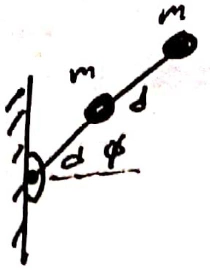
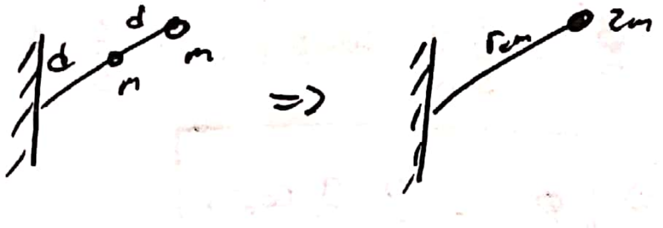
P2]



a) $\dot{\phi}(\phi)$

b) F_{z1} de la rótula en $\phi = 0$

a) Pasamos el sist de 2 partículas a uno con 1 masa igual a 2m:



Con r_{cm} Posición del centro de masa

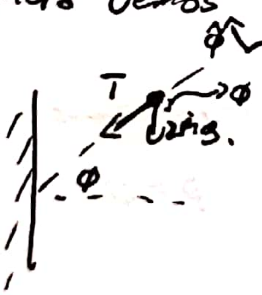
- Como las masas son iguales r_{cm} estará en el Punto medio: $r_{cm} = \frac{3}{2}d$

- Luego vemos su cinemática:

$$\vec{r} = \frac{3}{2}d\,\hat{\rho} \Rightarrow \vec{v} = \frac{3}{2}d\,\dot{\phi}\,\hat{\phi}$$

$$\Rightarrow \vec{a} = -\frac{3}{2}d\,\dot{\phi}^2\hat{\rho} + \frac{3}{2}d\,\ddot{\phi}\,\hat{\phi}$$

- Ahora vemos las fuerzas



$$\Sigma F = -T\hat{\rho} + 2mg(-\operatorname{sen}\phi\,\hat{\rho} - \cos\phi\,\hat{\phi})$$

$$= -(T + 2mg\operatorname{sen}\phi)\hat{\rho} - 2mg\cos\phi\,\hat{\phi}$$

- Hacemos $\vec{F} = 2m\vec{a}$ por componentes:

$\hat{\rho}$] $T + 2mg\operatorname{sen}\phi = \frac{3}{2}2md\,\dot{\phi}^2$

$\hat{\phi}$] $-2mg\cos\phi = \frac{3}{2}2md\,\ddot{\phi}$

• De la ec en $\hat{\phi}$] usamos el truco $\left(\ddot{\phi} = \dot{\phi}\,\frac{d\dot{\phi}}{d\phi} \right)$

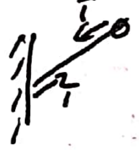
$$\Rightarrow \ddot{\phi} = \dot{\phi}\,\frac{d\dot{\phi}}{d\phi} = -\frac{2}{3}\frac{g}{d}\,\mathrm{sen}\,\phi$$

$$\Rightarrow \dot{\phi}\,d\dot{\phi} = -\frac{2}{3}\frac{g}{d}\,\mathrm{sen}\,\phi\,d\phi \quad \Big/ \int_0^{\dot{\phi}} \,,\, \int_{\pi/2}^{\phi}$$

$$\Rightarrow \frac{\dot{\phi}^2}{2} = \frac{2}{3}\frac{g}{d}\cos\phi\,\Big|_{\pi/2}^{\phi} = \frac{2}{3}\frac{g}{d}(\cos\phi - 0)$$

$$\Rightarrow \boxed{\dot{\phi}^2 = \frac{4}{3}\frac{g}{d}\cos\phi}$$

b) Por acción y reacción la Fza de la rótula será T en sentido opuesto:



• Usamos la ec en $\hat{\rho}$ y despejamos T

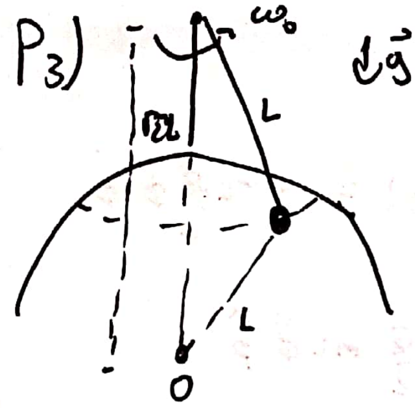
$$T = 3md\dot{\phi}^2 - 2mg\,\mathrm{sen}\,\phi$$

$$\Rightarrow T(\phi) = \cancel{3}m\cancel{d}\,\frac{4}{\cancel{3}}\frac{g}{\cancel{d}}\cos\phi - 2mg\,\mathrm{sen}\,\phi$$

$$= mg(4\cos\phi - 2\,\mathrm{sen}\,\phi) \quad \Big| \begin{array}{c}\text{Evaluamos} \\ \phi = 0\end{array}$$

$$T^*(0) = mg(4\,\cancel{\cos(0)}^{1} - 2\,\cancel{\mathrm{sen}(0)}^{0})$$

$$\Rightarrow \boxed{T(0) = 4mg}$$

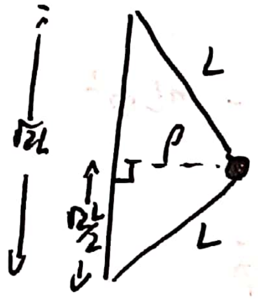
P3) 

$\downarrow \vec{g}$, $\vec{F}_v = -c\vec{v}$

- En $\underline{t=0}$, ω_0 es tal que $N=0$

a) Encontrar $\dot{\phi}(\phi)$ donde ϕ es el ángulo barrido

- Usaremos coordenadas cilíndricas, hacemos geometría para encontrar ρ y $\vec{r}$, $\vec{v}$, $\vec{a}$

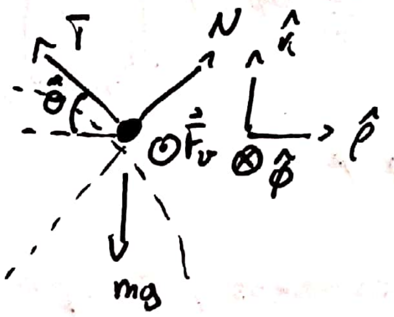


$\Rightarrow \rho = \frac{\sqrt{2}}{2} L$, $z = \frac{\sqrt{2}}{2} L$

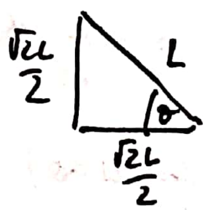
$\Rightarrow \vec{r} = \frac{\sqrt{2}}{2} L \hat{\rho} + \frac{\sqrt{2}}{2} L \hat{k} \Rightarrow \vec{v} = \frac{\sqrt{2}}{2} L \dot{\phi} \hat{\phi}$

$\Rightarrow \vec{a} = -\frac{\sqrt{2}}{2} L \dot{\phi}^2 \hat{\rho} + \frac{\sqrt{2}}{2} L \ddot{\phi} \hat{\phi}$

- Vemos las fuerzas sobre la masa



$\Rightarrow$ El ángulo θ forma el siguiente triángulo:



$\Rightarrow \theta = \frac{\pi}{4}$

$$\Rightarrow \Sigma F = T\left(-\cos(\pi/4)\hat{\rho} + \text{sen}(\pi/4)\hat{k}\right)$$
$$+ N\left(\cos(\pi/4)\hat{k} + \text{sen}(\pi/4)\hat{\rho}\right)$$
$$- c\frac{\sqrt{2}}{2} L \dot{\phi} \hat{\phi} - mg\hat{k}$$

- Hacemos $\vec{F} = m\vec{a}$

$$\Rightarrow \left(-\frac{\sqrt{2}}{2}T + N\frac{\sqrt{2}}{2}\right)\hat{\rho} + \left(T\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} - mg\right)\hat{z} - cL\frac{\sqrt{2}}{2}\dot{\phi}\,\hat{\phi}$$

$$= -\frac{\sqrt{2}}{2}mL\dot{\phi}^2\hat{\rho} + \frac{\sqrt{2}}{2}mL\ddot{\phi}\,\hat{\phi}$$

- De la ec en $\hat{z}]$

$$\hat{z}]\quad T\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} - mg = 0 \Rightarrow T = \sqrt{2}mg - N \quad \star$$

- Del resto tenemos:

$$\hat{\rho}]\quad -\frac{\cancel{\sqrt{2}}}{\cancel{2}}T + N\frac{\cancel{\sqrt{2}}}{\cancel{2}} = -\frac{\cancel{\sqrt{2}}}{\cancel{2}}mL\dot{\phi}^2 \Rightarrow -T + N = -mL\dot{\phi}^2$$

$$\hat{\phi}]\quad -c\cancel{L}\frac{\cancel{\sqrt{2}}}{\cancel{2}}\dot{\phi} = \frac{\cancel{\sqrt{2}}}{\cancel{2}}m\cancel{L}\ddot{\phi} \Rightarrow \ddot{\phi} = \frac{-c}{m}\dot{\phi}$$

- Resolvemos la ec en $\hat{\phi}$ usando $\ddot{\phi} = \dot{\phi}\frac{d\dot{\phi}}{d\phi}$

$$\Rightarrow \ddot{\phi} = \cancel{\dot{\phi}}\frac{d\dot{\phi}}{d\phi} = \frac{-c}{m}\cancel{\dot{\phi}} \Rightarrow d\dot{\phi} = \frac{-c}{m}\,d\phi \quad \Big/ \int_{\dot{\phi}_0}^{\dot{\phi}}, \int_0^{\phi}$$

$$\Rightarrow \dot{\phi} - \dot{\phi}_0 = -\frac{c}{m}\phi \quad \star\star$$

- Pero $\dot{\phi}_0$ (w_0 en el enunciado) no lo sabemos, reemplazamos $\star$ en la ec en $\hat{\rho}$

$$\Rightarrow -mL\dot{\phi}^2 = N - \sqrt{2}mg + N \Rightarrow \dot{\phi}^2 = \frac{2N - \sqrt{2}mg}{-mL}$$

- En $t=0$, $N=0$

$$\Rightarrow \dot{\phi}_0^2 = \frac{0 - \sqrt{2}mg}{-mL} \Rightarrow \dot{\phi}_0^2 = \sqrt{2}\frac{g}{L}$$

• Reemplazamos en $\star\star$

$$\Rightarrow \dot{\phi} = \dot{\phi}_0 - \frac{c}{m}\phi$$

$$\Rightarrow \boxed{\dot{\phi}(\phi) = \sqrt{\sqrt{2}\frac{g}{L}} - \frac{c}{m}\phi}$$

b) Encontrar $T(\phi)$

• Reescribimos la ec en $\hat{\rho}$ como: $N = T - mL\dot{\phi}^2$ y reemplazamos en $\star$

$$\Rightarrow T = \sqrt{2}mg - T + mL\dot{\phi}^2$$

$$\Rightarrow T = \frac{1}{2}\left[\sqrt{2}mg + mL\dot{\phi}^2\right]$$ $\dot{\phi}(\phi)$ de a)

$$= \frac{1}{2}\left(\sqrt{2}mg + mL\left(\sqrt{\sqrt{2}\frac{g}{L}} - \frac{c}{m}\phi\right)^2\right)$$

$$\Rightarrow \boxed{T = \frac{\sqrt{2}}{2}mg + \frac{mL}{2}\left(\sqrt{\sqrt{2}\frac{g}{L}} - \frac{c}{m}\phi\right)^2}$$

c) Obtener ϕ_{tot} donde m se detiene:

• De la ec $\dot{\phi}(\phi)$ hacemos $\dot{\phi}(\phi_{tot}) = 0$

$$\Rightarrow 0 = \sqrt{\sqrt{2}\frac{g}{L}} - \frac{c}{m}\phi_{tot} \Rightarrow \boxed{\phi_{tot} = \frac{m}{c}\sqrt{\sqrt{2}\frac{g}{L}}}$$