

$$E(g(x)) = \sum_{\underline{x} \in \mathcal{R}_x} g(\underline{x}) P(X = \underline{x})$$

$$g(\underline{x}) = c \quad \forall \underline{x}$$

$$E(c) = E(g(x)) = \sum_{\underline{x} \in \mathcal{R}_x} g(\underline{x}) P(X = \underline{x}) \xrightarrow{c}$$

$$= \sum_{\underline{x}} c P(X = \underline{x})$$

$$= C \sum_{x \in R_x} P(X=x) = C$$

↑

$$\mathbb{E}(\cancel{X}) = cte$$

$$\mathbb{E}(C) = C \quad \checkmark \checkmark$$

$$\mathbb{E}(\mathbb{E}(X)) = \mathbb{E}(X)$$

X, Y v. l. n's: discretas

$$\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$

$$= \sum_{x \in R_X} \sum_{y \in R_Y} (x + y) P(X = x, Y = y)$$

$$\begin{aligned}
 & \sum_{R_x} \sum_{R_y} x P(X=x, Y=y) \\
 & \quad (1) \quad + \sum_{R_x} \sum_{R_y} y P(X=x, Y=y)
 \end{aligned}$$

(1) $\sum_{x \in R_X} x \sum_{y \in R_Y} P(X=x, Y=y)$

$\rightarrow \sum_{y \in R_Y} P(X=x | Y=y) P(Y=y)$

$\rightarrow P(X=x)$

$$(1) \sum_{x \in \mathcal{R}_X} x \mathbb{P}(X=x) \\ = \mathbb{E}(X)$$

Y caso $n^2 \log n$

$$\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$

$$aX + bY$$

(★)

$$\mathbb{E}(Z + W) = \mathbb{E}(aX) + \mathbb{E}(bY)$$

$$(\star) \Rightarrow \sum a x P_X(x) = a \mathbb{E}(X)$$

$$\begin{aligned} E(ax + by) &= E(ax) + E(by) \\ &= aE(x) + bE(y) \end{aligned}$$

$$\mu = E(X)$$

$$\text{var}(X) = E((X - \mu)^2)$$

$$= E(X^2 - 2\mu X + \mu^2)$$

$$= E(X^2) - 2\mu E(X) + E(\underline{\underline{\mu^2}})$$

$$= E(x^2) - 2\mu \underbrace{E(x)} + \mu^2$$

$$= E(x^2) - 2\overset{\mu}{\mu}^2 + \mu^2$$

$$= E(x^2) - E(x)^2$$

...

$$\boxed{P_2} \quad \underline{\underline{E(X) = 0}}$$

$$\sum_{\substack{x \in \mathbb{R}_x \\ x \geq 0}} x P(X=x) = 0 \cdot P(X=0) + \sum_{x \geq 1} x \cancel{P(X=x)} \rightarrow 0 = 0$$

$$\sum_{x \geq 0} P(X=x) = 1$$

$$P(X=0) + \sum_{x \geq 1} P(X=x) = 1$$

$$P(X=0) = 1$$

P4 $X \sim \text{Geom}(p)$, $E(X) = \frac{1}{p}$

$$P(X=k) = (1-p)^{k-1} p$$

$k = 1, 2, \dots$

$P \in (0, 1) \Rightarrow 1 - P \in (0, 1)$

$$0 \leq (1 - P)^{k-1} \cdot P \leq 1 \quad \forall k$$

$\leq 1 \quad \leq 1$

$$\begin{aligned}
 \sum_{k=1}^{\infty} (1-p)^{k-1} p &= p \sum_{k=1}^{\infty} (1-p)^{k-1} \\
 &= p \sum_{k=0}^{\infty} (1-p)^k = p \cdot \left(\frac{1}{1-(1-p)} \right)
 \end{aligned}$$

The diagram shows the derivation of the geometric series sum. Red arrows indicate the shift of the index k from 1 to 0 in the second step.

$$= P\left(\frac{1}{p}\right) = 1$$

$$1 - (1 - ?) = p$$

$$E(X) = \sum_{k \geq 1} k P(X = k)$$

$$\overline{H}(x) = \sum_{k \geq 1} \frac{k(1-p)^{k-1}}{p}$$

$$\sum_{k=0}^{\infty} x^k =$$

$$\frac{1}{1-x}$$

$\frac{d}{dx}$

$$\sum_{k=1}^{\infty} \frac{k x^{k-1}}{(1-x)^2}$$

$$\frac{1}{(1-x)^2}$$

$$\Rightarrow E(x) = \left(\frac{1}{\sum (1 - (1 - p))^2} \right) \cdot p$$

$$= \frac{p}{p^2} = \frac{1}{p}$$

$$\sum_{k \geq 0} x^k = \frac{1}{1-x}, \quad x \in (0,1)$$

$$\sum_{k \geq 0} \frac{x^k}{k!} = e^x, \quad x \in \mathbb{R}$$