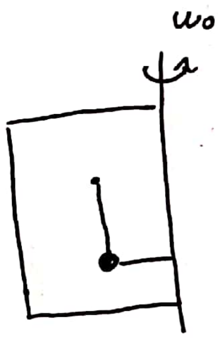
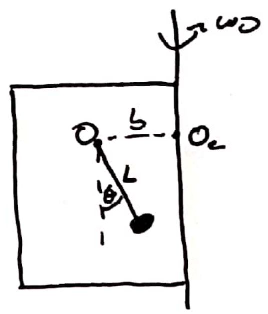
P2

a)

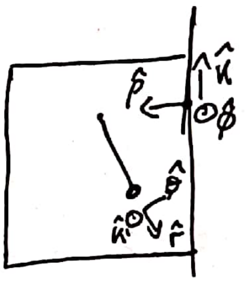


b), c), d)



a) $N=0$, ya que no hay acel. tangencial

b) Usamos un sist estático en O_e (polar) y uno no inercial en O (polar)



-> Tenemos para el sist fijo

$$\vec{R} = b\hat{\rho} \Rightarrow \ddot{\vec{R}} = -b\omega_0^2\hat{\rho} \quad ; \quad \Omega = \omega_0\hat{k}$$

-> Para el sist no inercial

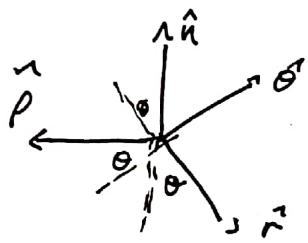
$$\vec{r}\,' = L\hat{r} \Rightarrow \dot{\vec{r}}\,' = L\dot{\theta}\hat{\theta} \Rightarrow \ddot{\vec{r}}\,' = -L\dot{\theta}^2\hat{r} + L\ddot{\theta}\hat{\theta}$$

-> La relación entre vectores unitarios serán:

$$\hat{\rho} = -\operatorname{sen}\theta\,\hat{r} - \cos\theta\,\hat{\theta}$$

$$\hat{k} = -\cos\theta\,\hat{r} + \operatorname{sen}\theta\,\hat{\theta}$$

$$\hat{\phi} = \hat{k}'$$



-> Calculamos las Pseudo fuerzas

$$* \; m\ddot{\vec{R}} = -mb\omega_0^2(-\operatorname{sen}\theta\,\hat{r} - \cos\theta\,\hat{\theta})$$

$$* \; m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}\,') = m\omega_0\vec{\Omega}\times\left((-\cos\theta\,\hat{r}+\operatorname{sen}\theta\,\hat{\theta})\times L\hat{r}\right) = m\omega_0^2(-\cos\theta\,\hat{r}+\operatorname{sen}\theta\,\hat{\theta})\times(-\operatorname{sen}\theta\,\hat{k}')L$$

$$= -Lm\omega_0^2(\cos\theta\operatorname{sen}\theta\,\hat{\theta} + \operatorname{sen}^2\theta\,\hat{r})$$

$$* \; 2m\vec{\Omega}\times\dot{\vec{r}}\,' = 2m\omega_0L\dot{\theta}(-\cos\theta\,\hat{r} + \operatorname{sen}\theta\,\hat{\theta})\times\hat{\theta}$$

$$= -2mL\omega_0\dot{\theta}\cos\theta\,\hat{k}'$$

-> Vemos las fuerzas

$$\Sigma \vec F = -T\hat r + N\hat k' - mg\hat k$$
$$= -T\hat r + N\hat k' - mg(-\cos\theta\,\hat r + \operatorname{sen}\theta\,\hat\theta)$$

-> Escribimos la ec de Movimiento

$$m(-L\dot\theta^2\hat r + L\ddot\theta\hat\theta) = -T\hat r + N\hat k' - mg(-\cos\theta\,\hat r + \operatorname{sen}\theta\,\hat\theta) + mb\omega_0^2(-\operatorname{sen}\theta\,\hat r - \cos\theta\,\hat\theta)$$
$$+ Lm\omega_0^2(\cos\theta\operatorname{sen}\theta\,\hat\theta + \operatorname{sen}^2\theta\,\hat r)$$
$$+ 2mL\omega_0\dot\theta\cos\theta\,\hat k'$$

=> $\hat r$] $\quad -mL\dot\theta^2 = -T + mg\cos\theta - mb\omega_0^2\operatorname{sen}\theta + Lm\omega_0^2\operatorname{sen}^2\theta$

$\hat\theta$] $\quad mL\ddot\theta = -mg\operatorname{sen}\theta - mb\omega_0^2\cos\theta + Lm\omega_0^2\cos\theta\operatorname{sen}\theta$

$\hat k'$] $\quad 0 = N + 2mL\omega_0\dot\theta\cos\theta$

-> Usamos la ec en $\hat\theta$] con $\ddot\theta = \dot\theta\frac{d\dot\theta}{d\theta}$

$$\Rightarrow \dot\theta\, d\dot\theta = -\frac{g}{L}\operatorname{sen}\theta\, d\theta - \frac{b}{L}\omega_0^2\cos\theta\, d\theta + \omega_0^2\cos\theta\operatorname{sen}\theta\, d\theta \quad \Big/ \int_0^{\dot\theta}, \int_0^{\theta}$$

$$\Rightarrow \frac{\dot\theta^2}{2} = \frac{g}{L}(\cos\theta - 1) - \frac{b\omega_0^2}{L}\operatorname{sen}\theta + \omega_0^2\frac{\operatorname{sen}^2\theta}{2}$$

$$\Rightarrow \dot\theta = \left(\frac{2g}{L}(\cos\theta-1) - \frac{2b\omega_0^2}{L}\operatorname{sen}\theta + \omega_0^2\operatorname{sen}^2\theta\right)^{1/2}$$

-> La velocidad relativa será $\boxed{\dot{\vec r}' = L\dot\theta\hat\theta}$

c) Sacamos la Normal de la ec en $\hat k'$]

$$\boxed{N = -2mL\omega_0\cos\theta\,\dot\theta}$$

d) Notamos que $N=0$ si $\boxed{\theta = \frac{\pi}{2}}$, $\boxed{\theta = 0}$