Aux #22.

10/Julio/2020

$$\sum \vec{F}_{ext} = \dot{\vec{P}}_{cm} = M \cdot \vec{a}_{cm}$$

$$\vec{\tau}_o = \sum \vec{r} \times \vec{F} \longrightarrow \vec{\tau}_o = \frac{d\vec{L}_o}{dt} = I_o \cdot \ddot{\theta}$$

$$\vec{L}_o = \vec{R}_{cm} \times M \cdot \vec{V}_{cm} + \vec{L}_{cm}$$

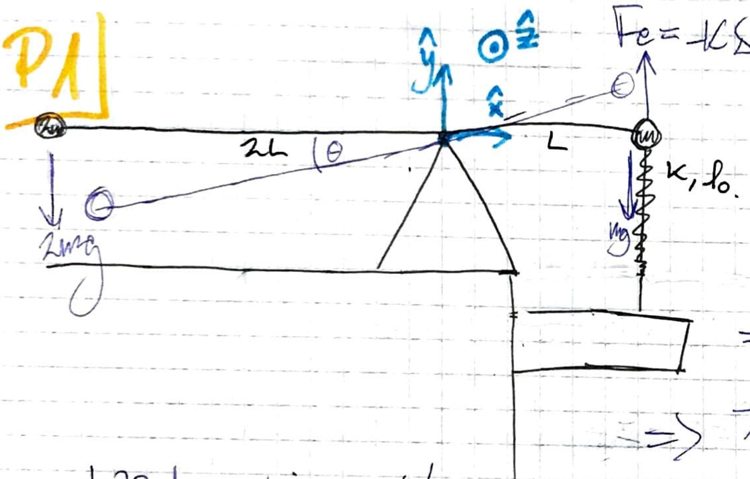
$$I_{cm} = \int_0^M r^2 dm = \int_0^M r^2 D \cdot dV$$

Steiner: $I_o = I_{cm} + MR^2$

$$K_{rot} = \frac{1}{2} I_o \cdot \dot{\theta}^2 \quad , \quad K_{tras} = \frac{1}{2} m \vec{V}^2$$

$$U_g = mgh \quad , \quad U_e = \frac{1}{2} k \delta^2$$

P1



a) δ inicial

$\downarrow g$

$$\vec{\tau}_o = \sum \vec{r} \times \vec{F}$$

$$= 2L(-\hat{x}) \times 2mg(-\hat{y}) + L(\hat{x}) \times mg(-\hat{y}) + L(\hat{x}) \times \vec{F}_e(\hat{y})$$

$$= 4mgL(\hat{z}) + -Lmg(\hat{z}) + -Lk\delta(\hat{z})$$

$$\Rightarrow \vec{\tau}_o = (3mgL - Lk\delta)\hat{z}$$

Inicialmente está en reposo $\Rightarrow$ NO está rotando $\Leftrightarrow$ $\vec{\tau}_o = 0$.

$$3mg\cancel{L} - \cancel{L}k\delta = 0$$

$$\boxed{\delta = \frac{3mg}{k}}$$

b) frecuencia pequeñas oscilaciones. (ENERGÍA)

Energía Cinética: $K = \underset{(2m)}{K_1} + \underset{(m)}{K_2}$

$$K = \frac{1}{\cancel{2}} \cdot \cancel{2}m \cdot \vec{V}_1^2 + \frac{1}{2} m \cdot \vec{V}_2^2$$

$$K = m \cdot (2L \cdot \dot{\theta})^2 + \frac{1}{2} m (L\dot{\theta})^2$$

$$K = mL^2 \dot{\theta}^2 \left(4 + \frac{1}{2}\right) = \frac{9}{2} mL^2 \dot{\theta}^2$$

$\vec{V}$ en polares

$$\vec{V} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k}$$

$$\dot{z} = \dot{\rho} = 0$$

$$\Rightarrow \vec{V} = \rho \cdot \dot{\phi}\hat{\phi}$$

$$U(\theta) = \overbrace{+mgL\cdot sen\theta}^{m\ sube} - \overbrace{2mg\cdot(2L\cdot sen\theta)}^{2m\ baja} + \frac{1}{2}K\left[(\cancel{l_0}+\Delta) + L\cdot sen\theta) - \cancel{l_0}\right]^2$$

$$U(\theta) = -3mgL\,sen\theta + \frac{1}{2}k\left[\Delta + L\,sen\theta\right]^2$$

$$\Rightarrow E(\theta) = K + U(\theta) = \frac{9}{2}mL^2\dot{\theta}^2 - 3mgL\,sen\theta + \frac{1}{2}k(\Delta + L\cdot sen\theta)^2.$$

⊛HINT: $E(\theta) = \frac{1}{2}\alpha\cdot\dot{\theta}^2 + U(\theta) \quad\Rightarrow\quad w^2 = \frac{U''(\theta_{eq})}{\alpha}$

$$\Rightarrow w^2 = \frac{U''(\theta_{eq})}{\alpha} = \frac{U''(0)}{9mL^2} \longrightarrow \begin{array}{l}\theta_{eq} = 0\\ \alpha = 9mL^2\end{array}$$

$$U'(\theta) = -3mgL\cos\theta + k(\Delta + L\,sen\theta)\cdot L\cdot\cos\theta.$$

$$U''(\theta) = +3mgL\cdot sen\theta - kL(\Delta + L\,sen\theta)\cdot sen\theta + kL\cos\theta\cdot L\cos\theta.$$

$$U''(\theta) = 3mgL\cdot sen\theta - kL(\Delta + L\cdot sen\theta)\cdot sen\theta + kL^2\cdot\cos^2\theta.$$

$$U''(\theta_{eq}) = U''(0) = 0 - 0 + kL^2.$$

$$\Rightarrow w^2 = \frac{k\cancel{L^2}}{9m\cancel{L^2}} \Rightarrow \boxed{w^2 = \frac{k}{9m}}$$

© $\vec{V}_{máx}$ de 2m si el resorte se corta desde $\theta = 0$

$\vec{V}_{máx} \iff K_{máx} \Rightarrow$ Por conservación de E. $U(\theta)$ mínimo.

$$U'(\theta^*) = 0 = \left[-3mgL_{\cos} + k(\Delta + L\cdot sen\theta)\right]\cdot\cos\theta$$

$$\Rightarrow \cos\theta^* = 0 \Rightarrow \boxed{\theta^* = \oplus\frac{\pi}{2}} \longrightarrow$$ tiene sentido: es donde 2m está más abajo.

$$E(\theta=0) = E(\theta = \tfrac{\pi}{2}) \longrightarrow E\ elástica = 0$$

$$\cancel{\frac{1}{2}k\Delta^2 = \frac{9}{2}mL^2\dot{\theta}^2 - 3mgL\cdot 1 + \frac{1}{2}k(\Delta + L\cdot 1)^2}$$

$$\cancel{\frac{9}{2}mL^2\dot{\theta}^2 = 3mgL + \frac{1}{2}k\left[\left(\frac{3mg}{k} + L\right)^2 - \Delta^2\right]}$$

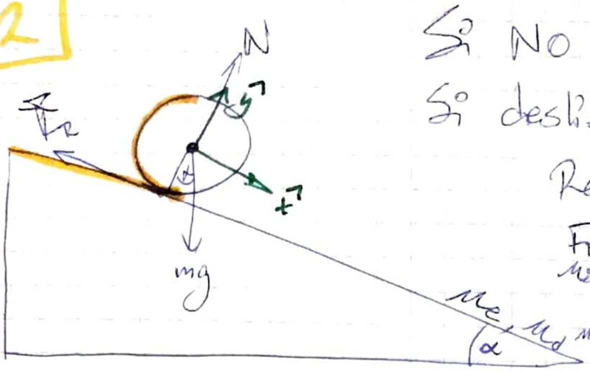
$$\cancel{\frac{9}{2}mL^2\dot{\theta}^2 = 3mgL + \frac{1}{2}k\left[-\Delta^2 - 2\Delta L - L^2 + \Delta^2\right]}$$

$$\cancel{\frac{9}{2}mL^2\dot{\theta}^2 = 3mgL - kL\cdot\frac{3mg}{k}}$$

$$0 = \frac{3}{2}\cancel{m}\cdot\vec{V}_m^2 - 3\cancel{m}gL.$$

$$\Rightarrow \boxed{\vec{V}_m^2 = \sqrt{\frac{2gL}{3}}}$$

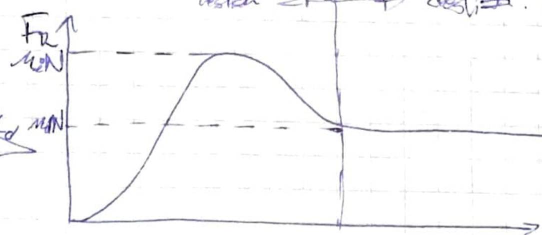
P2



Si NO desliza: μ_e, F_R incógnita.

Si desliza: μ_d, $F_R = \mu_d N$

Recordar: No desliza ⟷ desliza.



① Si NO desliza y $F_R < \mu_e N$ ⇒ se queden sin deslizar

② Si NO desliza. ⇒ y $F_R > \mu_e N$ Va a deslizar.

① DESLIZA: Ecs. de movimiento: → cómo se traslada.

$\hat{x}$) $m \cdot \ddot{x}_{cm} = mg \cdot \operatorname{sen}\alpha - F_R$.

$\hat{y}$) $m \cdot \ddot{y}_{cm} = N - mg \cdot \cos\alpha = 0 \Rightarrow N = mg \cdot \cos\alpha$.

$\Rightarrow m\ddot{x}_{cm} = mg \cdot \operatorname{sen}\alpha - \mu_d \cdot mg\cos\alpha$.

$I_{cm} \cdot \ddot{\phi} = \dot{\vec{L}}_{cm} = \vec{\tau}^{ext}_{cm}$ → cómo rota.

$\vec{\tau}^{ext}_{cm} = R(-\hat{y}) \times F_R(-\hat{x}) = R \cdot \mu_d \cdot mg\cos\alpha \cdot \hat{z} = R \cdot F_R \cdot \hat{z}$.

$\Rightarrow I_{cm} \cdot \ddot{\phi} = R \cdot \mu_d \cdot mg \cdot \cos\alpha$

② NO DESLIZA:

$\boxed{\text{Condición Rodadura perfecta. } x_{cm} = R \cdot \phi}$

Ahora queremos encontrar F_R.

$\Rightarrow \ddot{x}_{cm} = R \cdot \ddot{\phi}$

$\Rightarrow I_{cm} \cdot \ddot{\phi} = R \cdot F_R$ → incógnita.

$\rightarrow m\ddot{x}_{cm} = mR\ddot{\phi} = mg\operatorname{sen}\alpha - F_R /\cdot R$

Sumando:

$(+)\ (mR^2 + I_{cm})\ddot{\phi} = mg \cdot \operatorname{sen}\alpha \cdot R \Rightarrow \boxed{\ddot{\phi} = \frac{mg \cdot \operatorname{sen}\alpha}{mR^2 + I_{cm}}}$

$\Rightarrow mR\ddot{\phi} = mg\operatorname{sen}\alpha - F_R$

Reemplazando $\ddot{\phi} \Rightarrow mR\left(\frac{mg \cdot \operatorname{sen}\alpha}{mR^2 + I_{cm}}\right) - mg \cdot \operatorname{sen}\alpha = -F_R$.

$\Rightarrow \boxed{F_R = mg\operatorname{sen}\alpha\left[\frac{mR}{mR^2 + I_{cm}} - 1\right]}$

Ahora queda todo conocido.