P1

$$T' = K(T_A(t) - T(t))$$

$$T' + KT(t) = KT_A(t) \quad / \mu$$

$$\mu T' + \mu K T = \mu K T_A$$

Buscamos μ tal que $\mu K = \mu'$ pues así

$$\mu T' + \mu' T = \mu K T_A$$

$$(\mu T)' = \mu K T_A$$

Este μ es e^{Kt}

$$(e^{Kt} T)' = e^{Kt} K T_A \quad / \int$$

$$e^{Kt} T = K \int e^{Kt} T_A(t) dt + c$$

$$T(t) = Ke^{-Kt} \int e^{Kt} T_A(t) dt + ce^{-Kt}$$

$$T_A(t) = T_A^0 + A\operatorname{sen}(\omega t)$$

$$T(t) = \cancel{K}\cancel{e^{-Kt}} T_A^0 \frac{\cancel{e^{Kt}}}{\cancel{K}} + AKe^{-Kt} \int \operatorname{sen}(\omega t) e^{Kt} dt + ce^{-Kt}$$

$$T(t) = T_A^0 + AK \cdot \frac{K \operatorname{sen}(\omega t) - \omega \cos(\omega t)}{K^2 + \omega^2} + ce^{-Kt}$$

$$T_0 = T_A^0 + AK \cdot \frac{-\omega}{K^2 + \omega^2} + c \Rightarrow c = (T_0 - T_A^0) + \frac{AK\omega}{K^2 + \omega^2}$$

P2) a) $y'' = a\sqrt{1+y'^2}$

Sea $u = y'$ $\quad u' = a\sqrt{1+u^2}$

$$\int \frac{du}{\sqrt{1+u^2}} = \int a\,dx$$

Haciendo la sustitución
$s = \tan^{-1}(u)$
$u = \tan(s)$

$$\operatorname{arcsenh}(u) = ax + c$$

$$u = \operatorname{senh}(ax+c)$$

$$y' = \operatorname{senh}(ax+c)$$

$$y = \frac{\cosh(ax+c)}{a} + \tilde{c}$$

$$(*)\ y(0) = \frac{\cosh(c)}{a} + \tilde{c} = 0$$

$$y'(0) = \operatorname{senh}(c) = 0 \Rightarrow c = 0$$

Volviendo a (*) $y(0) = \frac{1}{a} + \tilde{c} = 0 \Rightarrow \tilde{c} = -\frac{1}{a}$

$$y(x) = \frac{\cosh(ax) - 1}{a}$$

b)

$$y' = -2\pi x y^2$$

$$\frac{y'}{y^2} = -2\pi x$$

$$\int \frac{dy}{y^2} = \int -2\pi x \, dx$$

$$-\frac{1}{y} = \frac{-\cancel{2}\pi x^2}{\cancel{2}} + c$$

$$\frac{1}{y} = \pi x^2 + c$$

$$y = \frac{1}{\pi x^2 + c}$$

$$1 = \int_{\mathbb{R}} y(x) dx = \int_{\mathbb{R}} \frac{1}{\pi x^2 + c} dx \qquad c = \pi \tilde{c}$$

$$= \frac{1}{\pi} \int_{\mathbb{R}} \frac{1}{x^2 + \tilde{c}} dx$$

$$= \frac{1}{\pi} \left. \frac{\operatorname{arctg}(x/\sqrt{\tilde{c}})}{\sqrt{\tilde{c}}} \right|_{-\infty}^{\infty}$$

$$= \frac{1}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{\sqrt{\tilde{c}}} \Rightarrow \tilde{c} = 1 \Rightarrow c = \pi$$

$$y(x) = \frac{1}{\pi(x^2+1)}$$

c) $y(1+xy)\,dx - x\,dy = 0$

$M(x,y) = y(1+xy) \qquad N(x,y) = -x$

$\frac{\partial M}{\partial y} = 1 + 2xy \qquad \frac{\partial N}{\partial x} = -1$

No es exacta :(, ¿cómo encontrar el factor integrante?
En general es difícil, pero en estos 2 casos siguientes es fácil

① Si $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x) \Rightarrow \mu = e^{\int^x f(s)ds}$

② Si $-\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = g(y) \Rightarrow \mu = e^{\int^x g(s)ds}$

En este caso pasa ② pues $\frac{-(2+2xy)}{y(1+xy)} = -\frac{2}{y} = g(y)$

$\mu = e^{\int -\frac{2}{y}} = e^{-2\ln(y)} = 1/y^2$

$\tilde{M} = \frac{1+xy}{y} \qquad \tilde{N} = -\frac{x}{y^2}$

$\frac{\partial u}{\partial x} = \frac{1}{y} + x \; / \int dx \qquad u = \frac{x}{y} + \frac{x^2}{2} + h(y)$

$\frac{\partial u}{\partial y} = -\frac{x}{y^2} + h'(y) = \tilde{N} \Rightarrow h'(y) = 0$

$\Rightarrow u = \frac{x}{y} + \frac{x^2}{2} + c$ Luego se iguala $u = \tilde{c}$ y despeja $y(x)$

$y(x) = \frac{2x}{c - x^2}$ //

Forma b: $y(1+xy)\,dx - x\,dy = 0$

$$\Updownarrow$$

$$y' = \frac{y}{x}(1+xy)$$

$$y' - \frac{y}{x} - y^2 = 0 \qquad \text{Ec Bernoulli} \qquad u = 1/y, \quad u' = -y'/y^2$$

Dividiendo en y^2 tenemos $\dfrac{y'}{y^2} - \dfrac{1}{x}\cdot\dfrac{1}{y} - 1 = 0$

$$-u' - \frac{1}{x}u - 1 = 0$$

$$u' + \frac{1}{x}u + 1 = 0$$

$$u' + \frac{1}{x}u = -1 \quad / \mu$$

μ tal que $\mu' = \frac{1}{x}\mu \Rightarrow \int \frac{\mu'}{\mu} = \int \frac{1}{x} \Rightarrow \ln \mu = \ln x$, $\mu = x$

$$\Rightarrow (xu)' = -x \quad / \int$$

$$xu = -\frac{x^2}{2} + c$$

$$u = -\frac{x}{2} + \frac{c}{x} = \frac{c - x^2}{2x}$$

$$y(x) = \frac{2x}{c - x^2}$$

c) $\circledast \quad y' = xy^2 + y + 1/x^2$

Verifiquemos que $y_1 = \frac{-1}{x}$ es solución

$$y_1' = \frac{1}{x^2} \overset{?}{=} x \cdot \cancel{\left(\frac{-1}{x}\right)^2} - \cancel{\frac{1}{x}} + \frac{1}{x^2}$$

Esta ecuación se llama EDO de Riccati.
La solución la buscaré en funciones del estilo $\tilde{y} = y_1 + 1/u$
Para esto necesito encontrar $u(x)$

$$\dot{\tilde{y}} = \frac{1}{x^2} - \frac{1}{u^2} u'$$

Sustituyendo en $\circledast$

$$\cancel{\frac{1}{x^2}} - \frac{u'}{u^2} = x\left(\frac{-1}{x} + \frac{1}{u}\right)^2 + \left(\frac{-1}{x} + \frac{1}{u}\right) + \cancel{\frac{1}{x^2}}$$

$$-\frac{u'}{u^2} = x\left(\frac{1}{x^2} - \frac{2}{xu} + \frac{1}{u^2}\right) - \frac{1}{x} + \frac{1}{u}$$

$$\frac{-u'}{u^2} = \cancel{\frac{1}{x}} - \frac{2}{u} + \frac{x}{u^2} - \cancel{\frac{1}{x}} + \frac{1}{u} \quad / \cdot u^2$$

$$-u' = -u + x \quad / \cdot -1$$

$$u' = u - x \Rightarrow u' - u = -x$$

$$u' - u = -x \quad / \mu = e^{-x}$$

$$(e^{-x}u)' = -e^{-x}x \quad / \int$$

$$e^{-x}u = -\int x e^{-x}\,dx$$

$$u(x) = -e^{x}\int \underbrace{x}_{u}\,\underbrace{e^{-x}dx}_{dv} \qquad \begin{array}{l} du = dx \\ v = -e^{-x} \end{array}$$

$$u(x) = -e^{x}\left[-xe^{-x} + \int e^{-x}\,dx + c\right]$$

$$= -e^{x}\left[(-x-1)e^{-x} + c\right]$$

$$= (1+x) + ce^{x}$$

$$\Rightarrow y(x) = -\frac{1}{x} + \frac{1}{1+x+ce^{x}}$$