Pandeo:

$$\frac{\partial^4 y}{\partial x^4} + \frac{P}{EI}\cdot\frac{\partial^2 y}{\partial x^2} = \frac{W(x)}{EI}$$

- Caso $W(x) = 0$

$\Rightarrow$ Solución:

$$\hat{y}(x) = C_1 \cdot \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cdot \cos\left(\sqrt{\frac{P}{EI}}\,x\right) + C_3 x + C_4$$

- Carga excéntrica:

$\Rightarrow$ Solución:

$$\hat{y}(x) = C_1 \cdot \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cdot \cos\left(\sqrt{\frac{P}{EI}}\,x\right) - e$$

Condiciones de borde:

> Pasadores en los extremos:

(1) $\hat{y}(0) = 0$ (3) $\hat{y}(L) = 0$

(2) $\frac{\partial^2 \hat{y}(0)}{\partial x^2} = 0$ (4) $\frac{\partial^2 \hat{y}(L)}{\partial x^2} = 0$

(1) $\Rightarrow C_2 + C_4 = 0 \Rightarrow \boxed{C_2 = -C_4}$

(2) $\Rightarrow -C_2 \cdot \frac{P}{EI} = 0 \Rightarrow \boxed{C_2 = 0 \Rightarrow C_4 = 0}$

(3) $\Rightarrow C_1 \cdot \text{sen}\left(\sqrt{\frac{P}{EI}}\cdot L\right) + C_3 \cdot L = 0$ (5)

(4) $\Rightarrow -C_1 \frac{P}{EI} \cdot \text{sen}\left(\sqrt{\frac{P}{EI}}\cdot L\right) = 0$ (6)

(6) en (5) $\Rightarrow C_3 = 0$

(6) $\Rightarrow C_1 = 0 \vee \text{sen}\left(\sqrt{\frac{P}{EI}}\cdot L\right) = 0$ * Si $C_1 = 0$, no hay pandeo.

$\Rightarrow \text{sen}\left(\sqrt{\frac{P}{EI}}\cdot L\right) = 0 \Rightarrow \boxed{\sqrt{\frac{P}{EI}}\cdot L = m\pi}$

↳ si, $\text{sen}\left(\sqrt{\frac{P}{EI}}\right) = 0$; C_1 podría tender a infinito $\Rightarrow$ hay pandeo.

> Empotrado en los extremos, con $\lambda = \sqrt{\frac{P}{EI}}$

1 $\hat{y}(0)=0$ 3 $\hat{y}(L)=0$

2 $\frac{\partial \hat{y}(0)}{\partial x}=0$ 4 $\frac{\partial \hat{y}(L)}{\partial x}=0$

1 $\Rightarrow C_2 + C_4 = 0 \Rightarrow \boxed{C_2 = -C_4}$ 5

2 $\Rightarrow \boxed{C_1 \cdot \lambda + C_3 = 0}$ 6

3 $\Rightarrow C_1 \cdot \text{sen}(\lambda \cdot L) + C_2 \cdot \cos(\lambda L) + C_3 \cdot L + C_4 = 0$

↳ Usando 5 y 6 $\Leftrightarrow C_1 \cdot [\text{sen}(\lambda L) - \lambda L] + C_2[\cos(\lambda L) - 1] = 0$ 7

4 $\Rightarrow C_1 \cdot \lambda \cdot \cos(\lambda L) - C_2 \cdot \lambda \cdot \sin(\lambda L) - \overbrace{C_1 \cdot \lambda}^{C_3} = 0$

$\Leftrightarrow C_1 \cdot [\cos(\lambda L) - 1] - C_2 \cdot \text{sen}(\lambda L) = 0$ 8

7 y 8 en forma matricial.

$$\overbrace{\begin{bmatrix} \text{sen}(\lambda L) - \lambda L & \cos(\lambda L) - 1 \\ \cos(\lambda L) - 1 & -\text{sen}(\lambda L) \end{bmatrix}}^{\mathbb{M}} \cdot \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \vec{0}$$

* Si $\vec{C} = \vec{0}$; No hay pandeo.
* Para que haya pandeo, la matriz $\mathbb{M}$ debe ser no invertible $\Leftrightarrow \det(\mathbb{M}) = 0$.

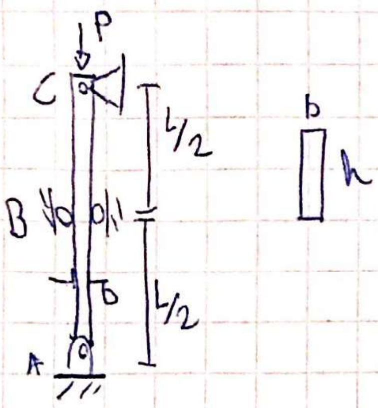
$$\det(\mathbb{M}) = \lambda L \cdot \text{sen}(\lambda L) - \text{sen}^2(\lambda L) - [\cos^2(\lambda L) - 2\cos(\lambda L) + 1] = 0$$

$$= \lambda L \cdot \text{sen}(\lambda L) + 2\cos(\lambda L) = 2$$

↳ La igualdad se cumple para $\cos(\lambda L) = 1 \wedge \text{sen}(\lambda L) = 0$

$\Rightarrow \lambda L = 2 \cdot m \cdot \pi \Leftrightarrow \boxed{\sqrt{\frac{P}{EI}} \cdot L = 2\pi \cdot m}$

P4)



$\Rightarrow$ Plano x-y

$$P_{cr} = \left(\frac{n\pi}{L}\right)^2 E I_z$$

$$P_{cr}^{(1)} = \left(\frac{2\pi}{L}\right)^2 E \cdot I_z = \left(\frac{2\pi}{L}\right)^2 E \frac{h b^3}{12}$$

Plano x-z

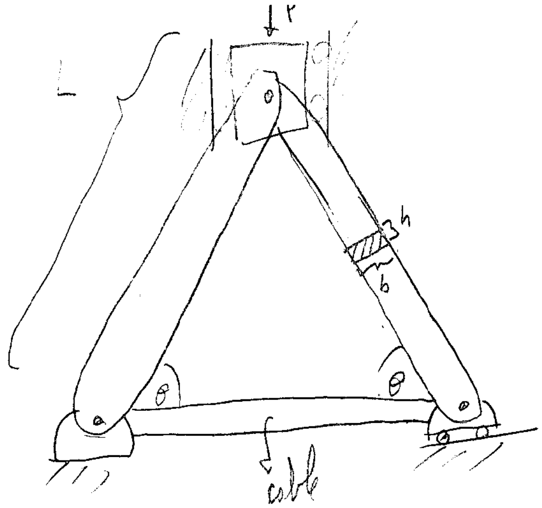
$$P_{cr}^{(2)} = \left(\frac{2\pi}{L}\right)^2 E I_y = \left(\frac{2\pi}{L}\right)^3 E \frac{b h^3}{12}$$

$\Rightarrow P_{cr}^{(1)} = P_{cr}^{(2)}$

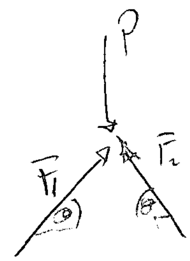
$$\Rightarrow \cancel{\left(\frac{2\pi}{L}\right)^2} \cancel{E} \frac{h b^3}{\cancel{12}} = \cancel{\left(\frac{2\pi}{L}\right)^2} \cancel{E} \frac{b h^3}{\cancel{12}}$$

$$\Rightarrow \left(\frac{h}{b}\right)^2 = 1$$

P3



DCL unión



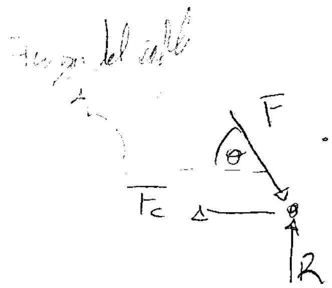
$\sum F_x: F_1\cos\theta = F_2\cos\theta$

$F_1 = F_2 = F$

$\sum F_y: 2F\sin\theta = P$

$F = \frac{P}{2\sin\theta}$

DCL pasador inferior derecho



$\sum F_x: F\cos\theta = F_c$

$\Rightarrow F_c = \frac{P\cos\theta}{2\sin\theta}$

Para que el cable no falle, $\sigma < \sigma_{adm} = \frac{\sigma_0}{FS}$

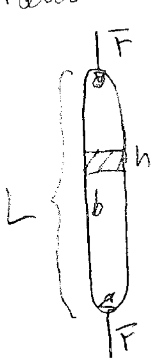
$\sigma = \frac{F_c}{A} \Rightarrow F_{c\,máx} = \frac{\sigma_0 A}{FS} = \frac{200\cdot 10^6 \cdot \pi\left(\frac{0,015}{2}\right)^2}{2} = 17671,5$ N

$\Rightarrow P_{máx} = \frac{2F_{c\,máx}\sin\theta}{\cos\theta} = 131902$ [N]

• Las barras pueden fallar por pandeo en 2 planos. Como ambas barras son iguales y tienen la misma carga F, se analiza solo una

① Pandeo x-y: Pasadores en los extremos

$\lambda = \sqrt{\frac{P}{EI_z}}$, $I_z = \frac{hb^3}{12}$



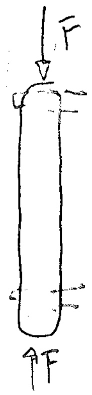
$\hat{y}(x) = C_1\sin(\lambda x) + C_2\cos(\lambda x) + C_3 x + C_4$

$\hat{y}(0) = \hat{y}(L) = 0$ $\Rightarrow F_{cr} = \frac{\pi^2 E I_z}{L^2} = \frac{P_{máx}}{2\sin\theta}$

$\hat{y}''(0) = \hat{y}''(L) = 0$

$\Rightarrow P_{máx} = \frac{2\pi^2 E I_z \sin\theta}{L^2} = 566040$ N

② Pandeo $x-z$



$\hat{y}(0) = \hat{y}(L) = 0$

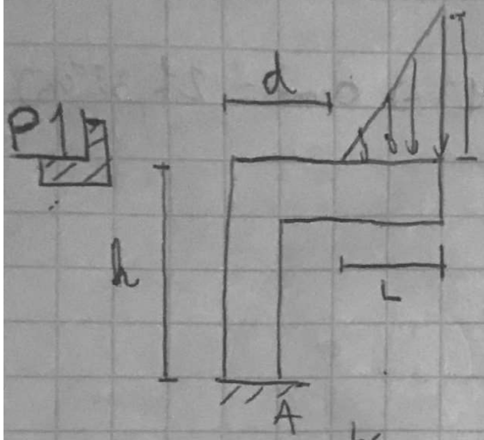
$\hat{y}'(0) = \hat{y}'(L) = 0$ $\longrightarrow$ $P_{cr} = \dfrac{4\pi^2 E I_y}{L^2} = \dfrac{P_{máx}}{2\sin\theta}$

$$P_{máx} = \frac{4\pi^2 E\, b^3h/12 \cdot 2\sin\theta}{L^2} = 203774\ N$$

$$P_{adm}^{x-y} = \frac{501431{,}6}{?} = 283020\ N$$

$$P_{adm}^{x-z} = \frac{2{,}264 \cdot 10^6}{?} = 101887\ N$$

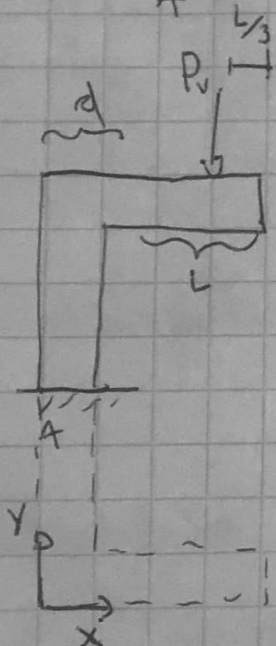
$\Rightarrow P_{adm} = 101887\ N$



Peso relito

$$V_v = L\,\frac{a}{2}\cdot 0{,}002^2 = a\cdot 10^{-3}$$

$$P_v = \rho g V_v = 77{,}42\,a$$



$$e = d + \frac{2L}{3}$$

$$\hat{y}(x) = C_1 \,\mathrm{sen}\left(\sqrt{\frac{P}{EI_z}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI_z}}\,x\right) - e$$

$P = P_v = 77{,}42\,a$, $E = 210\,GPa$

$$I_z = I_{z\phi_e} - I_{z\phi_i} = \frac{\pi}{64}\left(\phi_e^4 - \phi_i^4\right)$$

$$= 3{,}6756 \times 10^{-8}\ m^4$$

C.B

$\hat{y}(0) = 0$, $\hat{y}(2h) = 0$

$\Rightarrow \hat{y}(0) = 0 \Rightarrow C_2 = e$

$$\hat{y}(2h) = 0 \Rightarrow C_1 \sin\left(\sqrt{\frac{P}{EI}}\,2h\right) + e\cos\left(\sqrt{\frac{P}{EI}}\,2h\right) - e = 0$$

$\Rightarrow C_1 \to \infty$ ssi $\sqrt{\frac{P_v}{EI_z}}\,2h = n\pi \leftarrow$ ponder

$$\Rightarrow P_v = \frac{\pi^2 EI}{4h^2}$$

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Para impedir pandeo

$$P_v < \frac{\pi^2 E I_z}{4a} \rightarrow a < \frac{\pi^2 E I_z}{77{,}42 \cdot 4 \lambda^2} = 0{,}273 \rightarrow a_{max} = 27{,}33 \,[cm]$$