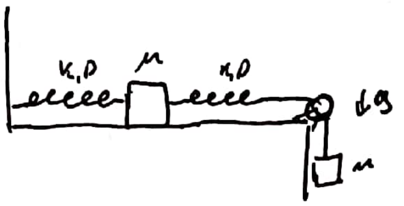
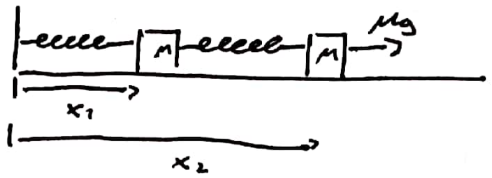
P1)



Equivalente $\Longleftrightarrow$



- Las ecuaciones de Movimiento serán

$$m\ddot{x}_1 = -F_{e1} + F_{e2}$$
$$m\ddot{x}_2 = -F_{e2} + Mg$$

Donde

$$F_{e1} = k(x_1 - D) \quad , \quad F_{e2} = k(x_2 - x_1 - D)$$

$$\Rightarrow m\ddot{x}_1 = -kx_1 + \cancel{kD} + kx_2 - kx_1 - \cancel{kD}$$
$$m\ddot{x}_2 = -kx_2 + kx_1 + kD + mg$$

Usamos

$$\omega_0^2 = k/m$$

$$\Rightarrow \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \omega_0^2 \underbrace{\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}}_{M} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \omega_0^2 D + mg \end{pmatrix}$$

- Estudiamos los modos normales de M

$$\rightarrow \det(M - \omega^2 \mathbb{I}) = (2-\omega^2)(1-\omega^2) - 1 \stackrel{!}{=} 0 \quad \Rightarrow \boxed{\omega_{\pm}^2 = \frac{\omega_0^2}{2}(3 \pm \sqrt{5})}$$

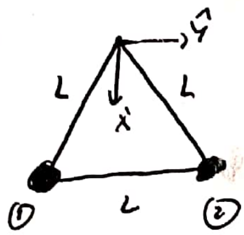
$\rightarrow$ Con vectores propios:

$$+) \quad \vec{v}_+ = \begin{pmatrix} -\frac{1}{2}(1+\sqrt{5}) \\ 1 \end{pmatrix} \qquad -) \quad \vec{v}_- = \begin{pmatrix} \frac{1}{2}(\sqrt{5}-1) \\ 1 \end{pmatrix}$$

$\Rightarrow$ Mov opuesto relativo

$\Rightarrow$ Mov en la misma dirección

P2



a) Sacamos el tensor de inercia

$$\vec{r}_1 = \frac{\sqrt{3}}{2} L \hat{x} - \frac{L}{2} \hat{y}$$

$$\vec{r}_2 = \frac{\sqrt{3}}{2} L \hat{x} + \frac{L}{2} \hat{y}$$

$$\Rightarrow \vec{r}_{cm} = \frac{\sqrt{3}}{4} L \hat{x}$$

$$I = \sum m_a (r_a^2 \delta_{ij} - r_{ai} r_{aj})$$

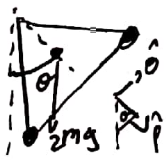
$$I_{xx} = m\left(L^2 - \tfrac{3}{4}\right) + m\left(L^2 - \tfrac{3}{4}\right) = \frac{mL^2}{2} \qquad I_{xy} = I_{xz} = I_{yz} = 0$$

$$I_{yy} = 2m\left(L^2 - \frac{L^2}{4}\right) = \frac{3}{2} mL^2$$

$$I_{zz} = 2mL^2$$

$$\Rightarrow I = mL^2 \begin{pmatrix} 1/2 & & \\ & 3/2 & \\ & & 2 \end{pmatrix}$$

b) En mov hacemos la correspondencia $\hat{x} \to \hat{\rho}$, $\hat{y} \to \hat{\theta}$, usamos la ec de torques



$$\to \vec{\tau} = \text{[illegible]} \times 2mg (\cos\theta \hat{\rho} - \operatorname{sen}\theta \hat{\theta})$$

$$= -\frac{\sqrt{3}}{2} Lmg \operatorname{sen}\theta \, \hat{z}$$

$$\to \text{Si } \vec{\omega} = \dot{\theta} \hat{z} \Rightarrow \vec{l} = I_{zz} \dot{\theta} \hat{z} \Rightarrow \dot{\vec{l}} = 2mL^2 \ddot{\theta} \hat{z}$$

$\to$ Hacemos $\vec{\tau} = \dot{\vec{l}}$

$$\Rightarrow \boxed{\ddot{\theta} = -\frac{\sqrt{3}}{4} \frac{g}{L} \operatorname{sen}\theta} \; *$$

Hacemos $\ddot{\theta} = \dot{\theta} \frac{d\dot{\theta}}{d\theta}$ e integramos

$$\int \dot{\theta} \, d\dot{\theta} = -\frac{\sqrt{3}}{4} \frac{g}{L} \int \operatorname{sen}\theta \, d\theta$$

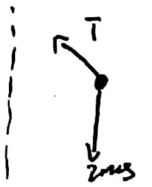
$$\frac{\dot{\theta}^2}{2} = \frac{\sqrt{3}}{4} \frac{g}{L} (\cos\theta - \cos\theta_0) \Rightarrow \boxed{\dot{\theta} = \left[\frac{\sqrt{3}}{2} \frac{g}{L} (\cos\theta - \cos\theta_0) \right]^{1/2}}$$

c) De la ec ✱ hacemos $\operatorname{sen}\theta \approx \theta$

$$\Rightarrow \ddot{\theta} + \frac{\sqrt{3}}{4}\frac{g}{L}\theta = 0 \quad \Rightarrow \boxed{\omega_0^2 = \frac{\sqrt{3}}{4}\frac{g}{L}}$$

Con $T = \frac{2\pi}{\omega_0}$

d) Para ver la reacción vemos las fuerzas sobre el CM.



→ Con T fuerza que conecta el sólido con la estructura

$$\Rightarrow \ddot{\vec{r}}_{cm} = -\frac{\sqrt{3}}{4}L\dot{\theta}^2\hat{\rho} + \frac{\sqrt{3}}{4}L\ddot{\theta}\hat{\theta}$$

$$\Rightarrow \Sigma F = -T\hat{\rho} + mg(\cos\theta\,\hat{\rho} - \operatorname{sen}\theta\,\hat{\theta})$$

→ Si vemos la ec de mov en $\hat{\rho}$

$$-\frac{\sqrt{3}}{2}mL\dot{\theta}^2 = -T + mg\cos\theta$$

La reacción de la estructura será $R = -T$ por acción y reacción

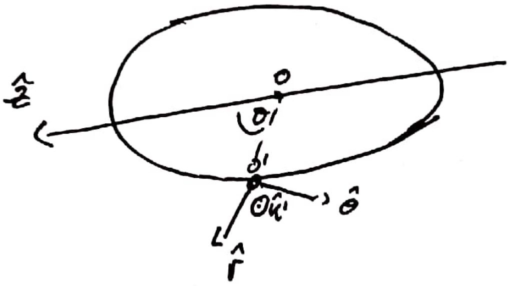
$$\Rightarrow R = -\frac{\sqrt{3}}{2}mL\dot{\theta}^2 - mg\cos\theta$$

$$\Rightarrow R = -\frac{\sqrt{3}}{2}mL\left[\frac{\sqrt{3}}{2}\frac{g}{L}(\cos\theta - \cos\theta_0)\right] - mg\cos\theta$$

$$= -\frac{3}{4}mg(\cos\theta - \cos\theta_0) - mg\cos\theta$$

$$\Rightarrow \boxed{R = mg\left[\frac{3}{4}\cos\theta_0 - \frac{7}{4}\cos\theta\right]}$$

P3]



→ En el sist fijo O

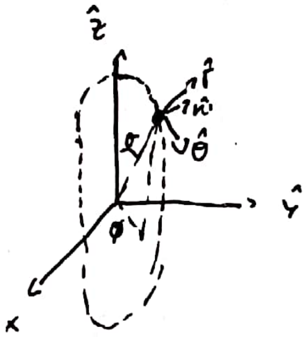
$\vec{\Omega} = -\omega_0 \hat{z}$; Gira en sentido opuesto

$\Rightarrow \vec{\Omega} = -\omega_0 (\operatorname{sen}\theta \hat{r} - \cos\theta \hat{\theta})$

→ En O'

$\vec{r}\,' = R\hat{r}$, $\dot{\vec{r}}\,' = R\dot{\theta}\hat{\theta}$, $\ddot{\vec{r}}\,' = -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}$

* Otra vista



→ Con este sistema la gravedad irá en dirección $\hat{x}$

$\hat{x} = \cos\phi (\operatorname{sen}\theta \hat{r} + \cos\theta \hat{\theta}) - \operatorname{sen}\phi \hat{n}'$

→ Recordar que nos interesa el instante de la figura

$\phi = \frac{\pi}{2}$, $\theta = \frac{\pi}{4}$

→ Las correspondencias al sistema fijo son de esféricas:

$\hat{r} = \operatorname{sen}\theta (\cos\phi \hat{x} + \operatorname{sen}\phi \hat{y}) + \cos\theta \hat{z}$, $\hat{\theta} = \cos\theta (\cos\phi \hat{x} + \operatorname{sen}\phi \hat{y}) - \operatorname{sen}\theta \hat{z}$

• Calculamos las fuerzas no inerciales:

→ $m\vec{\Omega} \times \vec{\Omega} \times \vec{r}\,' = -m\omega_0^2 R (\operatorname{sen}\theta \hat{r} - \cos\theta \hat{\theta}) \times [(\operatorname{sen}\theta \hat{r} - \cos\theta \hat{\theta}) \times \hat{r}]$

$= -m\omega_0^2 R \cos\theta (\operatorname{sen}\theta \hat{\theta} + \cos\theta \hat{r})$

→ $2m\vec{\Omega} \times \vec{v}\,' = 2m\omega_0 R\dot{\theta} (\operatorname{sen}\theta \hat{r} - \cos\theta \hat{\theta}) \times \hat{\theta}$

$= 2mR\omega_0 \dot{\theta} \operatorname{sen}\theta \hat{n}'$

• Las fuerzas externas serán:

$\Sigma \vec{F} = mg\hat{x} + N_r \hat{r} + N_n \hat{n}$

$= mg [\cos\phi (\operatorname{sen}\theta \hat{r} + \cos\theta \hat{\theta}) - \operatorname{sen}\phi \hat{n}'] + N_r \hat{r} + N_n \hat{n}$

• La ec de Movimiento será:

$$mR(-\dot{\theta}^2\hat{r}+\ddot{\theta}\hat{\theta}) = mg\left[\cos\phi(\operatorname{sen}\theta\,\hat{r}+\cos\theta\,\hat{\theta})-\operatorname{sen}\phi\,\hat{n}'\right] + N_r\,\hat{r} + N_n\,\hat{n}'$$

$$+ m\omega_0^2 R\cos\theta\,(\operatorname{sen}\theta\,\hat{\theta}+\cos\theta\,\hat{r}) - 2mR\,\omega_0\dot{\theta}\operatorname{sen}\theta\;\hat{n}'$$

• Vemos la ec en $\hat{\theta}]$

$$mR\ddot{\theta} = mg\cos\phi\cos\theta + m\omega_0^2 R\cos\theta\operatorname{sen}\theta$$

Evaluamos $\theta = \frac{\pi}{4}$, $\phi = \frac{\pi}{2}$

$$\cancel{mR}\ddot{\theta}_0 = \cancel{m}\omega_0^2\cancel{R}\cdot\left(\frac{\sqrt{2}}{2}\right)^2 \Rightarrow \boxed{\ddot{\theta}_0 = \frac{\omega_0^2}{2}}$$

• Escribimos la velocidad y aceleración en el sistema fijo

$$\dot{\vec{r}}_0' = R\,\dot{\theta}_0\,\hat{\theta} \Rightarrow \vec{v} = R\dot{\theta}_0\left(\cos\left(\tfrac{\pi}{4}\right)\left[\cos\left(\tfrac{\pi}{2}\right)\hat{x}+\operatorname{sen}\left(\tfrac{\pi}{2}\right)\hat{y}\right) - \operatorname{sen}\tfrac{\pi}{4}\,\hat{z}\right)$$

$$\boxed{\vec{v}_0 = \frac{\sqrt{2}}{2}R\dot{\theta}_0(\hat{y}-\hat{z})}$$

$$\ddot{\vec{r}}_0' = -R\dot{\theta}_0^2\hat{r} + R\ddot{\theta}_0\hat{\theta}$$

$$\Rightarrow \vec{a} = -R\dot{\theta}_0^2\left(\operatorname{sen}\theta_0(\cos\phi_0\hat{x}+\operatorname{sen}\phi_0\hat{y})+\cos\theta_0\hat{z}\right) + R\frac{\omega_0^2}{2}\left(\cos\theta\,(\cos\phi\,\hat{x}+\operatorname{sen}\phi\,\hat{y})-\operatorname{sen}\theta\,\hat{z}\right)$$

$$= -R\dot{\theta}_0^2\left(\frac{\sqrt{2}}{2}\hat{y}+\frac{\sqrt{2}}{2}\hat{z}\right) + R\frac{\omega_0^2}{2}\left(\frac{\sqrt{2}}{2}\hat{y}-\frac{\sqrt{2}}{2}\hat{z}\right)$$

$$\Rightarrow \boxed{\vec{a}_0 = \frac{\sqrt{2}R}{2}\left[\hat{y}\left(\dot{\theta}_0^2+\frac{\omega_0^2}{2}\right) + \hat{z}\left(\dot{\theta}_0^2-\frac{\omega_0^2}{2}\right)\right]}$$

• De la ec en $\hat{r}]$ despejamos N_r

$$-mR\dot{\theta}^2 = mg\cos\phi\operatorname{sen}\theta + N_r + m\omega_0^2R\cos^2\theta$$

Evaluamos

$$\Rightarrow \boxed{N_r = -mR\dot{\theta}_0^2 - \frac{m\omega_0^2 R}{2}}$$

• De la ec en $\hat{k}$ sacamos N_k

$$0 = N_k - 2mR\omega_0 \dot{\theta} \operatorname{sen}\theta - mg \operatorname{sen}\phi$$

$$\Rightarrow \boxed{N_k = 2mR\omega_0 \dot{\theta}_0 \frac{\sqrt{2}}{2} + mg}$$

• Finalmente el módulo de la reacción será:

$$\boxed{N = (N_r^2 + N_k^2)^{1/2}}$$