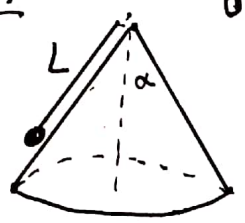
P3)



a) Det ω mínimo tal que la masa este en contacto

- En coordenadas esféricas:

$$\vec{r} = L\hat{r}, \quad \theta = \pi - \alpha$$

$\text{Sen}(\pi-\alpha) = \text{Sen}\,\alpha$
$\text{Cos}(\pi-\alpha) = -\text{Cos}\,\alpha$

$$\Rightarrow \vec{v} = L\dot{\phi}\,\text{sen}\alpha\,\hat{\phi}$$

$$\Rightarrow \vec{a} = -L\dot{\phi}^2\text{sen}^2\alpha\,\hat{r} + L\dot{\phi}^2\text{sen}\alpha\cos\alpha\,\hat{\theta} + L\,\text{sen}\alpha\,\ddot{\phi}\,\hat{\phi}$$

- Vemos las fuerzas

$$\sum \vec{F} = -T\hat{r} + N\hat{\theta} + mg(\cos\alpha\,\hat{r} + \text{sen}\alpha\,\hat{\theta})$$

- Ec de Mov:

$$\hat{r}\big] \quad -mL\dot{\phi}^2\text{sen}^2\alpha = -T + mg\cos\alpha$$

$$\hat{\theta}\big] \quad mL\dot{\phi}^2\text{sen}\alpha\cos\alpha = -N + mg\,\text{sen}\alpha$$

- Si imponemos $N=0$ en $\hat{\theta}$]:

$$mL\dot{\phi}_0^2\,\text{sen}\alpha\cos\alpha = mg\,\text{sen}\alpha$$

$$\Rightarrow \dot{\phi}_0^2 = \frac{g/L}{\cos\alpha}$$

- Luego la rapidez mínima será: $v_{min} = L\dot{\phi}_0\,\text{sen}\alpha$

$$\Rightarrow v_{min} = L\sqrt{\frac{g/L}{\cos\alpha}}\,\text{sen}\alpha$$

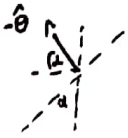
$$\Rightarrow \boxed{v_{min} = \sqrt{gL\,tg\alpha\,\text{sen}\alpha}}$$

b) Si la cuerda se recoge a $L/2$, det v_f, N, T

- Para conocer $\dot{\phi}_f$ usaremos la conservación del momento angular vertical

$$\vec{L} = m\vec{r}\times\vec{v} = m\, r\hat{r}\times r\dot{\phi}\,\text{sen}\alpha\,\hat{\phi} = m r^2\dot{\phi}\,\text{sen}\alpha\,(-\hat{\theta})$$

Descomponemos $(-\hat{\theta})$



$$-\hat{\theta} = \text{sen}\alpha\,\hat{z} + \cos\alpha\,\hat{\rho}$$

$$\Rightarrow L_{vert} = m r^2\dot{\phi}\,\text{sen}^2\alpha$$

- Luego nos ponemos en las condiciones iniciales y finales y las igualamos

$$L_{vert,0} = m L^2\dot{\phi}_0\,\text{sen}^2\alpha \quad , \quad L_{vert,f} = m\frac{L^2}{4}\dot{\phi}_f\,\text{sen}^2\alpha$$

$$\Rightarrow L_{vert,0} = L_{vert,f} \Rightarrow \dot{\phi}_0 = \frac{\dot{\phi}_f}{4} \Rightarrow \dot{\phi}_f = 4\sqrt{\frac{g/L}{\cos\alpha}}$$

- La velocidad final queda: $v_f = \frac{L}{2}\dot{\phi}_f\,\text{sen}\alpha \Rightarrow v_f = 2L\sqrt{\frac{g/L}{\cos\alpha}}\,\text{sen}\alpha$

$$\Rightarrow \boxed{v_f = \sqrt{4gL\,\text{tg}\alpha\,\text{sen}\alpha}}$$

- Las ecs de movimiento son iguales a la parte a) salvo los cambios $L \to L/2$, $\dot{\phi} \to \dot{\phi}_f$

$\hat{r}$] $T = m\frac{L}{2}\cdot\dot{\phi}_f^2\,\text{sen}^2\alpha + mg\cos\alpha$

$= m\frac{L}{2}\cdot 8\cdot\frac{g}{L\cos\alpha}\,\text{sen}^2\alpha + mg\cos\alpha \Rightarrow \boxed{T = mg\left(\frac{8 - 7\cos^2\alpha}{\cos\alpha}\right)}$

$\hat{\theta}$] $N = mg\,\text{sen}\alpha - m\frac{L}{2}\dot{\phi}^2\,\text{sen}\alpha\cos\alpha$

$= mg\,\text{sen}\alpha - m\frac{L}{2}\cdot 8\cdot\frac{g}{L\cos\alpha}\,\text{sen}\alpha\cos\alpha \Rightarrow \boxed{N = -7mg\,\text{sen}\alpha}$