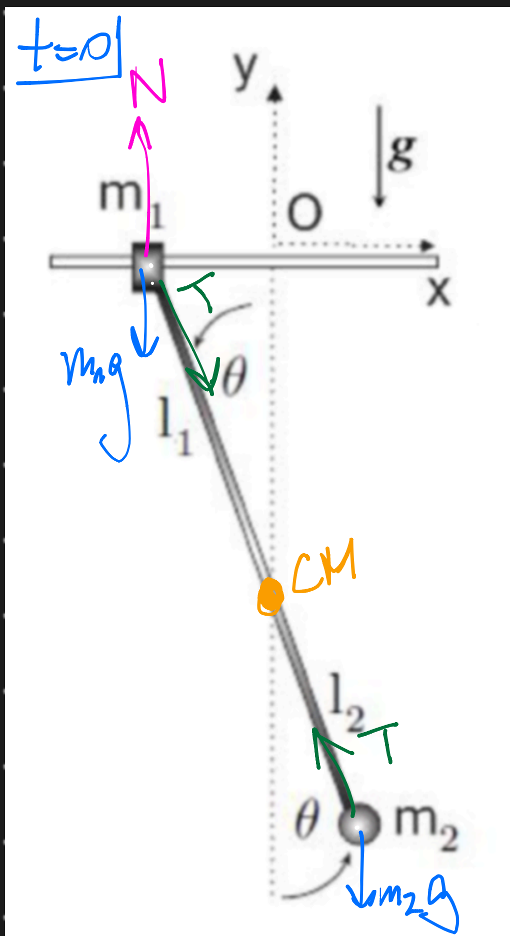
Let's Go Crazy
- Prince.

Auxiliar #20.

18 Junio 2021.

P1]



(a) $\dot{x}_{cm} = 0$

$\hat{y}]\ (m_1+m_2)\cdot\ddot{y}_{cm} = N - m_1 g - m_2 g$

$\hat{x}]\ (m_1+m_2)\cdot\ddot{x}_{cm} = 0$

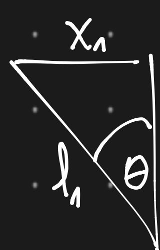
$\Rightarrow \ddot{x}_{cm} = 0.$ ⊛

Buscamos el CM.

$\vec{R}_{cm} = (x_{cm}, y_{cm}).$

$$\vec{R}_{cm} = \frac{1}{M}\cdot\sum_i \vec{r}_i\cdot m_i$$

$\Rightarrow x_{cm} = \frac{1}{m_1+m_2}\cdot(m_1 x_1 + m_2 x_2)$



$\Rightarrow |x_1| = l_1\cdot \text{sen}\,\theta.$

$\Rightarrow x_2 = l_2\cdot \text{sen}\,\theta$

$* M = m_1 + m_2$

$$\Rightarrow x_{cm} = \frac{1}{M}\cdot(m_2\cdot l_2\cdot \text{sen}\theta - m_1\cdot l_1 \text{sen}\theta).$$

$$x_{cm} = \frac{\text{sen}\theta}{M}(m_2\cdot l_2 - m_1\cdot l_1) \Big/ \frac{d}{dt}.$$

$$\dot{x}_{cm} = \frac{(m_2 l_2 - m_1 l_1)}{m} \cdot \cos\theta \cdot \dot{\theta} \quad / \quad \frac{d}{dt}$$

$$\ddot{x}_{cm} = \frac{(m_2 l_2 - m_1 l_1)}{m} \left(\cos\theta \cdot \ddot{\theta} - \operatorname{sen}\theta \cdot \dot{\theta}^2 \right) = 0$$

$$\Rightarrow \quad m_2 \cdot l_2 - m_1 \cdot l_1 = 0.$$

$$m_1 \cdot l_1 = m_2 \cdot l_2$$

En $\dot{x}_{cm} = \frac{(m_2 l_2 - m_1 l_1)}{m} \cdot \cos\theta \cdot \dot{\theta} = 0$

$\dot{x}_{cm} = 0 \iff$ el cm sólo se mueve en $\hat{y}$.

b) l_1, l_2; dem. que $\left(\frac{x_2}{l_2}\right)^2 + \left(\frac{y_2}{l}\right)^2 = 1$.

Sabemos que: ① $m_1 \cdot l_1 = m_2 \cdot l_2$.

② $l_1 + l_2 = l$.

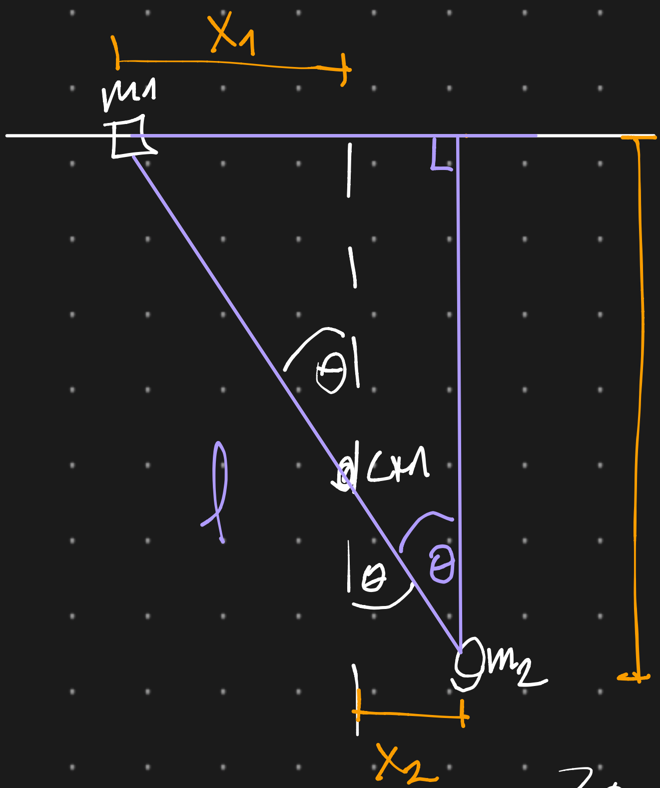
$\rightarrow$ De ①: $l_1 = \frac{m_2}{m_1} \cdot l_2$.

En ② $\frac{m_2}{m_1} \cdot l_2 + l_2 = l = l_2 \left(1 + \frac{m_2}{m_1}\right)$

$$\Rightarrow \quad l_2 = \frac{l}{1 + \frac{m_2}{m_1}}.$$

$\rightarrow$ ① $l_2 = \frac{m_1}{m_2} \cdot l_1 \quad \Rightarrow$ ② $l_1 + \frac{m_1}{m_2} \cdot l_1 = l$.

$$\Rightarrow l_1 = \frac{l}{1+\frac{m_1}{m_2}}.$$



$$\rightarrow \frac{x_2}{l_2} = \operatorname{sen}\theta$$

$$x_2 = l_2 \cdot \operatorname{sen}\theta.$$

$$\rightarrow \cos\theta = \frac{y_2}{l}.$$

$$\operatorname{sen}^2\theta + \cos^2\theta = 1.$$

$$\left(\frac{x_2}{l_2}\right)^2 + \left(\frac{y_2}{l}\right)^2 = 1.$$

ⓒ N que siente m_1.

$$M\ddot{y}_{cm} = N - M\cdot g$$

$$N = M(g + \ddot{y}_{cm})$$

necesitamos este.

$$y_{cm} = \frac{1}{M}(y_1\cdot m_1 + y_2 \cdot m_2)$$

$$= \frac{1}{M}\cdot(0 - m_2[l\cdot\cos\theta]).$$

$$\cos\theta = \frac{y_2}{l}.$$

$$y_{cm} = \frac{-m_2 \cdot l}{m} \cdot \cos\theta \quad / \frac{d}{dt}$$

$$\dot{y}_{cm} = \frac{-m_2 \cdot l}{m} \cdot \left(-\operatorname{sen}\theta \cdot \dot{\theta}\right) \quad / \frac{d}{dt}$$

$$\ddot{y}_{cm} = +\frac{m_2 \cdot l}{M} \cdot \left(\operatorname{sen}\theta \cdot \ddot{\theta} + \cos\theta \cdot \dot{\theta}^2\right)$$

Reemplazamos en N.

$$N = M \cdot (g + \ddot{y}_{cm}).$$

$$N = M \cdot g + m_2 \cdot l \left(\operatorname{sen}\theta \cdot \ddot{\theta} + \cos\theta \cdot \dot{\theta}^2\right).$$

d) Ec. de mov. para θ.

$$\sum \vec{\tau}_{CM} = I_{CM} \cdot \ddot{\theta}$$

$$I_{cm} = \sum_i m_i \cdot r_i^2$$

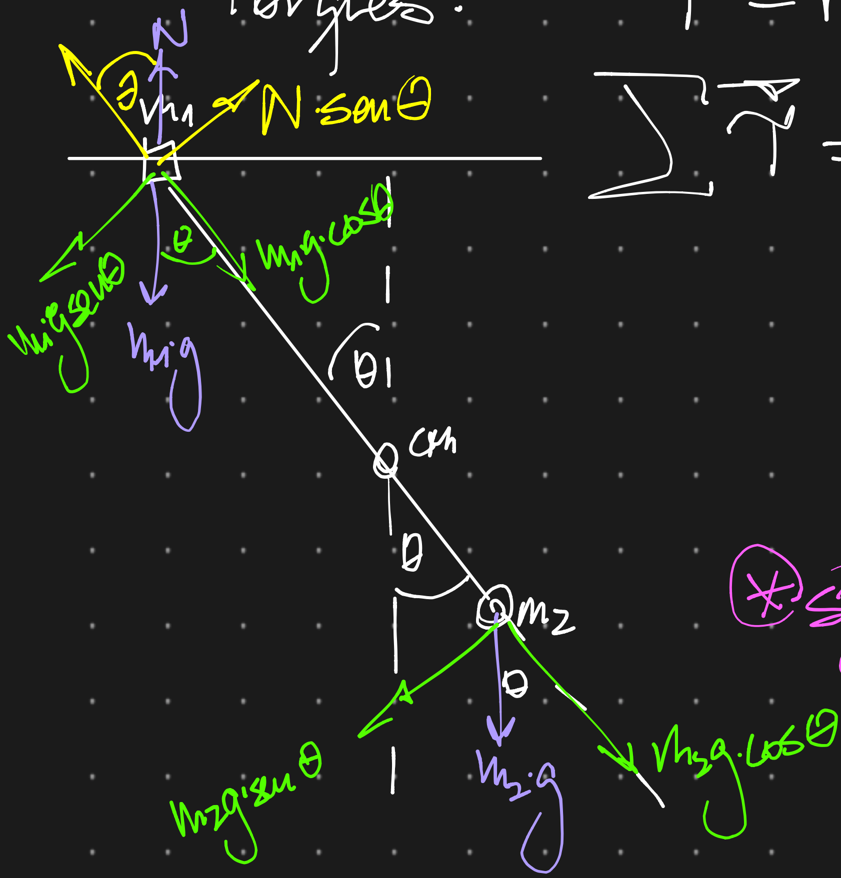
r_i: distancia de m_i al eje de giro.

$$\longrightarrow I_{cm} = m_1 \cdot r_1^2 + m_2 \cdot r_2^2$$

$$= m_1 \cdot l_1^2 + m_2 \cdot l_2^2 .$$

→ Torques:

$$\vec{\tau} = \vec{r} \times \vec{F}$$



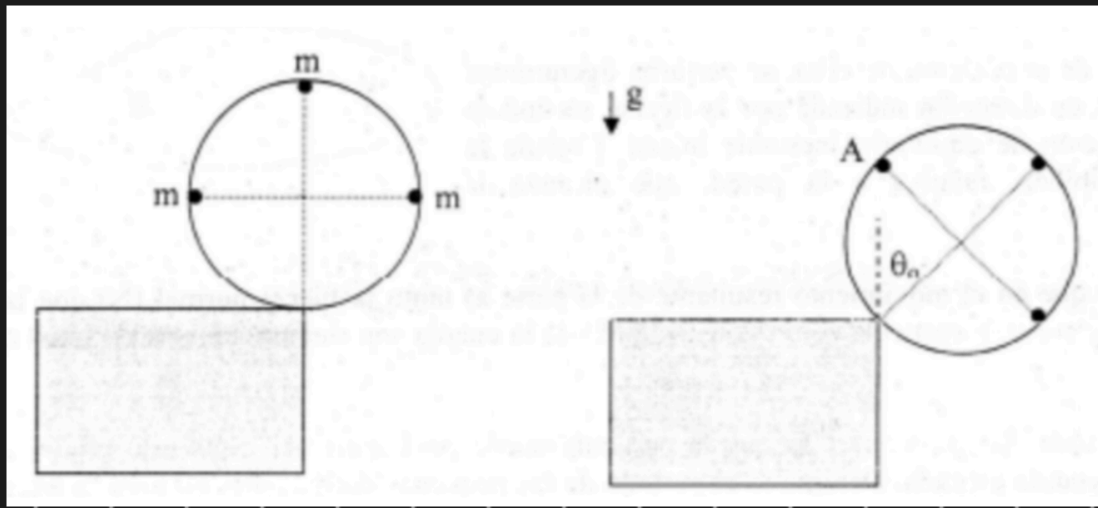
$$\sum \vec{\tau} = +l_1 \cdot m_1 \cdot g \cdot sen\theta$$
$$- l_2 \cdot m_2 \cdot g \cdot sen\theta$$
$$- N \cdot sen\theta \cdot l_1$$

(*) Signos con la regla de la (mano derecha)

$$\Rightarrow \sum \vec{\tau}_{cm} = I_{cm} \cdot \ddot{\theta}$$

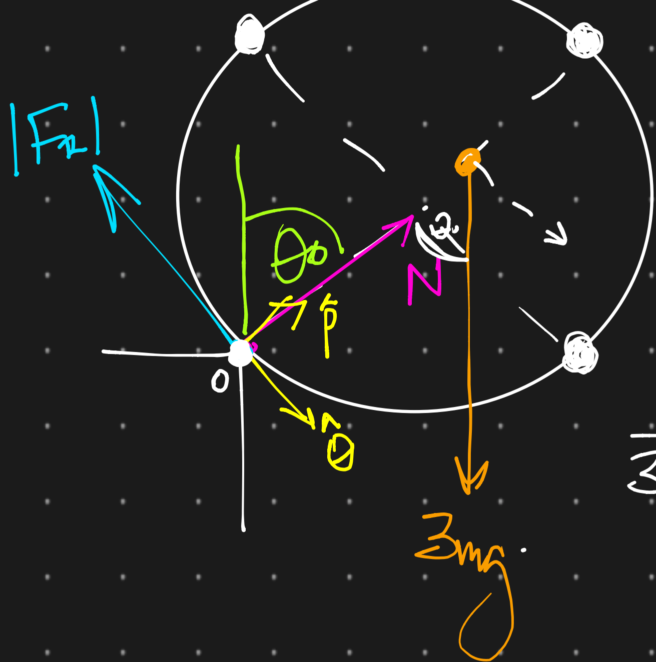
$$(l_1 m_1 g - l_2 m_2 g - N \cdot l_1) \cdot sen\theta = (m_1 \cdot l_1^2 + m_2 \cdot l_2^2) \cdot \ddot{\theta}$$

P2



ⓐ Normal y Fuerza de roce en el punto de apoyo.

$$3m\ddot{\vec{R}}_{cm} = N\hat{\rho} - F_R\hat{\theta} + 3mg\left(-\cos\theta\,\hat{\rho} + \sin\theta\,\hat{\theta}\right)$$



Buscamos el cm.

$$\vec{R}_{cm} = \frac{1}{3m}\left((R\hat{\rho} - \cancel{R\hat{\theta}})m + (R\hat{\rho} + \cancel{R\hat{\theta}})m + 2R\hat{\rho}m\right)$$

$$\vec{R}_{cm} = \frac{4}{3}R\hat{\rho} \quad / \frac{d}{dt}$$

$$\dot{\vec{R}}_{cm} = \frac{4}{3}R\dot{\theta}\hat{\theta} \quad / \frac{d}{dt}$$

$$\ddot{\vec{R}}_{cm} = \frac{4}{3}R\left(-\dot{\theta}^2\hat{\rho} + \ddot{\theta}\hat{\theta}\right)$$

$$= \frac{4}{3}R\left(\ddot{\theta}\hat{\theta} - \dot{\theta}^2\hat{\rho}\right)$$

Necesitamos expresiones para $\ddot{\theta}$ y $\dot{\theta}^2$.

$\Rightarrow$ hacemos $\sum \tau = I \cdot \ddot{\theta}$

La única fza. que ejerce torque es el peso.

$$\sum \vec{\tau} = -\frac{4}{3} R \hat{\rho} \times 3mg \cdot (\operatorname{sen}\theta_0 \cdot \hat{\theta} - \cos\theta_0 \hat{\rho})$$

$$= -4 R mg \operatorname{sen}\theta_0 \, \hat{z}.$$

$$\rightarrow I = m(2R)^2 + m \cdot (\sqrt{2} R)^2 \cdot 2$$

$$= 8mR^2.$$

$$\Rightarrow \sum \tau = I \cdot \ddot{\theta}$$

$$-\cancel{4} \cancel{R} \cancel{m} g \operatorname{sen}\theta_0 = {}_2\cancel{8} \cancel{m} R^{\cancel{2}} \cdot \ddot{\theta}$$

$$\ddot{\theta} = \frac{-g}{2R} \cdot \operatorname{sen}\theta \; (*) \quad \checkmark$$

trucazo

$$\int_0^{\dot{\theta}} \dot{\theta} \, d\dot{\theta} = \frac{-g}{2R} \cdot \int_0^{\theta} \operatorname{sen}\theta \, d\theta$$

$$\frac{\dot{\theta}^2}{2} = \frac{+g}{2R} \cdot (+\cos\theta) \Big|_0^{\theta} = \frac{g}{2R} \cdot (\cos\theta - 1).$$

$$\dot{\theta}^2 = \frac{g}{R} (\cos\theta - 1) \; (*)$$

Reemplazamos en la ec. de movimiento.

$$\hat{\rho}\,\Big|\; 3m \cdot \ddot{R}_{cm,\rho} = N - 3mg \cdot \cos\theta$$

$$\cancel{3}m \cdot \left(-\frac{4}{\cancel{3}} R \cdot \underbrace{\dot{\theta}^2}\right) = N - 3mg \cdot \cos\theta$$

reemplazamos $\dot{\theta}$

$$N = 8mg\cos - 4m\cancel{R} \cdot \frac{g}{\cancel{R}} (\cos\theta - 1)$$

$$N = -mg\cos\theta + 4mg$$

$$\hat{\theta}\,\Big|\; 3m \cdot \left(\frac{4}{\cancel{3}} \cdot R \cdot \ddot{\theta}\right) = -F_R + 3mg \cdot \sin\theta$$

reemplazamos

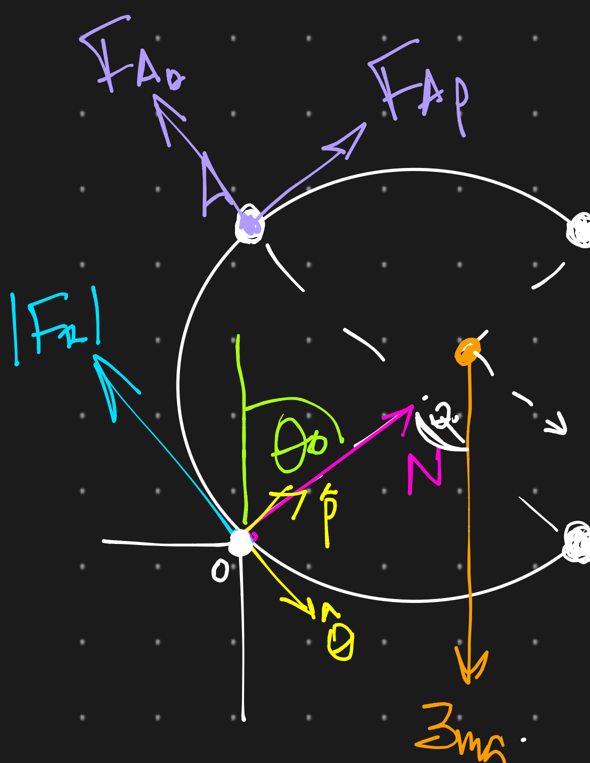
$$_2\cancel{4}m\cancel{R}\left(\frac{-g}{\cancel{2}\cancel{R}} \cdot \sin\theta\right) = -F_R + 3mg\sin\theta$$

$$F_R = 3mg\sin\theta + 2mg\sin\theta$$

$$F_R = 5mg\sin\theta$$

b) Suponiendo que $\theta_0 = \frac{\pi}{4}$, nos piden las fuerzas de adhesión. /?

Una fuerza de adhesión es lo mismo que una normal. Al final es la fuerza que siente A por estar apoyada en el aro.



Busquemos $\vec{r}_A$.

$$\vec{r}_A = R\hat{\rho} - R\hat{\theta} \quad / \frac{d}{dt}$$

$$\dot{\vec{r}}_A = R\dot{\theta}(\hat{\rho} + \hat{\theta}) \quad / \frac{d}{dt}$$

$$\ddot{\vec{r}}_A = R\left[(\ddot{\theta} - \dot{\theta}^2)\hat{\rho} + (\ddot{\theta} + \dot{\theta}^2)\hat{\theta}\right]$$

Ahora hacemos sumatoria de fuerzas.

$$m\ddot{\vec{r}}_A = mg(-\cos\theta\hat{\rho} + \operatorname{sen}\theta\hat{\theta}) + F_{A\rho}\hat{\rho} + F_{A\theta}\hat{\theta}$$

$$\hat{\rho}\,\big|\quad mR(\ddot{\theta} - \dot{\theta}^2) = F_{A\rho} - mg\cos\theta$$

De la parte anterior tenemos:

$$\ddot{\theta} = \frac{-g}{2R}\operatorname{sen}\theta \quad \text{y} \quad \dot{\theta}^2 = \frac{g}{R}(\cos\theta - 1)$$

Evaluamos en $\theta_0 = \frac{\pi}{4}$.

$$\ddot{\theta}\left(\frac{\pi}{4}\right) = \frac{-g\sqrt{2}}{4R} \quad y \quad \dot{\theta}^2\left(\frac{\pi}{4}\right) = \frac{g}{R}\left(\frac{\sqrt{2}}{2}-1\right).$$

En la ec. de mov:

$$m\cancel{R}\left(\frac{-g\sqrt{2}}{4\cancel{R}} - \frac{g}{\cancel{R}}\left(\frac{\sqrt{2}}{2}-1\right)\right) = F_{\Delta\rho} - mg\overbrace{\cos\theta}^{\frac{\sqrt{2}}{2}}$$

$$F_{\Delta\rho} = \cancel{mg\frac{\sqrt{2}}{2}} - mg\cdot\frac{\sqrt{2}}{4} - \cancel{mg\frac{\sqrt{2}}{2}} + mg.$$

$$F_{\Delta\rho} = mg\left(1 - \frac{\sqrt{2}}{4}\right)$$

De la ec. de mov. en $\hat{\theta}$ sacamos $F_{\Delta\theta}$.

$$\hat{\theta}\rfloor \quad mR\left(\ddot{\theta} + \dot{\theta}^2\right) = F_{\Delta\theta} + mg\,\mathrm{sen}\,\theta.$$

Reemplazamos $\ddot{\theta}$ y $\dot{\theta}^2$.

$$F_{\Delta\theta} + mg\underbrace{\mathrm{sen}\,\theta}_{\frac{\sqrt{2}}{2}} = m\cancel{R}\left(\frac{-g\sqrt{2}}{4\cancel{R}} + \frac{g}{\cancel{R}}\left(\frac{\sqrt{2}}{2}-1\right)\right)$$

$$F_{\Delta\theta} = \cancel{mg\frac{\sqrt{2}}{2}} - mg - mg\frac{\sqrt{2}}{4} - \cancel{mg\frac{\sqrt{2}}{2}}$$

$$F_{\Delta\theta} = -mg\left(1 + \frac{\sqrt{2}}{4}\right)$$

llegamos a lo que buscábamos :)