$$\mathbb{E}(g(X)) = \sum_{x \in R_Y} g(x)\,\mathbb{P}(X = x)$$

$$g(x) = c \quad \forall x$$

c

$$\mathbb{E}(c) = \mathbb{E}(g(X)) = \sum_{x \in R_X} g(x)\,\mathbb{P}(X = x)$$

$$= \sum_{x} c\,\mathbb{P}(X = x)$$

$$= c \sum_{x \in R_X} \mathbb{P}(X = x) = c$$

$$\mathbb{E}(X) = cte$$

$$\mathbb{E}(c) = c \quad \checkmark\checkmark$$

$$\mathbb{E}(\mathbb{E}(X)) = \mathbb{E}(X)$$

X, Y v.a's: discretas

$$\mathbb{E}(\overset{x}{X}+\overset{y}{Y}) = \mathbb{E}(X) + \mathbb{E}(Y)$$

$$= \sum_{x \in R_X} \sum_{y \in R_Y} (x+y)\, \mathbb{P}(X=x, Y=y)$$

$$= \underbrace{\sum_{\substack{R_X \\ x}} \sum_{R_Y} x \, \mathbb{P}(X=x, Y=y)}_{(1)} + \sum_{R_X} \sum_{\substack{R_Y \\ y}} y \, \mathbb{P}(X=x, Y=y)$$

(1) $\displaystyle\sum_{x \in R_X} x \sum_{R_Y} \mathbb{P}(X=x, Y=y)$

$\displaystyle\sum_{y \in R_Y} \mathbb{P}(X=x \mid Y=y)\,\mathbb{P}(Y=y)$

$\to \mathbb{P}(X=x)$

$$(1) \sum_{x \in R_X} x \, \mathbb{P}(X = x)$$

$$= \mathbb{E}(X)$$

Y caso análogo

$$\mathbb{E}(X+Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$

$$aX + bY$$

$$(\star)$$

$$\mathbb{E}(Z + W) = \mathbb{E}(aX) + \mathbb{E}(bY)$$

$$(\star) \hookrightarrow \sum a x \, \mathbb{P}_X(x) = a\,\mathbb{E}(X)$$

$$\mathbb{E}(aX+bY)$$

$$= \mathbb{E}(aX) + \mathbb{E}(bY)$$

$$= a\,\mathbb{E}(X) + b\,\mathbb{E}(Y)$$

$$\mu = \mathbb{E}(X)$$

$$\operatorname{var}(X) = \mathbb{E}\left(\left(X - \mu\right)^2\right)$$

$$= \mathbb{E}\left(X^2 - 2\mu X + \mu^2\right)$$

$$= \mathbb{E}(X^2) - 2\mu\,\mathbb{E}(X) + \mathbb{E}(\mu^2)$$

$$= \mathbb{E}(x^2) - 2\mu \underbrace{\mathbb{E}(x)}_{\mu} + \mu^2$$

$$= \mathbb{E}(x^2) - 2\mu^2 + \mu^2$$

$$= \mathbb{E}(x^2) - \mathbb{E}(x)^2$$

$\underline{P_2}|$ $\underline{\underline{\mathbb{E}(X) = 0}}$

$$\sum_{\substack{x \in R_X \\ x \geq 0}} x \, \mathbb{P}(X = x) = 0 \cdot \mathbb{P}(X=0) + \sum_{x \geq 1} x \, \underbrace{\mathbb{P}(X=x)}_{0}$$

$$= 0$$

$$\sum_{x \geq 0} \mathbb{P}(X=x) = 1$$

$$\mathbb{P}(X=0) + \cancelto{0}{\sum_{x \geq 1} \mathbb{P}(X=x)} = 1$$

$$\mathbb{P}(X=0) = 1$$

$\underline{P4}$

$X \sim \mathrm{Geom}(p)$, $p \in (0,1)$

$$\mathbb{P}(X=k) = (1-p)^{k-1} p$$

$k = 1, 2, \ldots$

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$$p \in (0,1) \Rightarrow 1-p \in (0,1)$$

$$0 \leq \underbrace{(1-p)^{k-1}}_{\leq 1} \cdot \underbrace{p}_{\leq 1} \leq 1 \quad \forall k$$

$$\sum_{k \geq 1} (1-p)^{k-1} p$$

$$= p \sum_{k \geq 1} (1-p)^{k-1}$$

$$= p \sum_{k \geq 0} (1-p)^{k} = p \cdot \left(\frac{1}{1-(1-p)} \right)$$

$$= p\left(\frac{1}{p}\right) = 1$$

$$1-(1-p) = p$$

$$\mathbb{E}(X) = \sum_{k \geq 1} k\,\mathbb{P}(X=k)$$

$$\mathbb{E}(X) = \sum_{k \geq 1} k(1-p)^{k-1} \cdot p$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \xrightarrow{\frac{d}{dx}} \sum_{\substack{k=1 \\ k=0}}^{\infty} k x^{k-1} = \frac{1}{(1-x)^2}$$

$$\Rightarrow \mathbb{E}(X) = \left(\frac{1}{(1-(1-p))^2}\right) \cdot p$$

$$= \frac{p}{p^2} = \frac{1}{p}$$

$$\sum_{k \geq 0} x^k = \frac{1}{1-x} \quad , \quad x \in (0,1)$$

$$\sum_{k \geq 0} \frac{x^k}{k!} = e^x \quad , \quad x \in \mathbb{R}$$