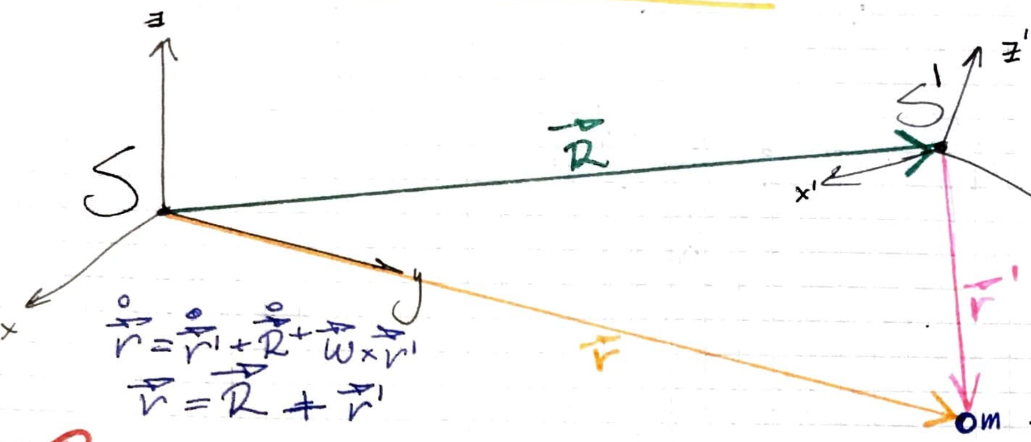
Aux #16.

12/Junio/2020.



$\ddot{\vec r} = \ddot{\vec r}' + \ddot{\vec R} + \vec\omega \times \vec r'$

$\vec r = \vec R + \vec r'$

$\vec R$: cómo S' se traslada c/r a S

$\vec\Omega$: cómo S' rota c/r a S.

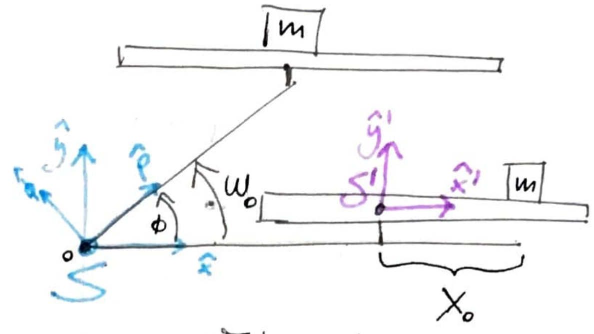
Pasos SRNI:

1. Definir S y S'
2. " sistema de coordenadas de S y S'.
3. Escribir vectores unitarios de S en función de los vectores unitarios de S'.
4. Calcular $\vec r'$, $\vec v'$, $\vec a'$.
5. Escribir fuerzas reales. → DCL en S'
6. Calcular $\vec\Omega$ y $\dot{\vec\Omega}$.
7. Calcular $\vec R$, $\dot{\vec R}$, $\ddot{\vec R}$.
8. Calcular fzas. ficticias.
9. Escribir ecuación de movimiento: $m\vec a' = \sum F_{reales} + \sum F_{ficticias}$

$$m\cdot\vec a' = \sum \vec F_{reales} - \underset{\text{lineal}}{m\cdot\vec a_0} - \underset{\text{centrífuga}}{m\cdot\vec\Omega\times(\vec\Omega\times\vec r')} - \underset{\text{Coriolis}}{2m\cdot\vec\Omega\times\vec v'} - \underset{\text{transversal}}{m\cdot\dot{\vec\Omega}\times\vec r'}$$

10. RESOLVER.

P1



$\downarrow g$

① S en O, S' en la plat. abajo

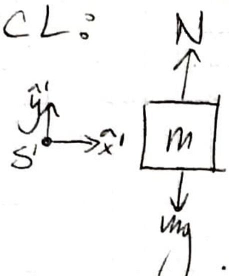
② S: polares.
S': cartesianas.

③ $\hat\rho = \cos\phi\,\hat x + \sin\phi\,\hat y$

$\hat\phi = -\sin\phi\,\hat x + \cos\phi\,\hat y$.

④ $\vec r\,' = x'\hat x' + y'\hat y'$, $\vec v\,' = \dot x'\hat x' + \dot y'\hat y'$, $\vec a\,' = \ddot x'\hat x' + \ddot y'\hat y'$

⑤ DCL:



$\Rightarrow \sum \vec F_{reales} = (N - mg)\,\hat y'$

⑥ $\vec\Omega = 0 \Rightarrow \dot{\vec\Omega} = 0$

⑦

$\rho = R \Rightarrow \dot\rho = \ddot\rho = 0$.

$\omega_0 = \dot\phi \Rightarrow \phi = \omega_0 t$, $\ddot\phi = 0$.

$\Rightarrow \vec R = \rho\hat\rho$, $\dot{\vec R} = \cancel{\dot\rho\hat\rho} + \rho\dot\phi\hat\phi = R\omega_0\hat\phi$

$\ddot{\vec R} = (\cancel{\ddot\rho} - \rho\dot\phi^2)\hat\rho + (\cancel{2\dot\rho\dot\phi} + \cancel{\rho\ddot\phi})\hat\phi$

$\ddot{\vec R} = -R\omega_0^2\hat\rho$

⑧ Fzas. ficticias:

Lineal: $-m\vec a_0 = -m\ddot{\vec R} = +mR\omega_0^2\hat\rho$.

Coriolis: $-2m\underbrace{\vec\Omega}_{=0}\times\vec v\,' = 0$

Centrífuga: $-m\vec\Omega\times(\vec\Omega\times\vec r\,') = 0$

Transversal: $-m\dot{\vec\Omega}\times\vec r\,' = 0$.

⑨ Ec. de movimiento: $m\vec a\,' = \vec F_{reales} + \vec F_{ficticias}$.

$m\ddot y'\hat y' + m\ddot x'\hat x' = (N - mg)\hat y' + mR\omega_0^2\hat\rho$

$m\ddot y'\hat y' + m\ddot x'\hat x' = (N - mg)\hat y' + mR\omega_0^2(\cos\phi\,\hat x + \sin\phi\,\hat y)$.

$\hat x'\rfloor$ $m\ddot x' = mR\omega_0^2\cos\phi$

$\hat y'\rfloor$ $m\ddot y' = N - mg + mR\omega_0^2\sin\phi$.

P1 ⓐ $x'_{máx}$:

$\hat{x}'$: $\cancel{m}\ddot{x}' = \cancel{m} R\omega_0^2 \cdot \cos\phi$) trucazo.

$$\int_0^{\dot{x}'_F} \dot{x}' \, d\dot{x}' = \int_0^{x'_F} R\omega_0^2 \cdot \cos\phi \, dx'$$

terminó de moverse.

$$0 = \frac{\dot{x}_F'^2}{2} = R\omega_0^2 \cdot \cos\phi \cdot x'_F$$

$$\Rightarrow \boxed{x'_{max} = 0}$$

ⓑ ω_0 máx tq m no se despega.

$$N > 0 \quad , \quad \ddot{y}' = 0.$$

$$0 = N - mg + mR\omega_0^2 \cdot \operatorname{sen}\phi$$

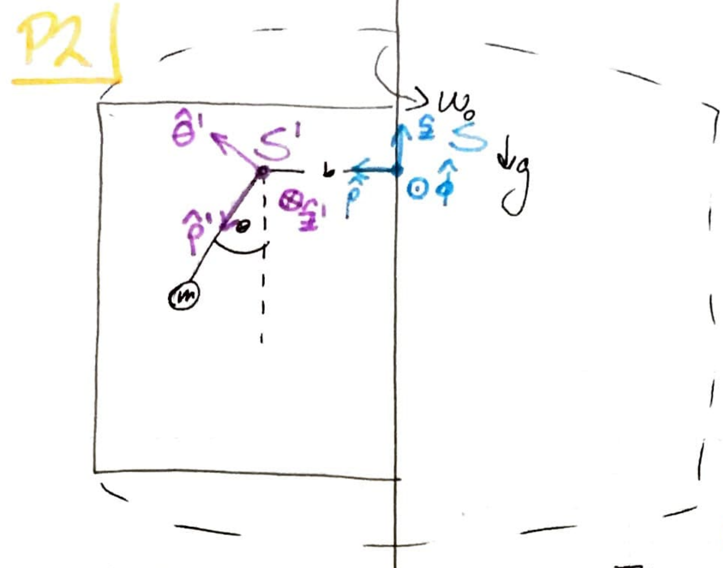
$$N = \cancel{m}g - \cancel{m}R\omega_0^2 \cdot \operatorname{sen}\phi > 0$$

$$-R\omega_0^2 \cdot \operatorname{sen}\phi > -g \quad / \cdot -1$$

$$R\omega_0^2 \cdot \operatorname{sen}\phi < g.$$

$$\boxed{\omega_0 < \sqrt{\frac{g}{R \cdot \operatorname{sen}\phi}}}$$

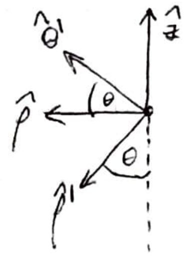
P2



① S, S'

② S: cilíndricas, S': polares.

③ $-\hat z' = \hat\phi$ $\quad \dot\phi = \omega_0$, $\phi = \omega_0 t$, $\ddot\phi = 0$



$$\hat z = -\cos\theta\,\hat\rho' + \operatorname{sen}\theta\,\hat\theta'$$
$$\hat\rho = \operatorname{sen}\theta\,\hat\rho' + \cos\theta\,\hat\theta'$$

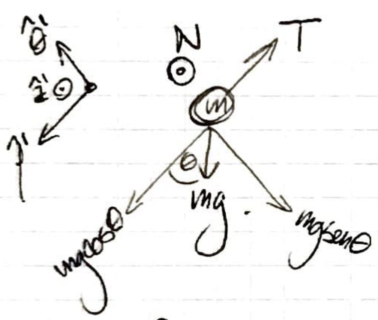
$\rho' = L \Rightarrow \dot\rho' = \ddot\rho' = 0$

$z' = 0 \Rightarrow \dot z' = \ddot z' = 0$

④ $\vec r' = L\hat\rho'$

$$\vec V' = \dot\rho\hat\rho' + \rho\dot\theta\hat\theta' + \dot z\hat z'$$
$$\vec V' = L\dot\theta\hat\theta'$$
$$\vec a' = -L\dot\theta^2\hat\rho' + L\ddot\theta\hat\theta'$$

⑤ Fzas reales:



$$\Rightarrow \sum F_{zas\ reales} = -T\hat\rho' + mg\cos\theta\,\hat\rho' - mg\operatorname{sen}\theta\,\hat\theta' + N\hat z'$$

⑥ $\vec\Omega$ y $\dot{\vec\Omega}$ $\Rightarrow$ $\vec\Omega = \omega_0\hat z$, $\quad \dot{\vec\Omega} = 0$

$$\vec\Omega = \omega_0(-\cos\theta\,\hat\rho' + \operatorname{sen}\theta\,\hat\theta')$$

⑦ $\vec R = b\hat\rho = b(\operatorname{sen}\hat\rho' + \cos\theta\,\hat\theta')$

$$\dot{\vec R} = \cancel{\dot b\hat\rho}_{0} + b\omega_0\hat\phi + \cancel{\dot z\hat z}_{0} = b\omega_0\hat\phi = -b\omega_0(\hat z')$$

$$\vec a_0 = \ddot{\vec R} = (-b\cdot\omega_0^2)\hat\rho + \cancel{b\ddot\phi}_{0}\hat\phi = -b\omega_0^2(\operatorname{sen}\theta\,\hat\rho' + \cos\theta\,\hat\theta').$$

⑧ $\rightarrow \vec F_{lineal} = -m\cdot\vec a_0 = +mb\omega_0^2(\operatorname{sen}\hat\rho' + \cos\theta\,\hat\theta')$

$\rightarrow \vec F_{centrífuga} = -m\cdot\vec\Omega\times(\vec\Omega\times\vec r') =$

1) $\vec\Omega\times\vec r' = \omega_0(-\cos\theta\,\hat\rho' + \operatorname{sen}\theta\,\hat\theta')\times L\hat\rho'$

$$\vec\Omega\times\vec r' = \omega_0\cdot L\cdot\begin{vmatrix}\hat\rho' & \hat\theta' & \hat z'\\ -\cos\theta & \operatorname{sen}\theta & 0\\ 1 & 0 & 0\end{vmatrix}$$

$$= \omega_0 L(0\hat\rho' + 0\hat\theta' + -\operatorname{sen}\theta\,\hat z') = -\omega_0 L\operatorname{sen}\theta\,\hat z'.$$

2) $\vec\Omega\times(\vec\Omega\times\vec r') = \omega_0(-\cos\theta\,\hat\rho' + \operatorname{sen}\theta\,\hat\theta')\times -\omega_0 L\operatorname{sen}\theta\,\hat z'$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = -w_0^2 L\cdot sen\theta \begin{vmatrix} \hat{\rho}' & \hat{\theta}' & \hat{z}' \\ -\cos\theta & sen\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= -w_0^2 L\cdot sen\theta\left[sen\theta\,\hat{\rho}' + (+\cos\theta\,\hat{\theta}') + 0\,\hat{z}'\right]$$

$$= -w_0^2 L\cdot\left[sen^2\theta\,\hat{\rho}' + sen\theta\cos\theta\,\hat{\theta}'\right].$$

$$\Rightarrow \vec{F}_{centrifuga} = -m\cdot\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = m w_0^2 L\left[sen^2\theta\,\hat{\rho}' + sen\theta\cos\theta\,\hat{\theta}'\right]$$

$$\rightarrow \vec{F}_{Coriolis} = -2m\,\vec{\Omega}\times\vec{V}' = -2m\,w_0(-\cos\theta\,\hat{\rho}' + sen\theta\,\hat{\theta}')\times L\dot{\theta}\,\hat{\theta}'$$

$$= -2m w_0 L\dot{\theta}\begin{vmatrix} \hat{\rho}' & \hat{\theta}' & \hat{z}' \\ -\cos\theta & sen\theta & 0 \\ 0 & 1 & 0 \end{vmatrix} = -2m w_0 L\dot{\theta}\left(0\,\hat{\rho}' + 0\,\hat{\theta}' + -\cos\theta\,\hat{z}'\right)$$

$$\vec{F}_{Coriolis} = 2m w_0 L\dot{\theta}\cos\theta\,\hat{z}'$$

$$\rightarrow \vec{F}_{transversal} = -m\,\dot{\vec{\Omega}}\times\vec{r}' = 0.$$

9 Ec de movimiento: $m\cdot\vec{a}' = F_{reales} + F_{ficticias}$.

$$m\cdot(-L\dot{\theta}^2\hat{\rho}' + L\ddot{\theta}\,\hat{\theta}') = (mg\cos\theta - T)\hat{\rho}' - mg\,sen\theta\,\hat{\theta}' \mp N\hat{z}'$$
$$+ m b w_0^2(sen\theta\,\hat{\rho}' + \cos\theta\,\hat{\theta}') + m w_0^2 L\left[sen^2\theta\,\hat{\rho}' + sen\theta\cdot\cos\theta\,\hat{\theta}'\right]$$
$$+ 2m w_0 L\dot{\theta}\cos\theta\,\hat{z}'.$$

10 RESOLVER.

a) $\vec{V}' = L\dot{\theta}\,\hat{\theta}'$ en función de θ, no de $\dot{\theta}$.

$\circledast\ sen(2\theta) = 2\,sen\theta\cdot\cos\theta$

$$\hat{\theta}'\,\Big|\quad \cancel{m}L\ddot{\theta} = -\cancel{m}g\,sen\theta + \cancel{m}b w_0^2\cos\theta + \cancel{m}w_0^2 L\cdot sen\theta\cdot\cos\theta.$$

$$\ddot{\theta} = -\frac{g}{L}\cdot sen\theta + \frac{b w_0^2}{L}\cdot\cos\theta + \frac{1}{2}w_0^2\cdot sen(2\theta)\quad \Big|\ \oint \text{TRUCAZO}$$

$$\int_0^{\dot{\theta}}\dot{\theta}\,d\dot{\theta} = \int_0^{\theta} -\frac{g}{L}\cdot sen\theta + \frac{b w_0^2}{L}\cdot\cos\theta + \frac{1}{2}w_0^2\cdot sen(2\theta)\ d\theta$$

$$\frac{\dot{\theta}^2}{2} = -\frac{g}{L}(1-\cos\theta) + \frac{b w_0^2}{L}\cdot sen\theta + \frac{1}{2}w_0^2(1-\cos(2\theta))\frac{1}{2}$$

$$\frac{\dot{\theta}^2}{2} = -\frac{g}{L}(1-\cos\theta) + \frac{b w_0^2}{L}\cdot sen\theta + \frac{1}{4}w_0^2(1-\cos(2\theta))$$

$$\Rightarrow \vec{V}' = L\dot{\theta}\,\hat{\theta}'\quad \text{con}$$

$$\dot{\theta} = \left[-\frac{2g}{L}(1-\cos\theta) + \frac{2b w_0^2}{L}\cdot sen\theta + \frac{1}{2}w_0^2(1-\cos(2\theta))\right]^{1/2}.$$

ⓑ $N(\theta)$ $\hat{z}'$) $0 = -N\hat{z}' + 2m\omega_0 L\dot{\theta}\cdot\cos\theta$.

$$N = 2m\omega_0 L\cdot\cos\theta\cdot\dot{\theta}$$

$$N = 2m\omega_0 L\cdot\cos\theta\cdot\left[\frac{-2g}{L}(1-\cos\theta) + \frac{2b\omega_0^2}{L}\cdot\operatorname{sen}\theta + \frac{1}{2}\omega_0^2(1-\cos(2\theta))\right]^{1/2}$$

ⓒ ¿Se despega? $N = 0$

$$2m\omega_0 L\cos\theta\cdot\dot{\theta} = 0.$$

1) $\cos\theta = 0.$ o 2) $\dot{\theta} = 0$

$$\boxed{\theta = \pi/2.}$$