Neural networks: Perceptron

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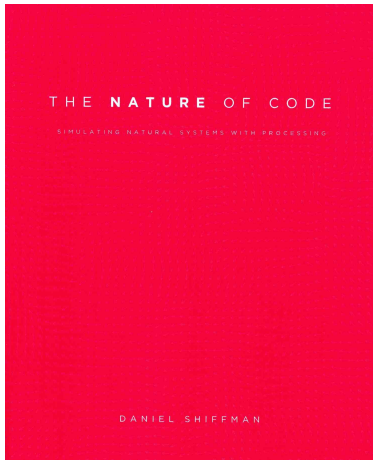
02/09/2020

Source

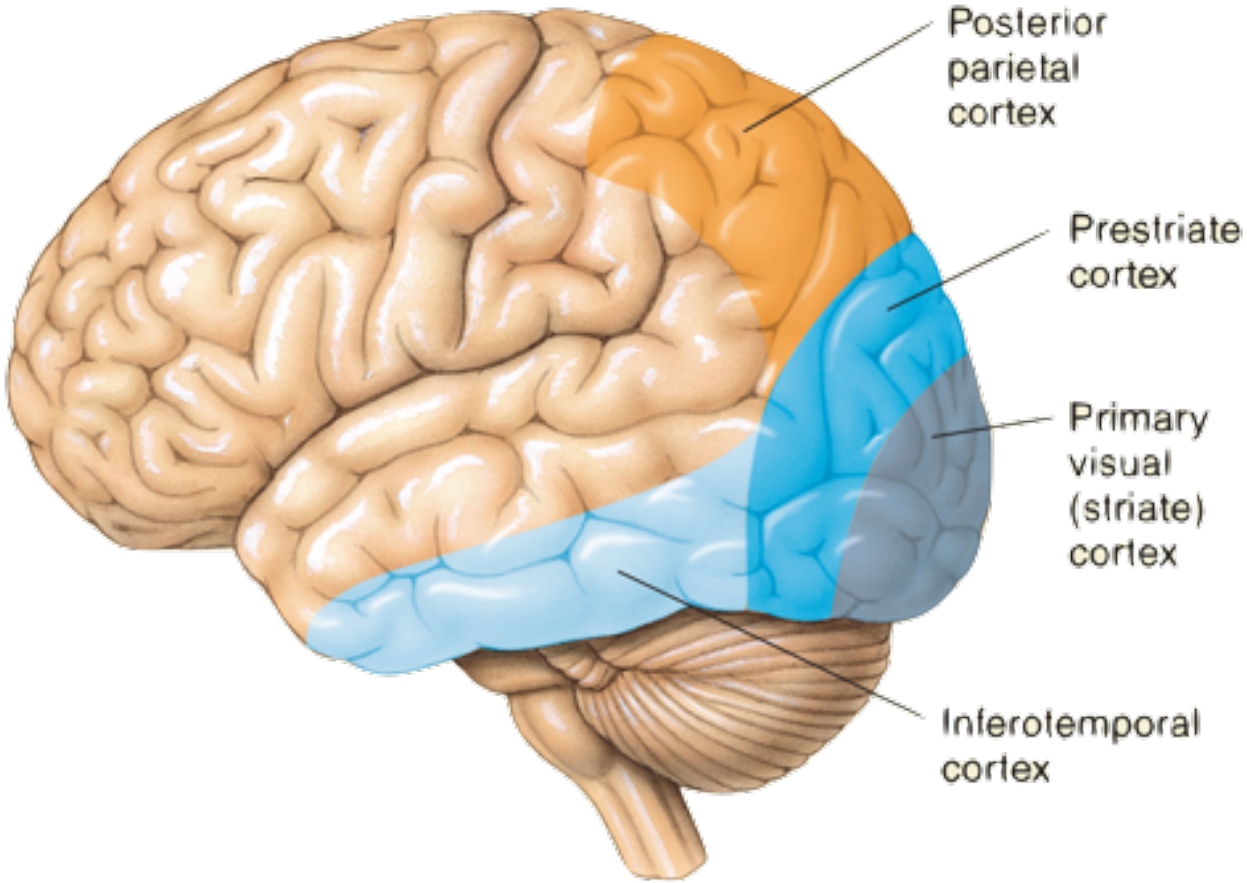
http://neuralnetworksanddeeplearning.com



Daniel Shiffman
http://natureofcode.com

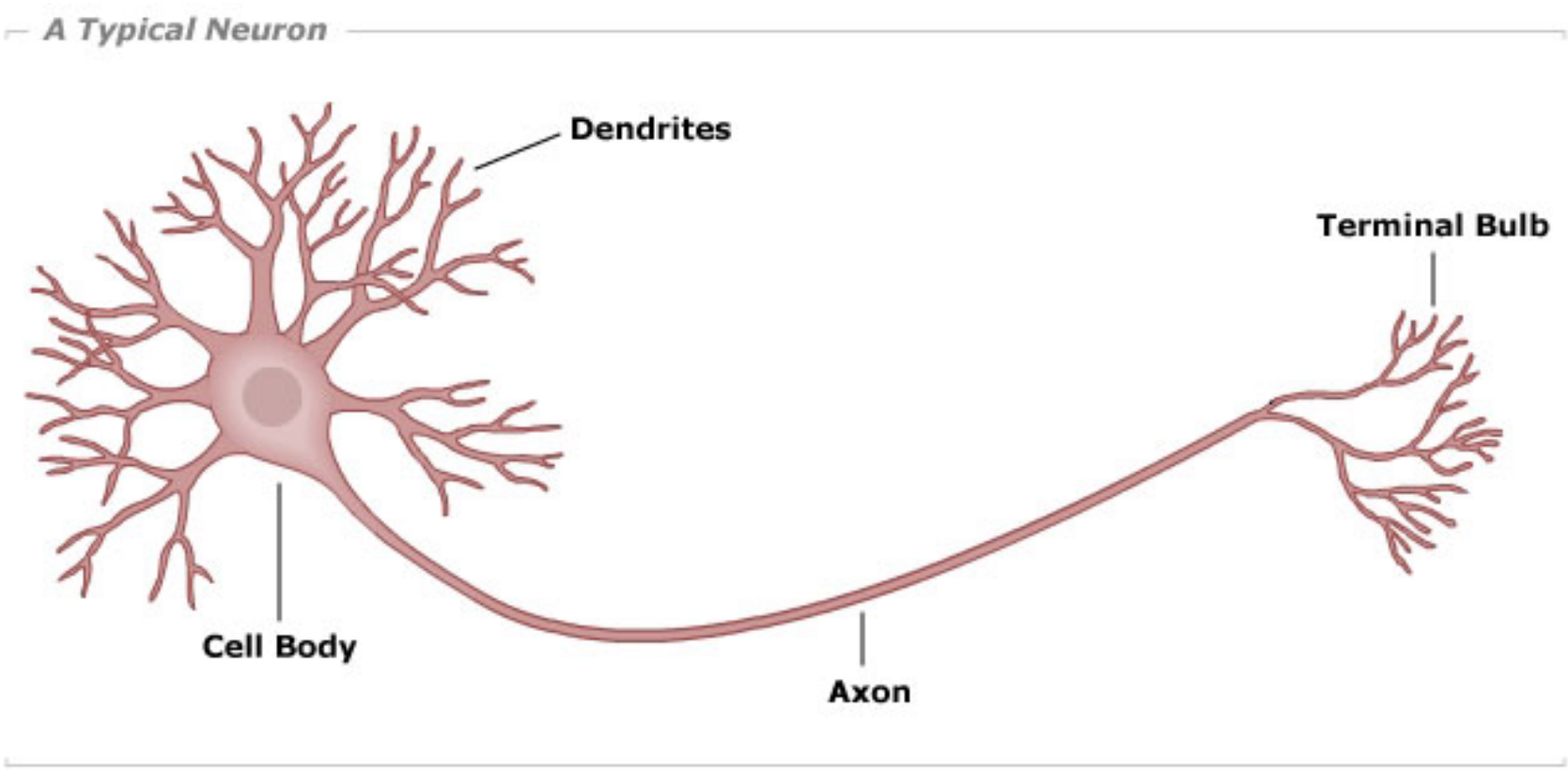
Outline

1. Biological connection
2. Model for decision-making
3. Perceptron
4. Formulating logical expressions
5. Exercises



The primary visual cortex is highly specialized for processing information about static and moving objects and is excellent in pattern recognition.

It contains 140 M of neurons, with tens of billions of connections



Dendrite: propagate electrochemical stimulation received from other neural cells

Axon: conducts electrical impulses away from the neuron

Logical calculus about nervous activity

"*A logical calculus of the ideas immanent in nervous activity*"

Warren S. Mcculloch, Walter Pitts, 1943

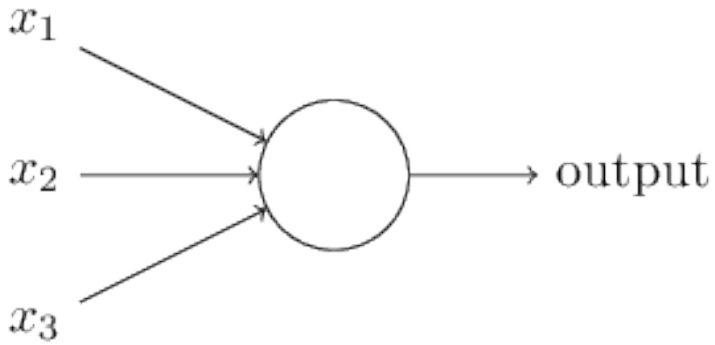
Bulletin of Mathematical Biophysics, Vol. 5, pp. 115-133 (1943)

> 14 000 citations

Perceptron

A *perceptron* is a kind of *artificial neuron*

Developed in the 50s and 60s by Frank Rosenblatt, Warren McCulloch, Walter Pitts



Takes several binary inputs, x1, x2, … and produces a single binary output

Perceptron

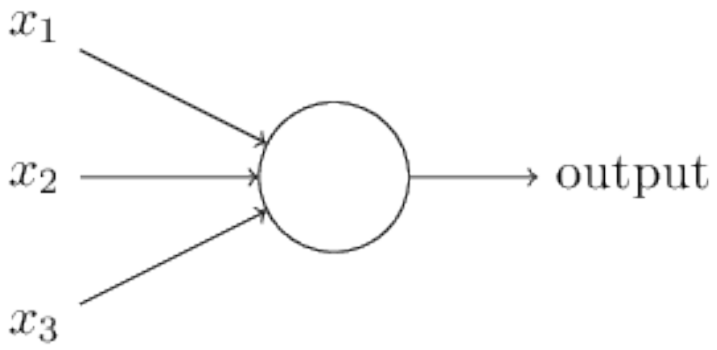
Rosenblatt proposed a simple rule to compute the output

Weights $(w_1, w_2, \ldots)$ are real numbers expressing the importance of the respective inputs to the output

The neuron's output, 0 or 1, is determined by whether the weighted sum $\sum_j w_j x_j$

is less or greater than a *threshold* value

Perceptron



This is how a perceptron works. A perceptron is a device that makes decisions by weighing up evidence

$$\text{output} = \begin{cases} 0 & \text{if } \sum_j w_j x_j \leq \text{ threshold} \\ 1 & \text{if } \sum_j w_j x_j > \text{ threshold} \end{cases}$$

Perceptron 

Suppose there is a great metal concert this weekend

You love metal, and you are wondering if you should go or not to the concert

You may want to make your decision by weighing up three factors:

- Is the weather good?
- Does your brother/sister/polol{o,a} want to accompany you?
- Is the festival near a metro stop? (You do not like driving)

Perceptron 

Suppose there is a great metal concert this weekend

You love metal, and you are wondering if you should go or not to the concert

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x1 Is the weather good?

x2 Does your brother/sister/polol{o,a} want to accompany you?

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Perceptron 

x1 Is the weather good?

x2 Does your brother/sister/polol{o,a} want to accompany you?

x3 Is the festival near a metro stop? (You do not like driving)

If you are a true-metal lover who love to share, then you may want to go even if the weather is bad and there is no stop near-by.

In that case, w1 = 2, w2 = 6, w3 = 2

Perceptron 

x1 Is the weather good?

x2 Does your brother/sister/polol{o,a} want to accompany you?

x3 Is the festival near a metro stop? (You do not like driving)

… or if you wish to not weak up your parents late in the evening: w1 = 1, w2 = 1, w3 = 8

Perceptron 

We are using the perceptron to model a simple decision-making.

If we pick 5 as our threshold, then we have the following condition:

$$(x1 * w1) + (x2 * w2) + (x3 * w3) > 5$$

If the condition is true, then the perceptron outputs 1, else it output 0

Perceptron 

Varying the weights and the threshold produces a new model of decision-making

x1 Is the weather good?

x2 Does your brother/sister/polol{o,a} want to accompany you?

x3 Is the festival near a metro stop? (You do not like driving)

w1 = 2	w1 = 2
w2 = 6	w2 = 6
w3 = 2	w3 = 2
t = 5	t = 2

Decreasing t means that you are more willing to the metal party

Simplifying the perceptron

The notation $\sum_j w_j x_j$ is cumbersome in some cases

For example, you may want to have a threshold per perceptron instead of a unique global threshold

First, we can write: $w \cdot x \equiv \sum_j w_j x_j$

Second, we can move the threshold to the other side of the equation: *threshold* is now named *bias*

The perception can be rewritten

$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

Simplifying the perceptron

You can think of the bias as a measure of how easy it is to get the perceptron to output a 1

In Biological terms: the bias is a measure of *how easy it is to get the perceptron to fire*

Big positive bias => easy for the perceptron to fire 1

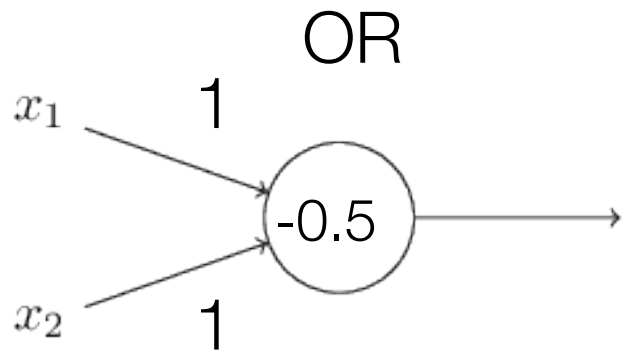
Very negative bias => difficult to fire 1

$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

Formulating logical equation

AND, OR are elementary logical functions

Consider the following perceptron:



$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

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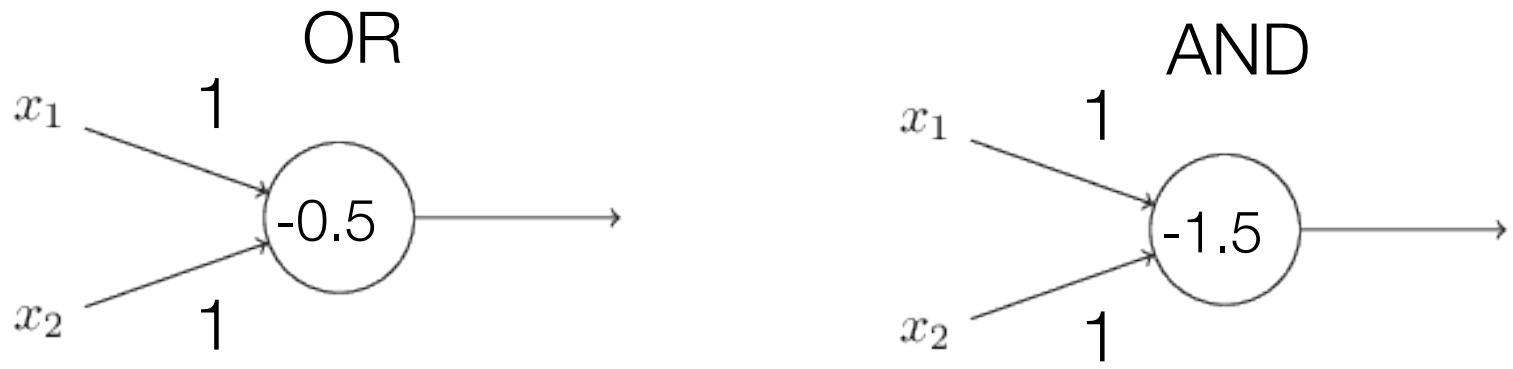
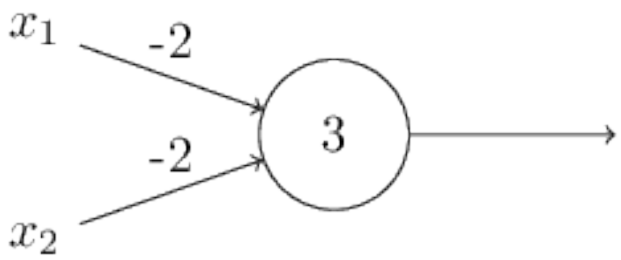
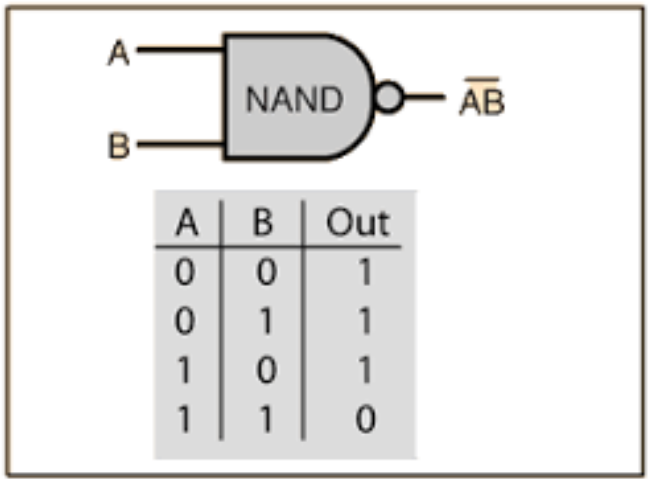


$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

Formulating logical equation

The following perceptron model the NAND logical gate

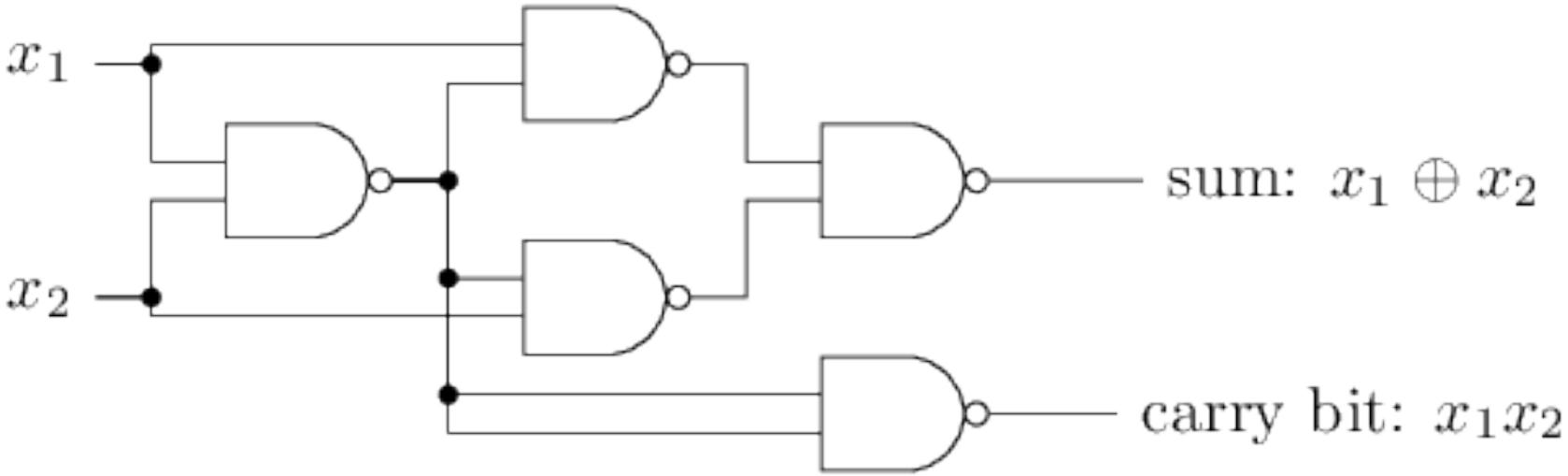




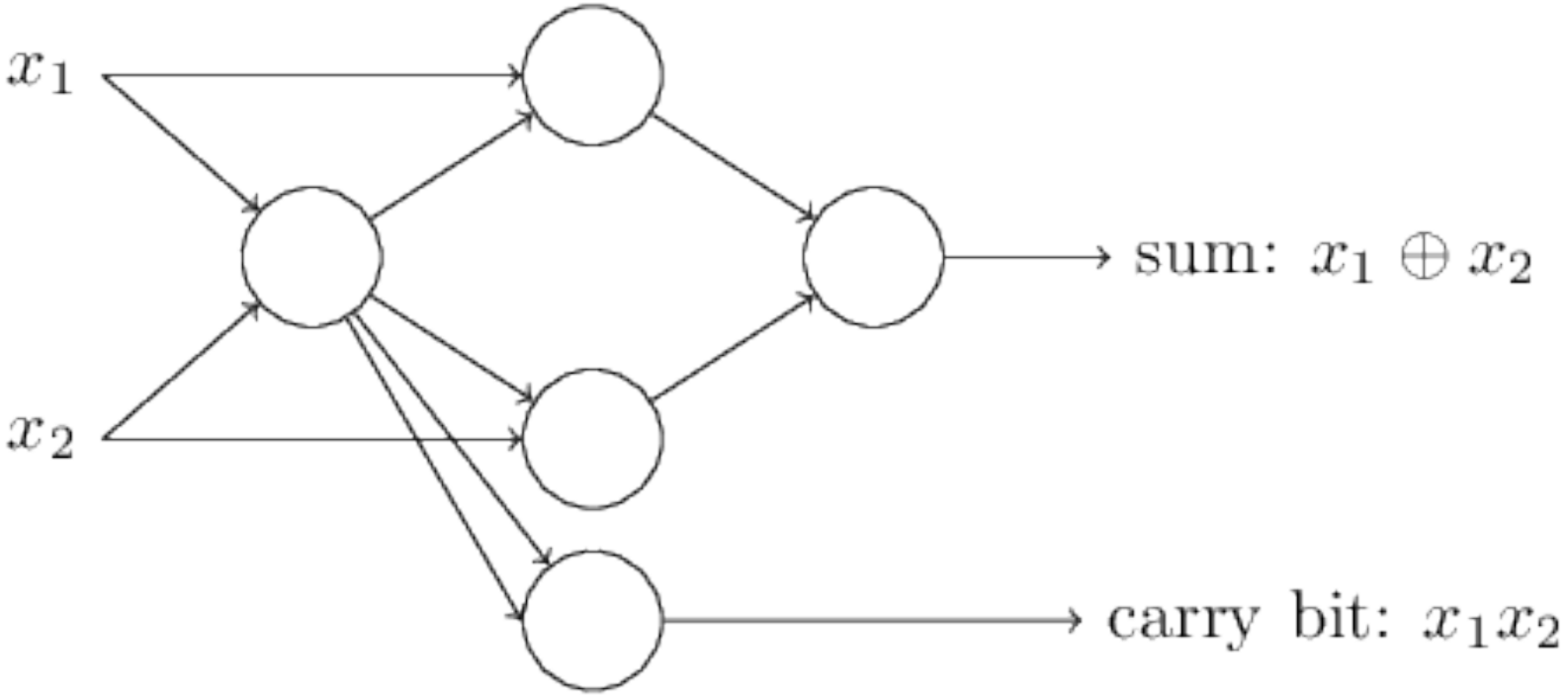
A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

Summing numbers

We can use NAND gates to build a circuit which adds two bits, x1 and x2.



Summing numbers



We can use NAND gates to build a circuit which adds two bits, x1 and x2.

Logical gates

The previous example demonstrates how a network of perceptrons can be used to simulate a circuit containing many NAND gates

The NAND gate is universal for computation:

It is the basic building block of the Arithmetic Logical Unit (ALU), which is at the core of a CPU

Exercises

- Provide the implementation of
  - a Perceptron, with the AND, OR, NAND behavior
  - The summing number gate
- Suben su código en GitHub
  - Comparten su codigo con Ignacio.
  - GitHub Id: islaterm
- Write unit test to test your code

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