AUXILIAR 4

pregunta 1 [control 2015]

Para S dada por $z = x^2 + y^2$, $z \leq 1$ calcule

$$\iint_S \frac{x^2}{z} \, ds.$$

$\left[\rho^2 \cos^2\theta + \rho^2 \operatorname{sen}^2\theta \right]$
$\rho^2 \cdot (1) = z$

$\varphi(\rho, \theta) \to$ coordenadas polares

$$x = \rho \cos\theta, \quad y = \rho \operatorname{sen}\theta, \quad z = \rho^2$$

$$\varphi(\rho, \theta) = (\rho\cos\theta, \rho\operatorname{sen}\theta, \rho^2)$$

$$\theta \in [0, 2\pi] \; ; \; \rho \in [0, 1]$$

$$ds = \| \varphi_\rho \times \varphi_\theta \| \, d\rho \, d\theta$$

$$\varphi_\rho = (\cos\theta, \operatorname{sen}\theta, 2\rho)$$

$$\varphi_\theta = (-\rho\operatorname{sen}\theta, \rho\cos\theta, 0)$$

$$\varphi_\rho \times \varphi_\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \operatorname{sen}\theta & 2\rho \\ -\rho\operatorname{sen}\theta & \rho\cos\theta & 0 \end{vmatrix}$$

$$\times = \hat{i}\left(-2\rho^2\cos\theta\right) - \hat{j}\left(2\rho^2\operatorname{sen}\theta\right) + \hat{k}\left(\rho\cos^2\theta + \rho\operatorname{sen}^2\theta\right)$$

$$\hat{k}\,\rho.$$

$$\|\cdot\| = \sqrt{4\rho^4 \cos^2\theta + 4\rho^4 \operatorname{sen}^2\theta + \rho^2}$$

$$\|\cdot\| = \sqrt{4\rho^4 + \rho^2} = \rho\sqrt{4\rho^2 + 1}.$$

$$\Rightarrow \quad ds = \rho\sqrt{4\rho^2+1}\; d\rho\, d\theta$$

$$\Rightarrow \quad \boxed{\iint_S \frac{x^2}{z}\, ds} =$$

$$\int_0^1 \int_0^{2\pi} \frac{\cancel{\rho^2} \cos^2\theta}{\cancel{\rho^2}}\, \rho\sqrt{4\rho^2+1}\; d\theta\, d\rho.$$

$$= \underbrace{\int_0^1 \rho\sqrt{4\rho^2+1}\; d\rho}_{1} \cdot \int_0^{2\pi} \cos^2\theta\; d\theta.$$

NOTA

$$1) \int_0^1 \rho\sqrt{4\rho^2+1}\; d\rho$$

CAMBIO D VARIABLE

$$u = 4\rho^2 + 1$$

$$du = 8\rho\, d\rho$$

$$d\rho = \frac{du}{8\rho}$$

PRIMITIVA

$$\int \rho\sqrt{u}\; \frac{du}{8\rho}$$

$$\Rightarrow \frac{1}{8}\int \sqrt{u}\; du = \frac{1}{8}\cdot\frac{2}{3}\, u^{3/2} + c$$

$$= \frac{1}{12}\cdot\left(4\rho^2+1\right)^{3/2}\Big|_0^1 = \frac{1}{12}\left[5^{3/2} - 1\right]$$

2) $$\int_0^{2\pi} \cos^2\theta \, d\theta$$

RECORDAR:

$$\cos^2\theta = \frac{\cos(2\theta)+1}{2}$$

$$= \frac{1}{2}\left[\int_0^{2\pi} \cos(2\theta)\, d\theta + \int_0^{2\pi} d\theta\right]$$

$$\cos(2\theta) = \cos^2\theta - \text{sen}^2\,\theta$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$2\cos^2\theta - 1$$

$$\frac{1}{\cancel{2}}\left[\frac{\text{sen}(2\theta)}{2}\Big|_0^{2\pi} + \cancel{2}\pi\right]$$

$$= \pi$$

$$\Rightarrow \iint_S \frac{x^2}{z}\, ds = \frac{1}{12}\left[5^{3/2} - 1\right]\cdot\pi \quad //$$

pregunta 2 [2009]

Para el campo:

$$\vec{F}(\vec{r}) = \underbrace{\left(2\theta + \sqrt{z+\rho^2}\right)}_{F_\rho}\hat{\rho} + \left(\frac{1}{\rho}\, e^{\theta^2}\right)\hat{\theta} + \underbrace{\left(\theta^2 + \log\left(1+z^2\right)\right)}_{F_z}\hat{z}$$

calcule rot $\vec{F}$

$$\text{rot } F = \frac{1}{h_u h_v h_w}\begin{vmatrix} h_u\hat{u} & h_v\hat{v} & h_w\hat{w} \\ \frac{\partial}{\partial u} & \partial_{\partial v} & \partial/\partial w \\ h_u F_u & h_v F_v & h_w F_w \end{vmatrix}$$

$h_\rho = 1$, $h_\theta = \rho$, $h_z = 1$

$$\text{rot } F = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\theta} & \hat{z} \\ \partial\rho & \boxed{\partial\theta} & \partial z \\ 2\theta + \sqrt{z+\rho^2} & \boxed{e^{\theta^2}} & \theta^2 + \log(1+z^2) \end{vmatrix}$$

$$\text{rot } \vec{F} = \frac{1}{\rho}\left[\hat{\rho}\,(2\theta - 0) - \rho\hat{\theta}\,(0-0) + \hat{z}\,(0-2)\right]$$

$$\Rightarrow \text{rot } \vec{F} = \frac{1}{\rho}\left[2\theta\,\hat{\rho} - 2\hat{z}\right] \,//$$

pregunta 3 [2010]

$$\vec{F} = \frac{2\cos\theta}{r^3}\,\hat{r} + \frac{\operatorname{sen}\theta}{r^3}\,\hat{\theta} \qquad F = 0$$

$$\operatorname{div} \vec{F} = \frac{1}{h_u h_v h_w}\left[\frac{\partial}{\partial u}\left(F_u h_v h_w\right) + \frac{\partial}{\partial v}\left[h_u F_v h_w\right] + \frac{\partial}{\partial w}\left[h_u h_v F_w\right] \right]$$

$$h_r = 1 \,, \quad h_\theta = r \,, \quad h_z = 1$$

$$\operatorname{div} F = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{2\cos\theta}{\cancel{r^3}\, r^2} \cdot \cancel{r} \right) + \frac{\partial}{\partial \theta}\left[\frac{\operatorname{sen}\theta}{r^3} \right] \right]$$

$$\operatorname{div} F = \frac{1}{r}\left[\frac{-2 \cdot 2\cos\theta}{r^3} + \frac{\cos\theta}{r^3} \right]$$

$$= \frac{1}{r} \cdot \frac{\left[-3\cos\theta\right]}{r^3}$$

$\text{rot}\ F = 0.$

$$\text{rot}\ F = \frac{1}{r}\begin{bmatrix} \hat{r} & r\hat{\theta} & \hat{z} \\ \partial r & \partial \theta & \partial z \\ F_r & rF_\theta & 0 \end{bmatrix}$$

$F_\theta = \frac{\text{sen}(\theta)}{r^{\cancel{2}}} \cdot \cancel{\phi}$

$= \frac{(-2)\cdot \text{sen}\,\theta}{r^3}$

$$= \text{rot}\ F = \frac{1}{r}\left[\hat{r}(0) - r\hat{\theta}(0) + \hat{z}\left(\cancel{\frac{-2\,\text{sen}\,\theta}{r^3}} - \cancel{\left(-\frac{2\,\text{sen}\,\theta}{r^3}\right)}\right)\right.$$

$F_r = \frac{2\cos\theta}{r^3}$

$-\frac{2\,\text{sen}\,\theta}{r^3}$

$\Rightarrow \text{rot}\ F = 0$ //