1

Matroca Sumatorias

a) $\sum_{j=1}^{m}\sum_{k=1}^{j}\left(k+\frac{2^j}{j}\right)=\sum_{j=1}^{m}\left(\sum_{k=1}^{j}k+\sum_{k=1}^{j}\frac{2^j}{j}\right)=\sum_{j=1}^{m}\left(\frac{j(j+1)}{2}+\frac{2^j}{j}\cdot\sum_{k=1}^{j}1\right)$ *

$$*=\sum_{j=1}^{m}\left(\frac{j(j+1)}{2}+\frac{2^j}{\cancel{j}}\cdot\cancel{j}\right)=\frac{1}{2}\sum_{j=1}^{m}\left(j^2+j+2^{j+1}\right)=\frac{1}{2}\left[\sum_{j=1}^{m}j^2+\sum_{j=1}^{m}j+\sum_{j=1}^{m}2^{j+1}\right]=$$

$$=\frac{1}{2}\left[\frac{m(m+1)(2m+1)}{6}+\frac{m(m+1)}{2}\right]+\sum_{j=1}^{m}2^{j}+\underset{2^0}{1}-\underset{2^0}{1}=\frac{m(m+1)}{4}\left[\frac{2m+1+3}{3}\right]+\frac{2^{m+1}-1}{2-1}-1$$

$$=\frac{m(m+1)(m+2)}{6}+2^{m+1}-2 \;//$$

b) $\sum_{k=1}^{m}\frac{1}{\sqrt{k(k+1)}(\sqrt{k+1}+\sqrt{k})}=\sum_{k=1}^{m}\frac{1}{\sqrt{k}(k+1)+\sqrt{k+1}\cdot k}\cdot\frac{(\sqrt{k}(k+1)-\sqrt{k+1}\,k)}{(\sqrt{k}(k+1)-\sqrt{k+1}\,k)}=$

$$=\sum_{k=1}^{m}\frac{\sqrt{k}(k+1)-\sqrt{k+1}\,k}{k(k+1)^2-(k+1)k^2}=\sum_{k=1}^{m}\frac{\sqrt{k}(k+1)-\sqrt{k+1}\,k}{\cancel{k^3}+2k^2+k-\cancel{k^3}-k^2}=\sum_{k=1}^{m}\frac{\sqrt{k}(k+1)-\sqrt{k+1}\,k}{k+k^2}$$

$$=\sum_{k=1}^{m}\frac{\sqrt{k}}{k}-\frac{\sqrt{k+1}}{k+1}\longrightarrow \text{TELESCÓPICA}=\frac{\sqrt{1}}{1}-\cancel{\frac{\sqrt{2}}{2}}+\cancel{\frac{\sqrt{2}}{2}}-\cancel{\frac{\sqrt{3}}{3}}+\cdots\cancel{\frac{\sqrt{m}}{m}}-\frac{\sqrt{m+1}}{m+1}$$

$$=1-\frac{\sqrt{m+1}}{m+1}\;//$$

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c) $\sum_{k=1}^{m}(k^2+1)k!$ preeliminar # $\sum_{k=1}^{m} k\,k!$

$$\sum_{k=1}^{m} k\,k! = \sum_{k=1}^{m}(k+1-1)k! = \sum_{k=1}^{m}(k+1)k! - k! = \sum_{k=1}^{m}(k+1)! - k!$$

$$= \cancel{2!} - 1! + \cancel{3!} - \cancel{2!} + \ldots + (m+1)! - m! = (m+1)! - 1! \;//$$

Luego $\sum_{k=1}^{m}(k^2+1)k! = \sum_{k=1}^{m}(k^2+2k+1-2k)k! = \sum_{k=1}^{m}[(k+1)^2-2k]k!$

$$= \sum_{k=1}^{m}(k+1)!(k+1) - 2k\,k! = \sum_{k=1}^{m}(k+1)!(k+1) - 2\sum_{k=1}^{m}k\,k!$$

→ no hace nada

$$= \sum_{k=2}^{m+1} k\,k! - 2(m+1)! + 2\cdot 1! = \sum_{k=2}^{m+1} k\,k! + \ldots - 2(m+1)! + 2$$

$$= \sum_{k=1}^{m+1} k\,k! - 1\cdot 1! - 2(m+1)! + 2 = (m+2)! - \cancel{1\cdot 1!} - \cancel{1\cdot 1!} - 2(m+1)! + \cancel{2}$$

$$= (m+1)m$$

P2| $\sum_{j=1}^{m} j \qquad \sum_{j=1}^{m-1} j + m \qquad \# \sum_{j=1}^{m-1} j = \sum_{i=1}^{m-1} i = \frac{m(m-1)}{2}$

a) $\displaystyle\sum_{k=1+\sum_{i=1}^{m-1} i}^{\sum_{j=1}^{m} j} 2k-1 = \sum_{k=1+\sum_{j=1}^{m-1} j}^{\sum_{j=1}^{m-1} j+m} 2k-1 = \sum_{k=1+\frac{m(m-1)}{2}}^{\frac{m(m-1)}{2}+m} 2k-1$

$$= \sum_{k=1}^{m} 2\left(k+\frac{m(m-1)}{2}\right)-1 = \sum_{k=1}^{m} 2k + m(m-1) - 1$$

$$= 2\frac{m(m+1)}{2} + (m-1+1)(m(m-1)) + (m-1+1)(-1)$$

$$= m(m+1) + m^2(m-1) - m$$

b) $\displaystyle\sum_{k=1}^{m} \frac{2k+3}{k(k+1)3^k} \qquad \# \frac{2k+3}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1} = \frac{k(A+B)+A}{k(k+1)} \Rightarrow \begin{array}{l} A=3 \\ B=-1 \end{array}$

$$= \sum_{k=1}^{m} \frac{3}{k}\cdot\frac{1}{3^k} - \frac{1}{k+1}\cdot\frac{1}{3^k} = \sum_{k=1}^{m} \frac{1}{k\,3^{k-1}} - \frac{1}{k+1}\cdot\frac{1}{3^k}$$

$$= \frac{1}{1\cdot 1} - \cancel{\frac{1}{2}\cdot\frac{1}{3}} + \cancel{\frac{1}{2\cdot 3}} - \cancel{\frac{1}{3\cdot 3^2}} + \text{———} + \cancel{\frac{1}{m\,3^{m-1}}} - \frac{1}{(m+1)}\cdot\frac{1}{3^m}$$

$$= 1 - \frac{1}{(m+1)3^m}$$

c) $\sum_{k=1}^{m} \frac{k^2+5k+5}{(k+4)!} = \sum_{k=1}^{m} \frac{k^2+6k+8-k-3}{(k+4)!} = \sum_{k=1}^{m} \frac{(k+2)(k+4)}{(k+4)!} - \frac{(k+3)}{(k+4)!}$ (4)

$= \sum_{k=1}^{m} \frac{k+2}{(k+3)!} - \frac{(k+3)}{(k+4)!} = \frac{3}{4!} - \cancel{\frac{4}{5!}} + \cancel{\frac{4}{5!}} - \cancel{\frac{5}{6!}} + \ldots$

$+ \cancel{\frac{m+1}{(m+2)!}} - \cancel{\frac{(m+2)}{(m+3)!}} + \cancel{\frac{(m+2)}{(m+3)!}} - \frac{(m+3)}{(m+4)!}$

$= \frac{3}{4!} - \frac{m+3}{(m+4)!} = \frac{3}{4!} - \frac{1}{(m+2)!(m+4)}$

$= \frac{\cancel{3}}{\cancel{3}\cdot 4\cdot 2} - \frac{1}{(m+2)!(m+4)}$

$= \frac{1}{8} - \frac{1}{(m+2)!(m+4)}$ //

P3)

a) $\sum_{j=3}^{m-1}(j+1)(j+2) = \sum_{j=3}^{m-1} j^2+3j+2 = \sum_{j=1}^{m-1} j^2 + 3\sum_{j=1}^{m-1} j + 2\sum_{j=3}^{m-1} 1 - 1^2 - 2^2 - 3\cdot 1 - 3\cdot 2$

$= \frac{(m-1)(m)(2(m-1)+1)}{6} + 3\frac{(m-1)(m)}{2} + 2\cdot(m-1-3+1)$

$-1-4-3-6$

b) $\sum_{k=3}^{m} b_k - b_{k-3} = \sum_{k=3}^{m} b_k \underbrace{- b_{k-1} + b_{k-1}}_{0} + \underbrace{b_{k-2} - b_{k-2}}_{0} - b_{k-3}$

$= \underbrace{\sum_{k=3}^{m} b_k - b_{k-1}}_{①} + \underbrace{\sum_{k=3}^{m} b_{k-1} - b_{k-2}}_{②} + \underbrace{\sum_{k=3}^{m} b_{k-2} - b_{k-3}}_{③}$

$= ①\ b_3 - b_2 + b_4 - b_3 + \_\_ + b_m - b_{m-1}$

$②\ b_2 - b_1 + b_3 - b_2 + \_\_\_ + b_{m-1} - b_{m-2}$

$③\ b_1 - b_0 + b_2 - b_1 + \_\_\_ + b_{m-2} - b_{m-3}$

$= b_m - b_2 + b_{m-1} - b_1 + b_{m-2} - b_0$ //

(6)

c) $\sum_{k=l}^{m} \left(\frac{2}{l}\right)^k =$

constante hay que ver que sea diferente de 2. pues

$\frac{2}{l} > 0 \; ; \; \forall \, l \in \mathbb{N}$

Si Si $l = 1$; pues $l \neq 0$

1) $\sum_{k=1}^{m} (2)^k = \frac{2^{m+1} - 1}{1} - 2^0$ //

Si $l = 2$

2) $\sum_{k=2}^{m} 1^k = m - 2 + 1$ // $; \; 1^k = 1 \, , \; \forall \, k \in \mathbb{N}$

3) Si: $l \geq 3$

$\sum_{k=l}^{m} \left(\frac{2}{l}\right)^k = \frac{\left(\frac{2}{l}\right)^{m+1} - 1}{\frac{2}{l} - 1} - \sum_{k=1}^{l-1} \left(\frac{2}{l}\right)^k$ //

7'

P4 $\displaystyle\sum_{k=m+1}^{2m} \frac{1}{k} = \sum_{j=1}^{2m} \frac{(-1)^{j+1}}{j}$ CASO base $m=1$ $\displaystyle\sum_{k=2}^{2} \frac{1}{k} = \frac{1}{2} = \sum_{j=1}^{2} \frac{(-1)^{j+1}}{j} = 1 - \frac{1}{2} = \frac{1}{2}$ //

$P(2m) \Rightarrow P(2m+2) \Leftrightarrow P(2(m+1))$

HI Para algún natual

luego $\displaystyle\sum_{k=m+1}^{2m} \frac{1}{k} = \sum_{j=1}^{2m} \frac{(-1)^{j+1}}{j}$ hipótesis

Veamos $P(2m+2) \Leftrightarrow \displaystyle\sum_{k=m+2}^{2m+2} \frac{1}{k} = \sum_{j=1}^{2m+2} \frac{(-1)^{j+1}}{j}$

Veamos
$$\sum_{j=1}^{2m+2} \frac{(-1)^{j+1}}{j} = \sum_{j=1}^{2m} \frac{(-1)^{j+1}}{j} + \frac{1}{2m+1} - \frac{1}{2m+2}$$
$$= \sum_{k=m+1}^{2m} \frac{1}{k} + \frac{1}{2m+1} - \frac{1}{2m+2}$$
$$= \sum_{k=m+1}^{2m+1} \frac{1}{k} - \frac{1}{2m+2} + \frac{2}{2m+2} - \frac{2}{2m+2}$$
$$= \sum_{k=m+1}^{2m+1} \frac{1}{k} + \frac{1}{2m+2} - \frac{1}{m+1} = \sum_{k=m+1}^{2m+2} \frac{1}{k} - \frac{1}{m+1} = \sum_{k=m+2}^{2m+2} \frac{1}{k} \quad //$$

P5

$$\sum_{i=1}^{m} u_i = 2m^2+3m \qquad \text{veamos} \quad \sum_{i=m+1}^{2m} u_i \quad ; \quad N = 2m$$

1) luego $$\sum_{i=m+1}^{N} u_i = \sum_{i=1}^{N} u_i - \sum_{i=1}^{m} u_i$$

$$= 2N^2+3N - 2m^2-3m$$

$$= 2(2m)^2+3(2m) - 2m^2-3m$$

$$= 8m^2+6m-2m^2-3m = 6m^2+3m \,//$$

2) $$u_m + \sum_{i=1}^{m-1} u_i = \sum_{i=1}^{m} u_i = 2m^2+3m$$

$$u_m = 2m^2+3m - \sum_{i=1}^{m-1} u_i = 2m^2+3m - (2(m-1)^2+3(m-1))$$

$$= \cancel{2m^2}+3m - \cancel{2m^2}+4m - 3m + 3 - 2$$

$$= 4m+1 \,//$$

P6|

$$\frac{1\cdot 2^1}{3!}, \frac{2\cdot 2^2}{4!}, \text{——} \frac{k\cdot 2^k}{(k+2)} \; ; \; k \in \{1, \text{——} m\}$$

$$\sum_{k=1}^{m} \frac{k\cdot 2^k}{(k+2)!} \quad ; \text{ luego } \frac{k}{(k+2)!} = \frac{A}{(k+2)!} + \frac{B}{(k+1)!}$$

$$\Rightarrow A = -2 \;\wedge\; B = 1 \text{ pues } k = A + B(k+2)$$

$$= \sum_{k=1}^{m} 2^k \cdot \frac{k}{(k+2)!} = \sum_{k=1}^{m} 2^k \left(\frac{1}{(k+1)!} - \frac{2}{(k+2)!} \right)$$

$$= \sum_{k=1}^{m} \frac{2^k}{(k+1)!} - \frac{2^{k+1}}{(k+2)!} = 1 - \frac{2^{m+1}}{(m+2)!}$$

Expanda telescópica

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P8| R: 1840

P9| Veamos que impares no multiplos de 3 menores a 6m son
impares menores a 6m − impares multiplos de 3 menores a 6m
↓
menos

Luego impares menores a 6m $\sum_{k=1}^{3m}(2k-1) = 2\sum_{k=1}^{3m} k - \sum_{k=1}^{3m} 1$

$= 9m^2$

Como $2(3m)-1 = 6m-1 < 6$, ~~mo~~ monotonía notable

Luego impares multiplos de 3 menores a 6m
podemos ver la sucesión 3, 9, 15, 21, 27, 33, —, 6m−3

Luego $\sum_{k=1}^{m} 3+6(k-1) = -3m + 6\frac{m(m+1)}{2} = 3m^2$

Luego $9m^2 - 3m^2 = 6m^2$ //

Ya terminamos!

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