auxiliar 5

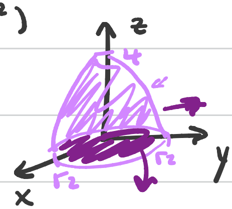
PREGUNTA 1:

$$F(x,y,z) = x\hat{i} + y\hat{j} + 2z\hat{k}$$ $\quad F_T = F_m + F_T$

a través de S que delimita al sólido

$$V := \{(x,y,z) \,/\, 0 \le z \le \underbrace{4 - 2x^2 - 2y^2}_{4 - 2(R^2)}\}$$

coordenadas cil.



$$\phi = (R\cos\theta,\ R\,\text{sen}\,\theta,\ 4 - 2R^2)$$

$\phi(R,\theta)$ $\quad R \in [0, \sqrt{2}]$

$\theta \in [0, 2\pi]$

$$\phi_R = (\cos\theta,\ \text{sen}\,\theta,\ -4R)$$

$$\phi_\theta = (-R\,\text{sen}\,\theta,\ R\cos\theta,\ 0)$$

$$\phi_R \times \phi_\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \text{sen}\,\theta & -4R \\ -R\,\text{sen}\,\theta & R\cos\theta & 0 \end{vmatrix}$$

$$\phi_R \times \phi_\theta = \hat{i}(4R^2\cos\theta) - \hat{j}(-4R^2\,\text{sen}\,\theta) + \hat{k}\underbrace{(R\cos^2\theta + R\,\text{sen}^2\theta)}_{R.}$$

$$\phi_R \times \phi_\theta = (4R^2\cos\theta,\ 4R^2\,\text{sen}\,\theta,\ R)$$

$$\phi_R \times \phi_\theta = (4R^2\cos\theta, \; 4R^2 \operatorname{sen}\theta, \; R)$$

$$\vec{F} = x, \; y, \; \underline{2z}$$

P //

$$F(R,\theta) = R\cos\theta, \; R\operatorname{sen}\theta, \; 2(4-2R^2)$$

$$F(R,\theta) = (R\cos\theta, \; R\operatorname{sen}\theta, \; \underline{8-4R^2})$$

$$P = (4R^3\cos^2\theta + 4R^3\operatorname{sen}^2\theta + 8R - 4R^3)$$

$$P = 4R^3(\underbrace{\cos^2\theta + \operatorname{sen}^2\theta}_{1} - 1) + 8R.$$

$$1 - 1 \to 0$$

$$P = 8R.$$

$$\int_0^{\sqrt{2}}\int_0^{2\pi} 8R \, d\theta \, dR = \int_0^{\sqrt{2}} 8R \cdot 2\pi$$

$$F_m = \frac{8}{\cancel{2}} R^2 \cdot \cancel{2}\pi \Big|_0^{\sqrt{2}} = \boxed{16\pi}$$

$\underline{F_T}$ $\Rightarrow$

$$\phi = (\underline{R\cos\theta}, \; \underline{R\operatorname{sen}\theta}, \; \underline{0})$$

$$\phi_R = \cos\theta, \; \operatorname{sen}\theta, \; 0$$

$$\phi_\theta = -R\operatorname{sen}\theta, \; R\cos\theta \; 0$$

$$\phi_R \times \phi_\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \operatorname{sen}\theta & 0 \\ -R\operatorname{sen}\theta & R\cos\theta & 0 \end{vmatrix}$$

$$= 0\hat{i} + 0\hat{j} + R\cos^2\theta + R\operatorname{sen}^2\theta$$

$$= (0, 0, R)$$

$$\iint_D \vec{F}(\phi(R,\theta)) \cdot (0,0,R)\, dR\, d\theta$$

$$= 0$$

$$F_T = 0$$

$$\therefore \underline{F_T = 16\pi} \,//$$

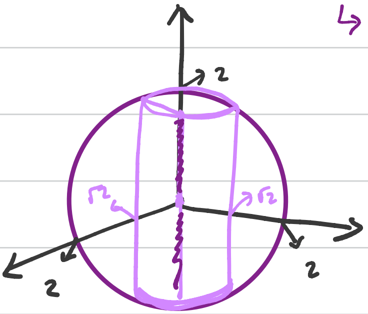
PREGUNTA 2:

$$\vec{V}(x,y,z) = (x-yz)\hat{i} + (y+xz)\hat{j} + (z+2xy)\hat{k}$$

S_1 : $x^2+y^2=2$ → dentro de : $x^2+y^2+z^2=4$

↳ $R=\sqrt{2}$

$$\iint_{S_1} \vec{V}\cdot\hat{n}\, dA \rightarrow \hat{n} \text{ normal interior}$$



$$\underbrace{x^2+y^2}+z^2=4$$

$$R^2+z^2=4$$

$$2+z^2=4 \longrightarrow z=\pm\sqrt{2}$$

$$z\in[-\sqrt{2},\sqrt{2}]$$

$$\theta\in[0,2\pi]$$

$$\phi(\theta,z) = (\sqrt{2}\cos\theta, \sqrt{2}\,\text{sen}\,\theta, z)$$

$$\phi_\theta = (-\sqrt{2}\,\text{sen}\,\theta, \sqrt{2}\cos\theta, 0)$$

$$\phi_z = (0, 0, 1)$$

$$\phi_\theta \times \phi_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sqrt{2}\,\text{sen}\,\theta & \sqrt{2}\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \hat{i}(\sqrt{2}\cos\theta) - \hat{j}(-\sqrt{2}\,\text{sen}\,\theta) + 0$$

$$\rightarrow \phi_\theta \times \phi_z = \underbrace{(\sqrt{2}\cos\theta, \sqrt{2}\,\text{sen}\,\theta, 0)}_{\|\cdot\|} \cdot (-1)$$

$$\hat{n} = \frac{(-\sqrt{2}\cos\theta\,,\,-\sqrt{2}\,\text{sen}\,\theta\,,\,0)}{\cancel{\|\cdot\|}} \quad \cdot \quad \cancel{\|\cdot\|}$$

$$\vec{V}(x,y,z) = (x - yz)\,\hat{i} + (y + xz)\,\hat{j} + (z + 2xy)\,\hat{k}$$

$$V(\theta, z) = \left(\sqrt{2}\cos\theta - \sqrt{2}\,\text{sen}\,z\right)\hat{i}$$
$$+ \left(\sqrt{2}\,\text{sen}\,\theta + \sqrt{2}\cos\theta\, z\right)\hat{j}$$
$$+ \left(z + 2\cos\theta\,\text{sen}\,\theta\right)$$

P

$$P = -2\cos^2\theta + \cancel{2\,\text{sen}\,\theta\, z\cos\theta}$$
$$-2\,\text{sen}^2\theta - \cancel{2\cos\theta\,\text{sen}\,\theta\, z} + 0$$

$$P = -2\,(\underbrace{\cos^2\theta + \text{sen}^2\theta}_{1})$$

$P = -2$.

$$\int_0^{2\pi}\int_{-\sqrt{2}}^{\sqrt{2}} -2\,dz\,d\theta = -2\pi \cdot 2 \cdot z\Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= -4\pi\left(\sqrt{2} + \sqrt{2}\right)$$

$$= -8\pi\sqrt{2} \quad //$$

PREGUNTA 3:

$T(x, y, z) = 3x^2 + 3z^2$ Ademas el

Flujo de calor $F = \boxed{-}k\nabla T$; $k = 1$

$S := x^2 + z^2 = 2 \quad 0 \leq y \leq 2$.

$F = -k\nabla T \qquad \nabla T = (6x, 0, 6z)$

$$\Rightarrow \vec{F} = (-6x, 0, -6z)$$

$\phi(\cdot, \cdot) \qquad x^2 + z^2 = 2.$

$R^2\cos^2\theta + R^2\operatorname{sen}^2\theta = 2$

$\Rightarrow R = \sqrt{2}. \leftarrow \qquad \theta \in [0, 2\pi]$

$y \in [0, 2)$.

$$\phi(\theta, y) = (\sqrt{2}\cos\theta, y, \sqrt{2}\operatorname{sen}\theta)$$

$$\phi_\theta = (-\sqrt{2}\operatorname{sen}\theta, 0, \sqrt{2}\cos\theta)$$

$$\phi_y = (0, 1, 0)$$

$$\phi_\theta \times \phi_y = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sqrt{2}\operatorname{sen}\theta & 0 & \sqrt{2}\cos\theta \\ 0 & 1 & 0 \end{vmatrix}$$

$$\hookrightarrow = \hat{i}(-\sqrt{2}\cos\theta) - \hat{j}(0) + \hat{k}(-\sqrt{2}\operatorname{sen}\theta)$$

$$= (-\sqrt{2}\cos\theta, 0, -\sqrt{2}\operatorname{sen}\theta)$$

$$(-6x, 0, -6z)$$

$$F(\phi(\theta, y)) = (-6 \cdot \sqrt{2}\cos\theta, \underline{0}, \underline{-6\sqrt{2}\operatorname{sen}\theta})$$

$$F(\phi(\theta, y)) \cdot (\phi_\theta \times \phi_y) = P.$$

$$P = 12\cos^2\theta + 12\operatorname{sen}^2\theta$$

$$P = 12.$$

$$\int_0^{2\pi}\int_0^{2} 12 \, dy \, d\theta = 12 \cdot 4\pi = \underline{\underline{48\pi}}$$