P1-b

$$P(I=I_1 \mid x<a) \underset{\text{(Bayes)}}{=} \frac{P(x<a \mid I=I_1)\, P(I=I_1)}{P(x<a)} \qquad 0.5p$$

Se $a \leq L_1$

- $P(x<a \mid I=I_1) = \dfrac{a}{L_1}$ 0.1p
- $P(x<a \mid I=I_2) = \dfrac{a}{L_2}$ 0.1p
- $P(I=I_1) = \dfrac{1}{2}$ (aleatoriamente) 0.1p
- $P(x<a) = P(x<a \mid I=I_1) P(I=I_1) + P(x<a \mid I=I_2) P(I=I_2)$

$$= \frac{a}{L_1} \cdot \frac{1}{2} + \frac{a}{L_2} \cdot \frac{1}{2} \qquad 0.1p$$

$$\Rightarrow P(I=I_1 \mid x<a) = \frac{\frac{a}{L_1} \cdot \frac{1}{2}}{\frac{a}{L_1} \cdot \frac{1}{2} + \frac{a}{L_2} \cdot \frac{1}{2}} = \frac{\frac{1}{L_1}}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_2}{L_1 + L_2} \qquad 0.1p$$

Se $L_1 < a \leq L_2$

- $P(x<a \mid I=I_1) = 1$ 0.1p
- $P(x<a \mid I=I_2) = \dfrac{a}{L_2}$ 0.1p
- $P(I=I_1) = \dfrac{1}{2}$ 0.1p
- $P(x<a) = 1 \cdot \dfrac{1}{2} + \dfrac{a}{L_2} \cdot \dfrac{1}{2}$ 0.1p

$$\Rightarrow P(I=I_1 \mid x<a) = \frac{1 \cdot \frac{1}{2}}{1 \cdot \frac{1}{2} + \frac{a}{L_2} \cdot \frac{1}{2}} = \frac{L_2}{L_2 + a} \qquad 0.1p$$

Sea $L_2 < a$

$P(x<a \mid I=I_1) = 1$ 0.1 p

$P(x<a \mid I=I_2) = 1$ 0.1 p

$P(I=I_1) = \frac{1}{2}$ 0.1 p

$P(x<a) = 1\cdot\frac{1}{2} + 1\cdot\frac{1}{2}$ 0.1 p

$$\Rightarrow P(I=I_1 \mid x<a) = \frac{1\cdot\frac{1}{2}}{1\cdot\frac{1}{2}+1\cdot\frac{1}{2}} = \frac{1}{2}$$ 0.1 p

¿Qué pasa si $x=a$?

Consideremos la aproximación

$x \in [a, a+\varepsilon]$

$$P(I=I_1 \mid x\in[a,a+\varepsilon]) = \frac{P(x\in[a,a+\varepsilon] \mid I=I_1)\, P(I=I_1)}{P(x\in[a,a+\varepsilon])}$$ 0.25 p

Sea $a \leq L_1$

$P(x\in[a,a+\varepsilon] \mid I=I_1) = \frac{\varepsilon}{L_1}$ 0.05 p

$P(x\in[a,a+\varepsilon] \mid I=I_2) = \frac{\varepsilon}{L_2}$ 0.05 p

$P(I=I_1) = \frac{1}{2}$ 0.05 p

$P(x\in[a,a+\varepsilon]) = \frac{\varepsilon}{L_1}\cdot\frac{1}{2} + \frac{\varepsilon}{L_2}\cdot\frac{1}{2}$ 0.05 p

$$\Rightarrow P(I=I_1 \mid x\in[a,a+\varepsilon]) = \frac{\frac{\varepsilon}{L_1}}{\frac{\varepsilon}{L_1}\frac{1}{2}+\frac{\varepsilon}{L_2}\cdot\frac{1}{2}} = \frac{\frac{1}{L_1}}{\frac{1}{L_1}+\frac{1}{L_2}} = \frac{L_2}{L_1+L_2}$$ 0.05 p

Sea $L_1 < a \leq L_2$

$P(X \in [a, a+\varepsilon] | F = I_1) = 0$, porque $a > L_1$

0.25 p

luego $P(I = I_1 | X \in [a, a+\varepsilon]) = 0$

Sea $L_2 < a$

$P(X \in [a, a+\varepsilon | F = I_1) = 0$; porque $a > L_2 > L_1$

luego $P(I = I_1 | X \in [a, a+\varepsilon]) = 0$ 0.25 p

(no se puede aplicar Bayes porque se indetermine)

Escaneado con CamScanner

P2)

a) $$P(Y > Kx) = \int_{-\infty}^{\infty}\left(\int_{Kx}^{\infty} f_X(x) f_Y(y)\,dy\right)dx \underset{TFC}{=} \int_{-\infty}^{\infty} f_X(x)\left(\int_{Kx}^{\infty} f_Y(y)\,dy\right)dx$$

$$= \int_{-\infty}^{\infty} f_X(x)\left[F_Y(\infty) - F_Y(Kx)\right]dx$$

$$= \int_{-\infty}^{\infty} f_X(x)\left[1 - F_Y(Kx)\right]dx$$ 1p (descuentos | 0.7p | 0.3p)

Si $X \sim exp(\alpha)$ $\Rightarrow$ $f_X(x) = \alpha e^{-\alpha x}$ $(x>0)$

$Y \sim exp(\beta)$ $\quad F_Y(y) = 1 - e^{-\beta x}$ $(y>0)$ 1p (descuentos | 0.7p | 0.3p)

luego $$P(Y>Kx) = \int_{-\infty}^{\infty} \alpha e^{-\alpha}\left[1 - 1 + e^{-\beta Kx}\right]dx$$

$$= \int_{-\infty}^{\infty} \alpha\, e^{-(\alpha+\beta K)x}\,dx$$

$$= -\frac{\alpha}{\alpha+\beta K} e^{-(\alpha+\beta K)x}\Big|_{0}^{\infty}$$

$$= \frac{\alpha}{\alpha+\beta K}$$ 1p (descuentos | 0.7p | 0.3p)

b) $z = \frac{X}{X+Y}$ $\Rightarrow$ $X = wz$ 0.25p

$w = X+Y$ $\qquad Y = w(1-z)$

$$|J| = \begin{vmatrix} \frac{\partial X}{\partial w} & \frac{\partial X}{\partial z} \\ \frac{\partial Y}{\partial w} & \frac{\partial Y}{\partial z} \end{vmatrix} = \begin{vmatrix} z & w \\ 1-z & -w \end{vmatrix} = |-zw - w + wz| = w \qquad 0.25p$$

$$f_{WZ}(w,z) = f_{XY}(wz, w(1-z))\,|J|$$
$$= \alpha e^{-\alpha wz} \alpha e^{-\alpha w} e^{\alpha wz}\, w$$
$$= \alpha^2 w e^{-\alpha w} \qquad 0.5p$$

Como $w = X+Y \wedge x, y > 0 \Rightarrow \infty > w > 0$ 0.25p

$z = \frac{X}{X+y} = 1 - \underbrace{\frac{y}{X+y}}_{\in [0,1]} \Rightarrow 1 > z > 0$ 0.25p

Entonces

$$f_Z(z) = \int_{-\infty}^{\infty} f_{WZ}(w,z)dw = \int_0^{\infty} \alpha^2 w e^{-\alpha w} dw = \alpha^2\left(-\frac{w}{\alpha} e^{-\alpha w}\Big|_0^{\infty} + \frac{1}{\alpha}\int_0^{\infty} e^{-\alpha w} dw\right) = \alpha^2\left(-\frac{1}{\alpha^2} e^{-\alpha w}\Big|_0^{\infty}\right) = 1$$

0.5p

$$f_W(w) = \int_{-\infty}^{\infty} f_{WZ}(w,z)dz = \int_0^1 \alpha^2 w e^{-\alpha w} dz = \alpha^2 w e^{-\alpha w} \qquad 0.5p$$

Como $f_{WZ}(w,z) = f_W(w) \cdot f_Z(z) \Rightarrow$ son independientes 0.5p

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