

Avx # 10: Energía Cinética:

$$K = \frac{1}{2} m \vec{V}^2$$

Energía Potencial: $U = mgh$

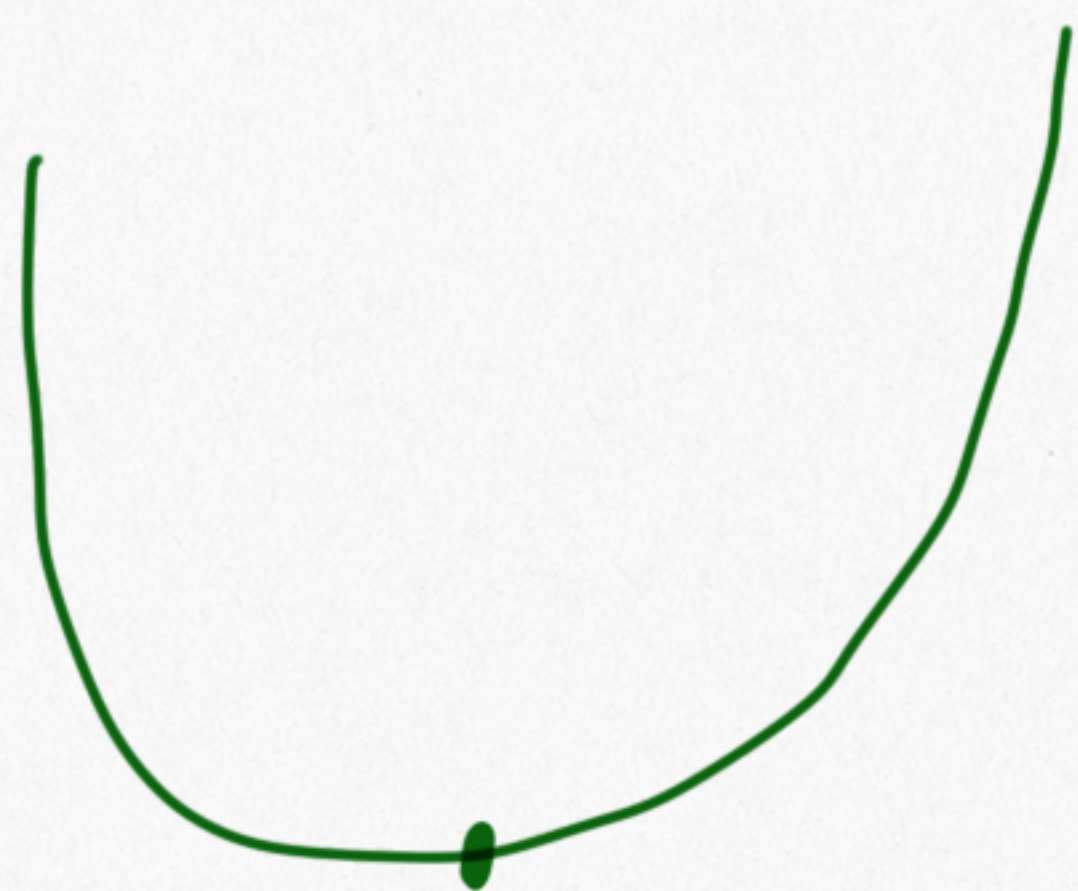
Pequeñas Oscilaciones: ω_0 : Frecuencia.

$$\textcircled{1} \quad \omega_0^2 = \frac{U''(x_{eq})}{m}$$

$$\textcircled{2} \text{ Taylor: } \ddot{x} + \omega_0^2 x = cte \quad \bullet$$

Puntos de equilibrio: $\frac{dU}{dx} = 0$

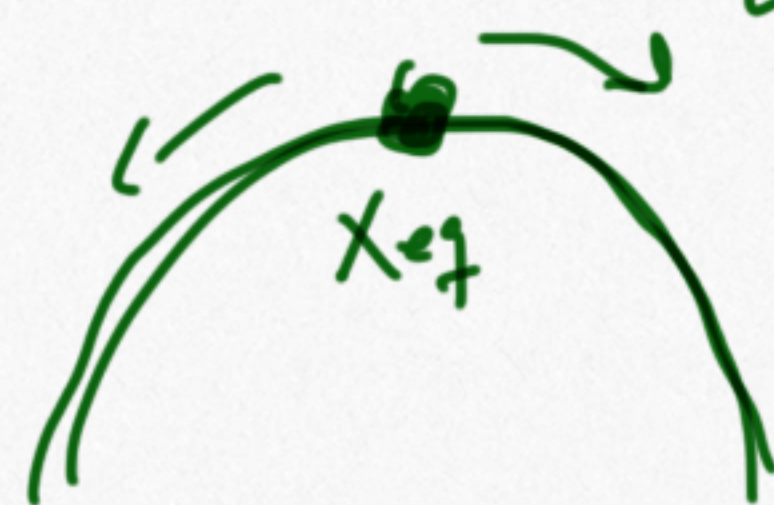
Estabilidad: $\frac{d^2U}{dx^2}(x_{eq})$



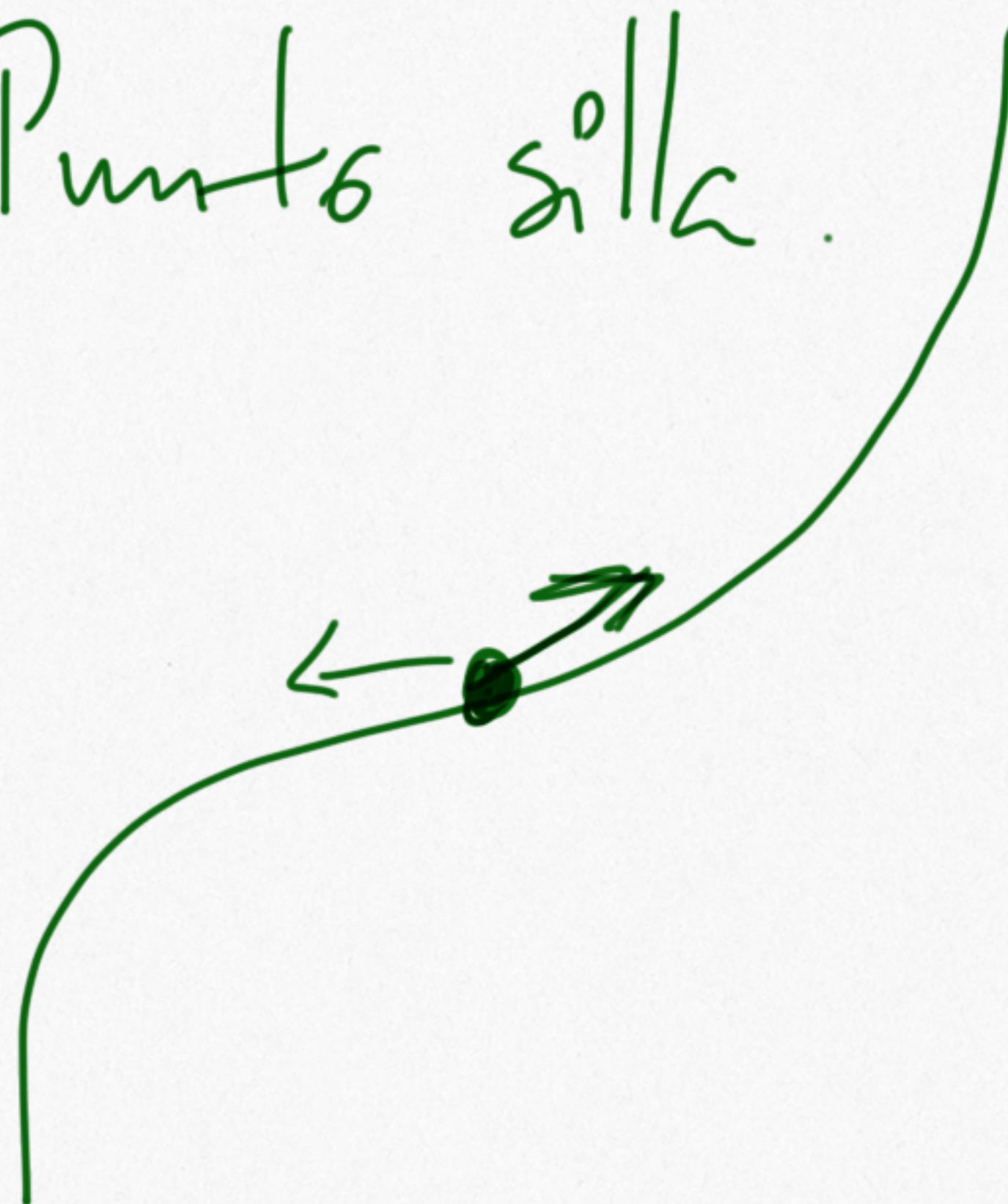
$$\frac{d^2 V}{dx^2}(x_{eq}) > 0$$

$\Rightarrow$ ESTABLE

$$\frac{d^2 V}{dx^2}(x_{eq}) < 0 \Rightarrow \text{INESTABLE}$$

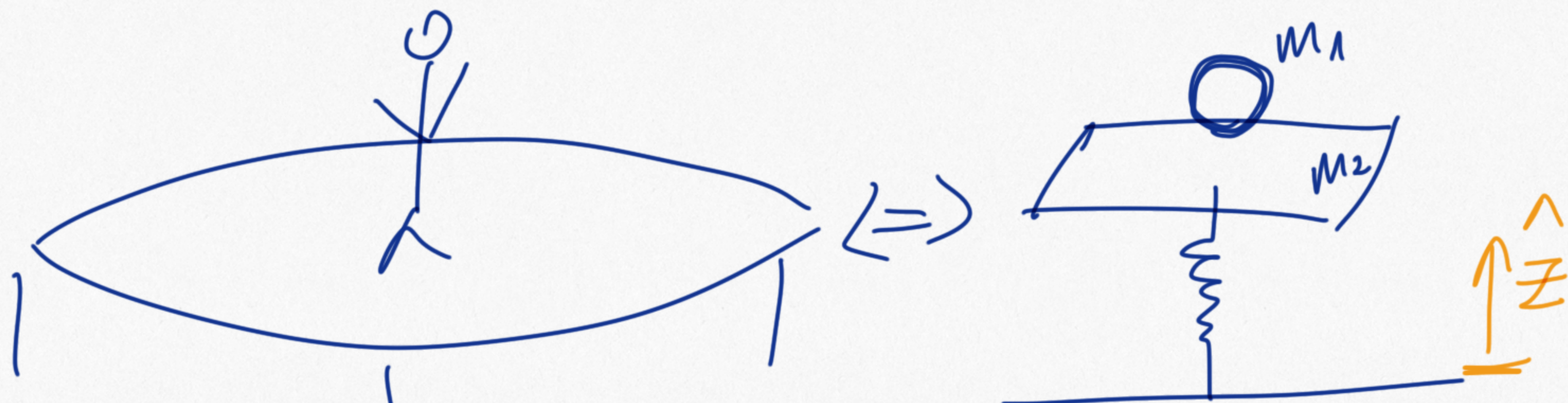


$$\frac{d^2 V}{dx^2}(x_{eq}) = 0 \Rightarrow \text{Punto silla.}$$



P_1 $\xrightarrow{\text{mure-O}} F_e$

$$F_e = (-K \cdot \Delta r)$$



(a)

Free-body diagram for mass m_1 : An upward arrow is labeled N and a downward arrow is labeled $m_1 g$.

$$(1) m_1 \ddot{z} = N - m_1 g \quad *$$

Free-body diagram for mass m_2 : An upward arrow is labeled F_e . Two downward arrows are labeled $m_2 g$ and N .

$$m_2 \ddot{z} = F_e - m_2 g - N$$

$$(2) m_2 \ddot{z} = \underbrace{-K(z - l_0)}_{<0} - m_2 g - N$$

>0

$$① m_1 \ddot{z} = N - m_1 g \quad / \cdot m_2$$

$$② m_2 \ddot{z} = -k(z - l_0) - N - m_2 g \quad / m_1$$

$$\cancel{m_1 m_2} \ddot{z} = m_2 N - \cancel{m_1 m_2} g$$

$$\cancel{m_1 m_2} \ddot{z} = -m_1 k(z - l_0) - m_1 N - \cancel{m_1 m_2} g$$

$$① \quad 0 = +m_1 k(z - l_0) + (m_2 + m_1) N$$

$$N = \frac{-m_1 k(z - l_0)}{m_1 + m_2}$$

$$② m_2 \ddot{z} = -k(z - l_0) + \frac{m_1 k(z - l_0)}{m_1 + m_2} - m_2 g$$

$$z = z_{eq} \Rightarrow \ddot{z} = 0$$

$$0 = -k(m_1 + m_2)(z - l_0) + m_1 k(z - l_0) - m_2(m_1 + m_2)g$$

$$0 = k(z - l_0) \left[+ \cancel{(m_1 + m_2)} + m_1 \right] - \cancel{m_2} (m_1 + m_2) g$$

$$\cancel{m_2} (m_1 + m_2) g = k(z_{eq} - l_0) (-\cancel{m_2})$$

$$-k z_{eq} = (m_1 + m_2) g + k \cdot l_0$$

$$z_{eq} = l_0 - \underbrace{\frac{(m_1 + m_2) g}{k}}$$

$$b) m_2 \ddot{z} = -k(z - l_0) - m_2 g - N$$

$$m_2 \ddot{z} = -k(z - l_0) - m_2 g + \frac{m_1 k (z - l_0)}{(m_1 + m_2)}$$

$$m_2 \ddot{z} = k(z - l_0) \left(\frac{m_1}{m_1 + m_2} - 1 \right) - m_2 g$$

$$\cancel{m_2} \ddot{z} = k(z - l_0) \left(\frac{-\cancel{m_2}}{m_1 + m_2} \right) - \cancel{m_2} g$$

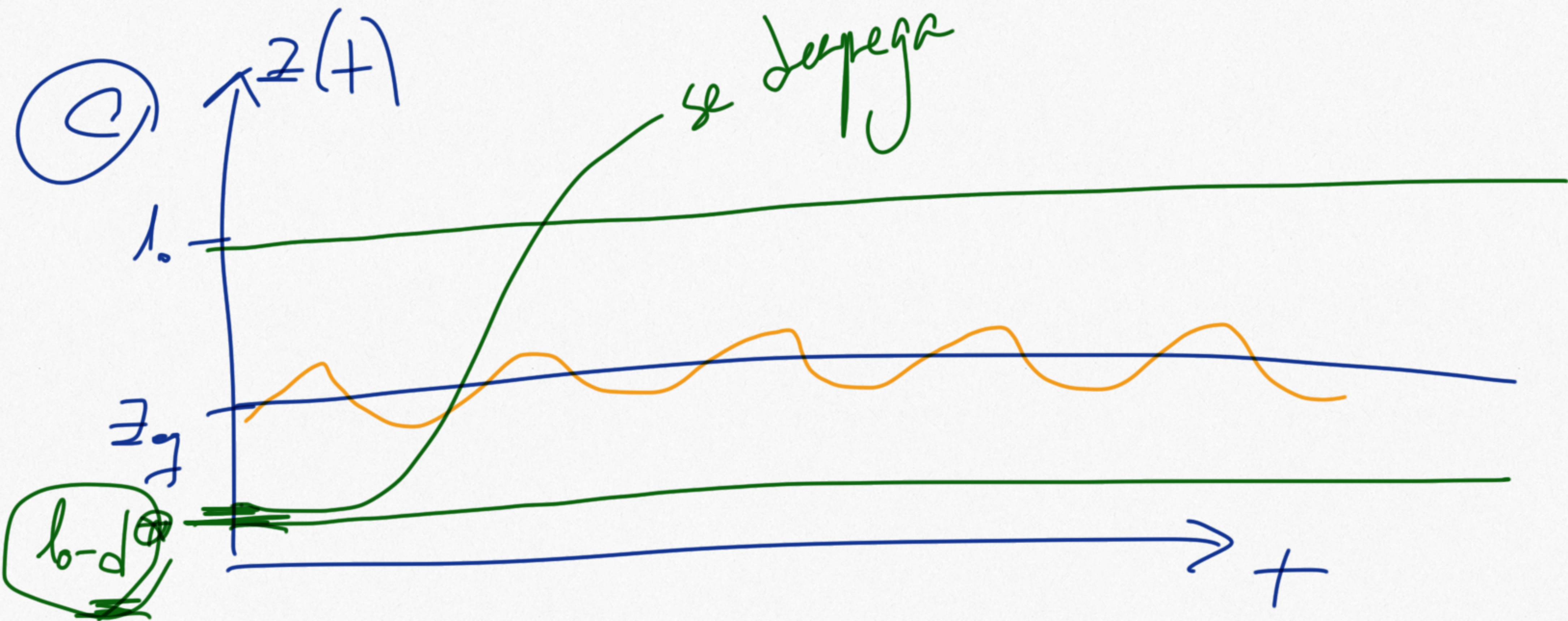
$$\ddot{z} = -k \frac{(z - l_0)}{(m_1 + m_2)} - g$$

$$\ddot{z} = \frac{-kz}{m_1 + m_2} + \frac{kl_0}{m_1 + m_2} - g$$

$$\ddot{z} + \left(\frac{\kappa}{m_1 + m_2} \right) z = \frac{\kappa l_0}{m_1 + m_2} - g$$

$$W_o^2 = \frac{K}{m_1 + m_2}$$

$$T = \frac{2\pi}{\omega_0} \quad T = 2\pi \sqrt{\frac{m_1 + m_2}{k}}$$



$$m_2 \ddot{z} = -k(z-l_0) - m_2 g + \underbrace{\frac{k m_1 (z-l_0)}{m_1 + m_2}}$$

$$N=0 \Rightarrow z=l_0$$

$$(m_1 + m_2) \ddot{z} = -k(z-l_0) - (m_1 + m_2)g$$

$$\int (m_1 + m_2) \dot{z} d\dot{z} = \int -k(z-l_0) - (m_1 + m_2)g dz + \boxed{E}$$

$E(l_0-l) = U_g + U_e$ cte? integrat

$$\vec{m} \ddot{\vec{z}} = \vec{F}$$

$$\int m \dot{\vec{z}} d\dot{\vec{z}} = \int (\vec{F}) d\vec{z}$$

$$\int m \dot{\vec{z}} d\dot{\vec{z}} = \int (-\nabla U) d\vec{z} = -U = E$$

$$\frac{(m_1 + m_2) \dot{z}^2}{2} = -k(z-l_0)^2 - (m_1 + m_2)g z + \overline{E}$$

$$0 = \boxed{E} - \frac{k d^2}{2} - (m_1 + m_2) g (l_0 - d)$$

$$0 = \frac{-k d^2}{2} - (m_1 + m_2) g (l_0 - d) + \underbrace{\left[g (m_1 + m_2) \cdot l_0 \right]}_{E_{\text{en } l_0}}$$

$$0 = \frac{-k d^2}{2} - \cancel{(m_1 + m_2) g \cdot l_0} + \cancel{(m_1 + m_2) g d} + \cancel{g (m_1 + m_2) l_0}$$

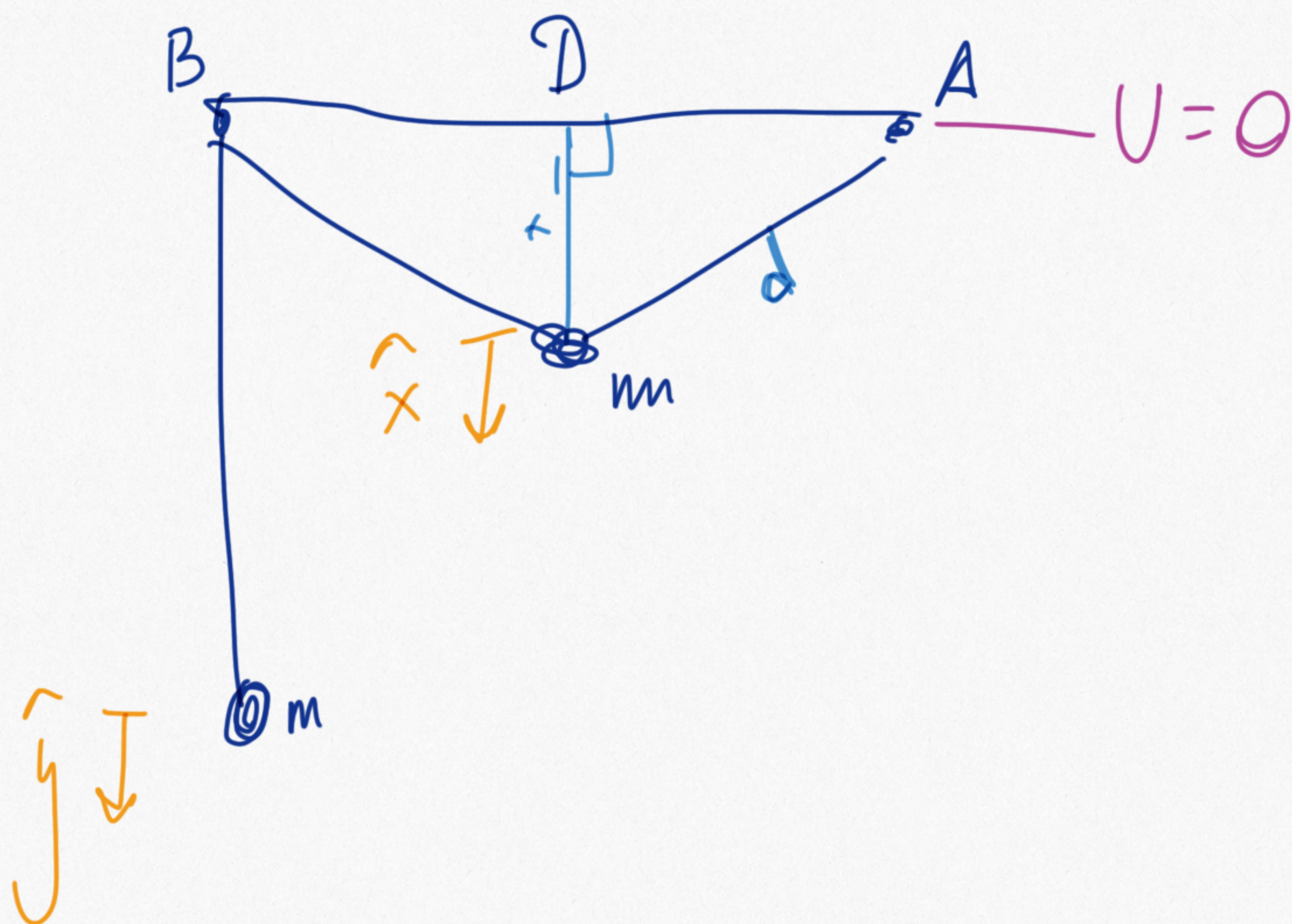
$$\frac{k d^2}{2} = (m_1 + m_2) g d$$

$$d = 2 \cdot \underbrace{\frac{(m_1 + m_2) g}{k}}_{\frac{1}{2} \text{ eq}}$$

$$z_{\text{max}} = l_0 - d$$

$$\boxed{z_{\text{max}} = l_0 + 2 \cdot \frac{(m_1 + m_2) g}{k}}$$

P21



$$U = -mgy - mgx$$

$$\frac{dU}{dx} = 0$$

$$\frac{d^2 U}{dx^2}(x_{eq}) \geq 0$$

$$K = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m (\dot{y})^2$$

$$K = \frac{1}{2} \cdot a(x) \dot{x}^2$$

$$K = \frac{1}{2} m \dot{x}^2$$

$$\Downarrow$$

$$\omega_0^2 = \frac{\frac{d^2 U}{dx^2}(x_{eq})}{m}$$

$$a(x) \approx a(x_{eq})$$

$$\omega_0^2 = \frac{\frac{d^2 U}{dx^2}(x_{eq})}{a(x_{eq})}$$

