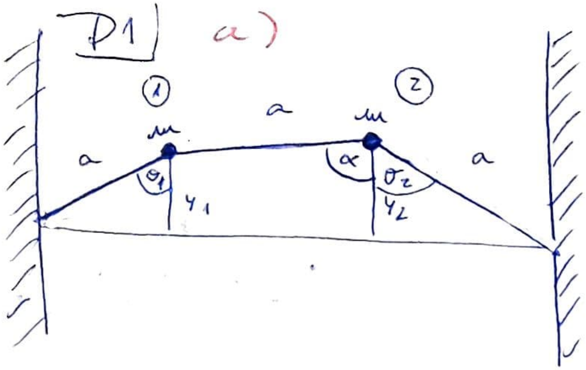
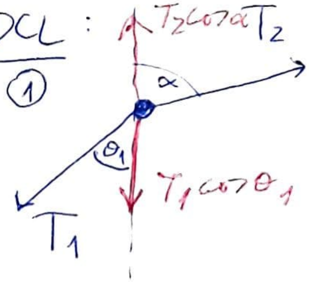
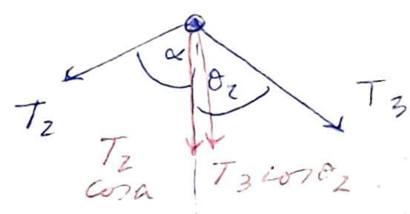
P1) a)



DCL ①

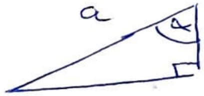


②



Geometría:

$\cos\theta_1 = \dfrac{y_1}{a}$



$y_2 - y_1$ $\quad \dfrac{y_2 - y_1}{a} = \cos\alpha$

$\cos\theta_2 = \dfrac{y_2}{a}$

$$\Rightarrow m\ddot{y}_1 = T_2\cos\alpha - T_1\cos\theta_1 = \frac{1}{a}\left[T_2(y_2-y_1) - T_1 y_1\right]$$

$$m\ddot{y}_2 = -T_2\cos\alpha - T_3\cos\theta_2 = \frac{1}{a}\left[-T_2(y_2-y_1) - T_3 y_2\right].$$

Como "no hay desplazamientos en x", podemos aproximar las tensiones horizontales:

$$-T_1\sin\theta_1 + T_2\sin\alpha = 0 \Rightarrow \sin\theta_1 \approx \sin\alpha \approx 1$$

$$-T_2\sin\alpha + T_3\sin\theta_2 = 0 \Rightarrow \sin\alpha \approx \sin\theta_2 \approx 1$$

$$\Rightarrow T_1 \approx T_2 \approx T_3 \,.\, \tau$$

∴ Las ecs. de movimiento quedan:

$$\boxed{\begin{aligned} m\ddot{y}_1 &= \frac{\tau}{a}\left[-2y_1 + y_2\right] \\ m\ddot{y}_2 &= \frac{\tau}{a}\left[y_1 - 2y_2\right] \end{aligned}} \Bigg\} \quad \underbrace{\begin{bmatrix}\ddot{y}_1 \\ \ddot{y}_2\end{bmatrix}} + \underbrace{\frac{\tau}{ma}\begin{bmatrix} 2 & -1 \\ -1 & 2\end{bmatrix}}\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Si resolvemos el sistema: $\dfrac{d^2}{dt^2}\vec{\xi} + \Omega^2 \cdot \vec{\xi} = 0.$

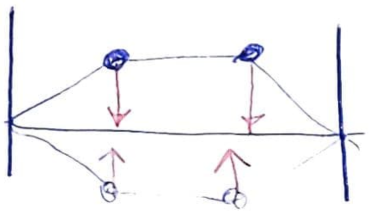
b) $\Omega^2 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \omega_0^2 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \Leftrightarrow \Omega^2 - \omega_0^2 I_{2\times 2} = 0.$

$$\begin{vmatrix} 2-\omega_0^2 & -1 \\ -1 & 2-\omega_0^2 \end{vmatrix} = 0 \Leftrightarrow (2-\omega_0)^2 - 1 = 0 \Rightarrow \begin{cases} \omega_{01}^2 = 1 \\ \omega_{02}^2 = 3 \end{cases}$$

$\therefore \boxed{\omega_1^2 = \frac{\tau}{ma}}$ y $\boxed{\omega_2^2 = \frac{3\tau}{ma}}$

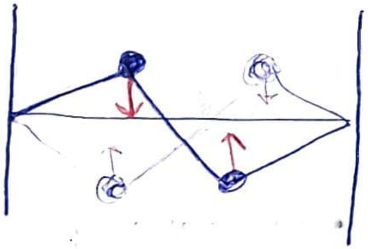
c) Para $\omega_{01}^2 = 1$: $\begin{bmatrix} 2-1 & -1 \\ -1 & 2-1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow e_1 = e_2$

$\therefore \boxed{\vec{e}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}} \Rightarrow$

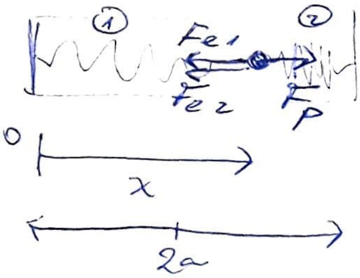


Para $\omega_{02}^2 = 3$: $\begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow e_1 = -e_2$

$\boxed{\vec{e}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}} \Rightarrow$



P2 a)



$F_{e_1} = k(x-a)$

$F_{e_2} = k((2a-x)-a) = k(a-x)$.

$F_p = f_0 \sin(\Omega t)$.

$m\ddot{x} = F_{e_1} - F_{e_2} + F_p$

$$\boxed{m\ddot{x} + 2k(x-a) = f_0 \sin(\Omega t)}$$

b) $x(0) = a$

$\dot{x}(0) = 0$

$$\ddot{x} + \underbrace{\frac{2k}{m}}_{\omega_0^2}(x-a) = \underbrace{\frac{f_0}{m}}_{f_0'}\sin(\Omega t).$$

¡debemos hacer un cambio de variable!

$u(t) = x(t) - a$

$\ddot{u}(t) = \ddot{x}(t)$

$\Rightarrow \ddot{u} + \omega_0^2 u = f_0' \sin(\Omega t) \rightarrow$ Ahora podemos usar el Hint (1)!

Solución:

$$u(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t) + \frac{f_0'}{\omega_0^2 - \Omega^2}\sin(\Omega t).$$

$\frac{d}{dt}$

$$\dot{u}(t) = -A\omega_0 \sin(\omega_0 t) + B\omega_0\cos(\omega_0 t) + \frac{f_0'\Omega}{\omega_0^2 - \Omega^2}\cos(\Omega t)$$

Condiciones iniciales:

si $x(0) = a \Rightarrow u(0) = 0$ y $\dot{x}(0) = 0 \Rightarrow \dot{u}(0) = 0$

1° $0 = A$

2° $0 = A\omega_0 \cdot 0 + B\omega_0 + \frac{f_0'\Omega}{\omega_0^2 - \Omega^2} \Rightarrow B = \frac{f_0'}{\Omega^2 - \omega_0^2}\frac{\Omega}{\omega_0}$.

$$\therefore\ u(t) + a = \boxed{x(t) = a + \frac{f_0'}{\Omega^2 - \omega_0^2}\left[\frac{\Omega}{\omega_0}\sin(\omega_0 t) - \sin(\Omega t)\right]}$$

c) $\omega_0 = \Omega = \sqrt{\frac{2k}{m}} \Rightarrow$ ¡Resonancia!

$x(T) = 2a$

$$a = \lim_{\omega_0 \to \Omega} \frac{f_0'}{\omega_0}\left[\frac{\Omega\sin(\omega_0 T) - \omega_0\sin(\Omega T)}{\Omega^2 - \omega_0^2}\right]$$

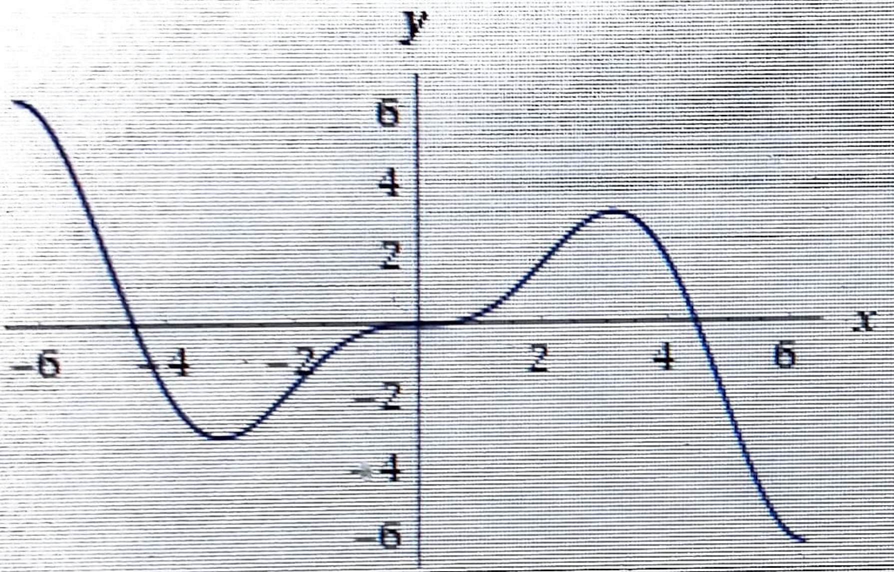
$$= \frac{f_0'}{\omega_0}\cdot\frac{1}{2\omega_0}\left[\sin(\omega_0 T) - \omega_0 T\cos(\omega_0 T)\right].$$

$$\Rightarrow \boxed{\sin(\omega_0 T) - \omega_0 T\cos(\omega_0 T) = \frac{2\omega_0^2 a}{f_0'}}$$ ✓

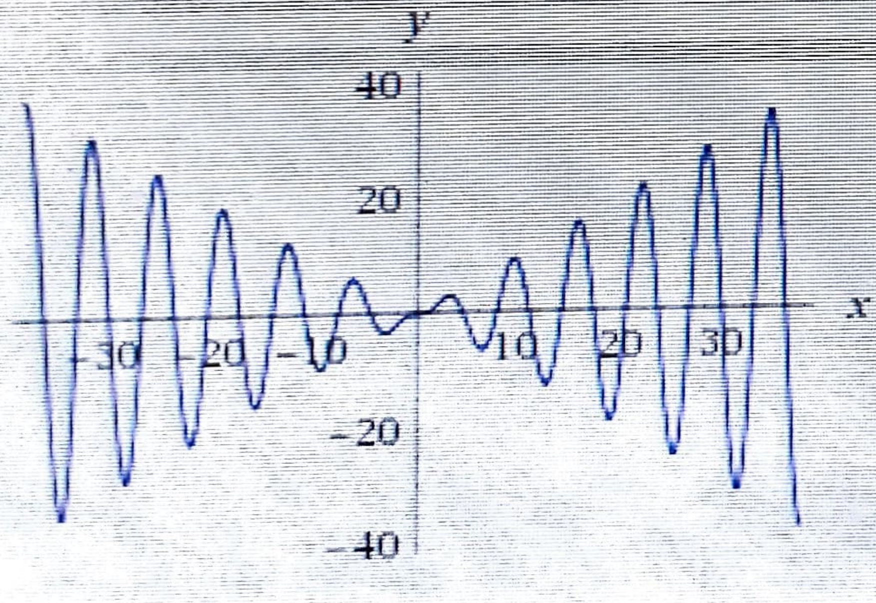
Input:

$\sin(x) - x\cos(x) = f(x)$

Plots:



(x from −6.3 to 6



(x from −37.7 to 37.7)