P1) a)

Encontrar las densidades marginales f_X y f_Y

$$f_{X,Y}(x,y) = \frac{x}{36} \mathbb{1}_{0<x<y<6}$$

$$\rightarrow f_X(x) = \int_{\mathbb{R}} \frac{x}{36} \mathbb{1}_{\cdot/\cdot}\, dy = \frac{x}{36} \int_{\mathbb{R}} \mathbb{1}_{0<x<6} \mathbb{1}_{x<y<6}\, dy = \frac{(6-x)x}{36} \mathbb{1}_{0<x<6}$$

$$f_Y(y) = \int_{\mathbb{R}} \frac{x}{36} \mathbb{1}_{0<y<6} \mathbb{1}_{0<x<y}\, dx = \frac{1}{36}\left(\frac{y^2}{2} - \frac{0^2}{2}\right) \mathbb{1}_y = \frac{y^2}{72} \mathbb{1}_{0<y<6} \text{//}$$

b) Para calcular las esp. condicionales, primero necesito las dens. condicionales:

$$f(x|y) = \frac{f_c(x,y)}{f_y(y)} = \frac{\frac{x}{36}\,1_{x,y}}{\frac{y^2}{72}\,1_y} = \frac{2x}{y^2}\,1_{0<x<y}$$

$$f(y|x) = \frac{\frac{x}{36}\,1_{x,y}}{\frac{(6-x)x}{36}\,1_x} = \frac{1}{6-x}\,1_{x<y<6}$$

Procedemos a calcular las esperanzas:

$$\mathbb{E}(X \mid Y=\bar{y}) = \int_{\mathbb{R}} x \, f(x \mid \bar{y})\, dx = \int_0^{\bar{y}} \frac{2x^2}{\bar{y}^2}\, dx = \frac{2}{\bar{y}^2}\,\frac{\bar{y}^3}{3} = \frac{2\bar{y}}{3}$$

$$\mathbb{E}(Y \mid X=\bar{x}) = \int_{\bar{x}}^{6} y \, \frac{1}{6-\bar{x}}\, dy = \frac{1}{6-\bar{x}}\left(\frac{6^2}{2} - \frac{\bar{x}^2}{2}\right) = \frac{6+\bar{x}}{2} \quad !!$$

c) Nos piden calcular ahora E(X) y E(Y)

$$E(X) = E(\underbrace{E(X|Y)}_{g(Y)}) = \int_{\mathbb{R}} g(y)\, f_Y(y)\, dy = \int_0^6 \frac{2y}{3} \cdot \frac{y^2}{72} = \frac{2}{3 \cdot 72}\left(\frac{6^4}{4}\right) = \frac{6^2}{12} = \underline{3}$$

$$E(Y) = \int_{\mathbb{R}} h(x)\, f_X(x)\, dx = \int_0^6 \frac{6+x}{2} \, \frac{(6-x)x}{36}\, dx = \underline{4{,}5}$$