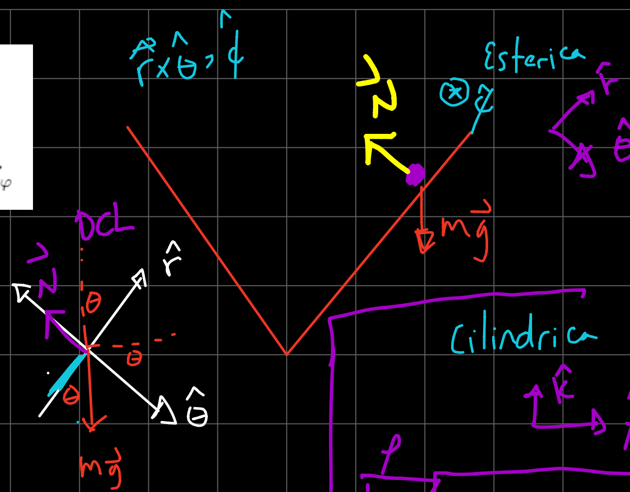
$$\begin{aligned}\vec{a} &= \left(\ddot{r} - r\dot{\theta}^2 - r\,\mathrm{sen}^2(\theta)\dot{\varphi}^2\right)\vec{u}_r \\ &+ \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\,\mathrm{sen}(\theta)\cos(\theta)\dot{\varphi}^2\right)\vec{u}_\theta \\ &+ \left(r\,\mathrm{sen}(\theta)\ddot{\varphi} + 2\dot{r}\dot{\varphi}\,\mathrm{sen}(\theta) + 2r\dot{\theta}\dot{\varphi}\cos(\theta)\right)\vec{u}_\varphi\end{aligned}$$



$$m\vec{g} = -mg\cos\theta\,\hat{r} + mg\,\mathrm{sen}\theta\,\hat{\theta}$$

$$\vec{N} = -N\,\hat{\theta}$$

Cilindrica

$\hat{k}$ $\hat{\rho}$

ρ, z, θ

$$\boxed{\tan\theta = \frac{\rho}{z}}$$

$$\boxed{z\tan\theta = \rho}$$

Escribimos las ec de mov.

Cono

$$\hat{r}\,|\; m(\ddot{r} - r\cancel{\dot{\theta}^2}^{\,0} - r\,\mathrm{sen}^2\theta\,\dot{\varphi}^2) = -mg\cos\theta$$

$$\hat{\theta}\,|\; m(r\cancel{\ddot{\theta}}^{\,0} + 2\dot{r}\cancel{\dot{\theta}}^{\,0} - r\,\mathrm{sen}\theta\cos\theta\,\dot{\varphi}^2) = -N + mg\,\mathrm{sen}\theta$$

$$\hat{\varphi}\,|\; m(\underline{r\,\mathrm{sen}\theta}\,\ddot{\varphi} + 2\dot{r}\dot{\varphi}\,\mathrm{sen}\theta + 2r\cancel{\dot{\theta}}^{\,0}\dot{\varphi}\cos\theta) = 0$$

Como no hay fzas en φ, escribo a_φ como

$$③\quad m\underbrace{\frac{1}{r\,\mathrm{sen}\theta}\cdot\frac{d}{dt}\left(r^2\,\mathrm{sen}^2\theta\,\dot{\varphi}\right)} = 0 \;\checkmark$$

/ Cons. momento angular

$$①\quad m(\ddot{r} - r\,\mathrm{sen}^2\theta\,\dot{\varphi}^2) = -mg\cos\theta \;\checkmark$$

✓ (Ec de MoV)

$$②\quad -mr\,\mathrm{sen}\theta\cos\theta\,\dot{\varphi}^2 = -N + mg\,\mathrm{sen}\theta \;\checkmark$$

Condición mov circular vel V_0 ← Conocido $\Rightarrow \dot{r} = \ddot{r} = 0$

De 1.- $+\cancel{m} r \operatorname{sen}^2\theta\, \dot{\varphi}^2 = +\cancel{m} g \cos\theta$ (4)

¿Como escribo V_0 en esteri cas?

$\dot{\varphi} = [1/s]$

$$\vec{V} = \cancel{\dot{r}}\,\hat{r} + r\dot{\varphi}\operatorname{sen}\theta\,\hat{\varphi} + r\cancel{\dot{\theta}}\,\hat{\theta}$$

$$V_0 = r\dot{\varphi}\operatorname{sen}\theta \quad (5)$$

(5) en (4) $\Rightarrow$ $\dfrac{V_0^2}{r} = g\cos\theta \Rightarrow \boxed{r_0 = \dfrac{V_0^2}{g\cos\theta}}$

Perturbaremos radialmente

$$③\quad m\,\frac{1}{r\operatorname{sen}\theta}\cdot\underbrace{\frac{d}{dt}\left(r^2\operatorname{sen}^2\theta\,\dot{\varphi}\right)}_{=0} = 0 \quad \boxed{\hat{\varphi}}$$

$$④\quad m\left(\ddot{r} - r\operatorname{sen}^2\theta\,\dot{\varphi}^2\right) = -mg\cos\theta \ \checkmark$$

Si perturbo, pasan 2 cosas r deja de ser cte y $\dot{\varphi}$ deja de ser cte

$$\Rightarrow r^2\operatorname{sen}^2\theta\,\dot{\varphi} = C$$

/ Usaremos la CI para encontrar c

$$r\operatorname{sen}\theta\cdot(r\operatorname{sen}\theta\,\dot{\varphi}) = C$$

/ CI $\Rightarrow$ $r = r_0$ $\quad r\operatorname{sen}\theta\,\dot{\varphi} = V_0$

$\boxed{r_0 \operatorname{sen}\theta \cdot V_0 = C} \Rightarrow C$ es conocido

$\dot\varphi = \dfrac{C}{r^2 \operatorname{sen}^2\theta}$, lo reemplazo en mi ec en $\hat{r}$

$\cancel{m}(\ddot r - r \operatorname{sen}^2\theta\, \dot\varphi^2) = -\cancel{m} g \cos\theta$ ✓

$\ddot r - r \cdot \operatorname{sen}^2\theta \cdot \dfrac{C^2}{r^4 \operatorname{sen}^4\theta} = -g\cos\theta$

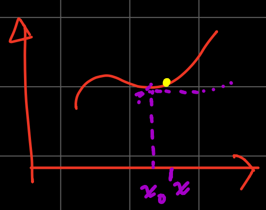
$\hat{r}$: $\boxed{\ddot r - \dfrac{1}{r^3} \cdot \dfrac{C^2}{\operatorname{sen}^2\theta} = -g\cos\theta}$ ← Desplaza el pto de eq,

$\ddot r + r\omega^2 = 0$

$r = r_0 + \varepsilon \Rightarrow \underline{\ddot r = \ddot\varepsilon}$

$f(x) \sim f(x_0) + (x-x_0)\cdot f'(x)\Big|_{x_0} + \underbrace{\dfrac{(x-x_0)^2}{2} f''(}_{O(\varepsilon^2)}$

$f(x) = \dfrac{1}{x^3}$ $\quad f'(x) = -3\,\dfrac{1}{x^4}$



$\dfrac{1}{r^3} = \dfrac{1}{r_0^3} + (r_0 + \varepsilon - r_0) \cdot \left(-3 \cdot \dfrac{1}{x^4}\Big|_{r_0}\right)$

$= \dfrac{1}{r_0^3} - \varepsilon \cdot \dfrac{3}{r_0^4}$ //

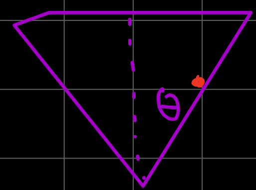
Nuesta nueva ec será

$$\ddot{\xi} - \left(\frac{1}{r_0^3} - \xi \cdot \frac{3}{r_0^4}\right) \cdot \frac{C^2}{sen^2\theta} = -g\cos\theta$$

$$\boxed{\ddot{\xi} + \xi \cdot \frac{3C^2}{r_0^4\, sen^2\theta} = \frac{1}{r_0^3}\frac{C^2}{sen^2\theta} - g\cos\theta}$$

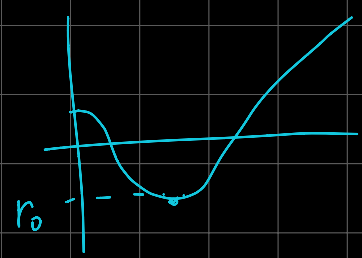
$$w^2 = \frac{3C^2}{r_0^4\, sen^2\theta} \qquad C = r_0\, sen\theta\, V_0$$

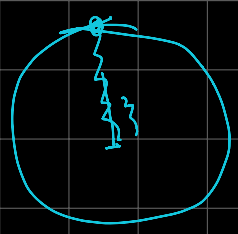
$$w^2 = \frac{3 \cdot V_0^2}{r_0^2} \quad //$$

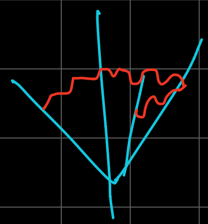


$$\boxed{\left.\frac{dU_{eff}}{dr}\right|_{r=r_0} = 0} \Leftarrow \text{Condición para } \underline{r_0} \; //$$

$$\boxed{V_0 = r_0\, sen\theta\, \dot{\varphi}}$$







$$\omega_{r_0} = \sqrt{\frac{U''_{eff}(r)}{\beta}}$$

Usualmente es la <u>masa</u>

$$\boxed{E(q,\dot q) = \boxed{\frac{1}{2}\beta\,\dot q^{2}} + U_{eff}(q)}$$

q es coordenada

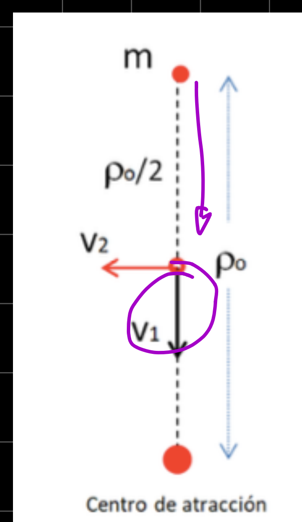
$r, \dot r$

$\theta, \dot\theta$

$$V_{esf} = \dot r\,\hat r + r\dot\theta\,\hat\theta + r\,\mathrm{sen}\,\theta\,\dot\phi\,\hat\phi$$

$$E = \frac{1}{2}m\dot r^{2} + \underbrace{\frac{1}{2}m(r\dot\theta)^{2} + \frac{1}{2}m(r\,\mathrm{sen}\,\theta\,\dot\phi)^{2} + U}_{U_{eff}}$$

$$V_{cil} = \dot r\,\hat r + r\dot\theta\,\hat\theta$$



P2) $F(\rho) = -K\rho$

Ec de Mov) Cilíndricas

$$m(\ddot\rho - \rho\dot\theta^{2}) = -K\rho$$

Cae en linea recta $\Rightarrow \dot\theta = 0$

$$m\ddot\rho = -K\rho_{\rho_0/2} \qquad / \quad \ddot\rho = \dot\rho\,\frac{d\dot\rho}{d\rho}$$

$$\int_0^{\dot\rho} \dot\rho\,\frac{d\dot\rho}{d\rho}\,d\rho = -\frac{K}{m}\int_{\rho_0} \rho\,d\rho$$

$$\frac{\dot\rho_f^{2}}{\ } = -\frac{K}{\ }\left[\frac{\rho^{2}}{\ }\right]^{\rho_0/2} = -\frac{K}{\ }\Bigg[\underbrace{\frac{\rho_0^{2}}{\ } - \rho^{2}}_{-\frac{3}{4}\rho_0^{2}}\Bigg]$$

$$\frac{\dot{\rho}^2}{\cancel{2}} = -\frac{K}{m}\left[\frac{\rho^2}{\cancel{2}}\Big|_{\rho_0}\right] = -\frac{K}{m}\left[\frac{\rho_0^2}{4} - \rho_0^2\right]$$

$$\dot{\rho}_f^2 = +\frac{K}{m}\cdot\left[+\frac{3}{4}\rho_0^2\right]$$

$$V_1 = \rho_0\sqrt{\frac{K}{m}}\cdot\frac{\sqrt{3}}{2} \Leftarrow \text{Velocidad en } \rho_0/2$$

$$V_2 = \frac{1}{\sqrt{3}}V_1 = \rho_0\sqrt{\frac{K}{m}}\cdot\frac{1}{2}$$

Como encuentro la velocidad maxima y la distancia minima? Para responder esto, sera util pensar en terminos de Energía, para ello veamos el potencial asociado a la fuerza del problema

$$F = -\nabla U$$

$$+K\rho = +\nabla U = \frac{d}{d\rho}U \qquad \Big/ \int \frac{d}{\cancel{d\rho}}U\cdot\cancel{d\rho} = \int K\rho\, d\rho$$

$$\boxed{U = \frac{1}{2}K\rho^2} \qquad U = K\frac{\rho^2}{2}$$

Mientras mas pequeño sea rho, mas pequeño es el potencial, tenemos tambien que E=K+U, por lo que habra mas energia cinetica (velocidad) mientras mas cerca estemos.
Entonces el rho_min y v_max ocurren simultaneamente.
Para encontrarlos usaremos, conservacion de energìa y conservacion de momento angular (ausencia de fuerzas en phi_tongo)
Notamos que en p_min la velocidad es solo tangencial.

Por ec en $\hat{\theta}$ $\quad m(\rho\ddot{\theta} + 2\dot{\rho}\dot{\theta}) = 0$

$$\frac{1}{\rho}\frac{d}{dt}(\rho^2\dot{\theta}) = 0$$

$$\rho^2\dot{\theta} = C$$

Necesitamos encontrar C, usamos instante

① $\boxed{\rho V_T = C}$ / $\overset{\text{inicial}}{\text{Inicialmente}}$ $\rho = \frac{\rho_0}{2}$ y $V_T = V_2$

$$C = \frac{\rho_0}{2} \cdot \rho_0 \sqrt{\frac{k}{m}} \cdot \frac{1}{2} = \frac{\rho_0^2}{4} \sqrt{\frac{k}{m}} \quad //$$

$U = \frac{1}{2} k \rho^2$

② $E_i = E_f$

$$E_i = \frac{1}{2} m V_1^2 + \frac{1}{2} m V_2^2 + \frac{1}{2} k \left(\frac{\rho_0}{2}\right)^2 \quad / \; V_1 = \frac{\sqrt{3}}{2} \rho_0 \sqrt{\frac{k}{m}}$$

$$= \frac{1}{2} \cdot \left[\cancel{m} \cdot \frac{3}{4} \rho_0^2 \frac{k}{\cancel{m}} + \cancel{m} \frac{\rho_0^2}{4} \frac{k}{\cancel{m}} + k \frac{\rho_0^2}{4} \right]$$

$$E_i = \frac{1}{2} \rho_0^2 k \left[\frac{5}{4} \right]$$

$$E_f = \frac{1}{2} m V_{max}^2 + \frac{1}{2} k \rho_{min}^2$$

V_{max} es solo tangencial $\quad C = V_{T\,max} \cdot \rho_{min}$

$$\overbrace{\frac{\rho_0^2}{4} \cdot \sqrt{\frac{k}{m}}}^{C} = \rho_{min} \cdot V_{max} \quad (1)$$

$$\overbrace{\frac{5}{8} k \rho_0^2}^{E_i} = \frac{1}{2} m V_{max}^2 + \frac{1}{2} k \rho_{min}^2 \quad (2)$$

$\rho_{min} = a \cdot \rho_0 \qquad V_{max} = b \cdot \rho_0 \sqrt{\frac{k}{m}}$

$$\frac{1}{4}\cdot \cancel{\rho_0^2}\cancel{\sqrt{\frac{K}{m}}} = a\cdot\cancel{\rho_0}\cdot b\,\cancel{\rho_0}\cancel{\sqrt{\frac{K}{m}}} \Rightarrow \frac{1}{4} = a\cdot b \quad (3)$$

$$\frac{5}{8}\cancel{K}\cancel{\rho_0^2} = \frac{1}{2}\cancel{m}\cdot b^2\cancel{\rho_0^2}\cdot\frac{\cancel{K}}{\cancel{m}} + \frac{1}{2}\cancel{K}\,a^2\cancel{\rho_0^2} \Rightarrow \frac{5}{4} = a^2+b^2 \quad (4)$$

$2\cdot(3)+(4)$

$$\frac{2}{4}+\frac{5}{4} = (a+b)^2$$

$$\sqrt{\frac{7}{4}} = a+b \quad (5)$$

$-2(3)+(4)$

$$-\frac{2}{4}+\frac{5}{4} = (a-b)^2$$

$$\sqrt{\frac{3}{4}} = a-b \quad (6)$$

$(5)+(6)$ $$\frac{\sqrt{7}}{2}+\frac{\sqrt{3}}{2} = 2a \Rightarrow a = \frac{\sqrt{7}+\sqrt{3}}{4}$$

$(5)-(6)$ $$\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2} = 2b \qquad b = \frac{\sqrt{7}-\sqrt{3}}{4}$$

$$\boxed{\rho_{min} = a\,\rho_0 \qquad V_{max} = b\cdot\rho_0\sqrt{\frac{K}{m}}}$$