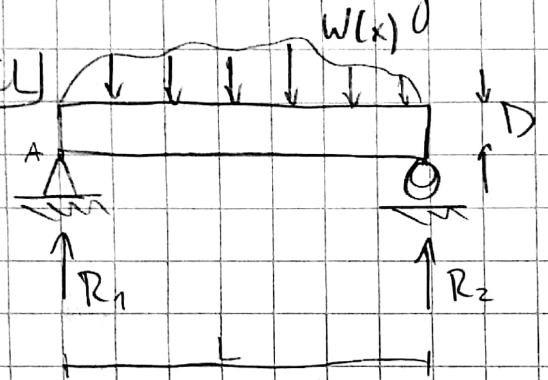
Aux 5: Corte

P1

$$\tau_{xy} = \frac{V(x)}{I_z t} \cdot \int_y^c \xi \, dA$$

DCL



$$\Sigma F_y: R_1 + R_2 - \underbrace{\int_0^L w(x)\,dx}_{P} = 0 \quad (1)$$

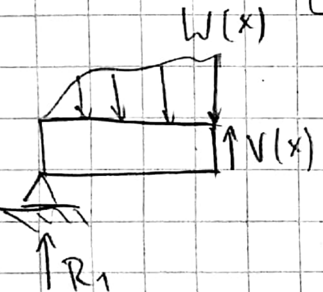
$$\Rightarrow R_1 + R_2 = P$$

$$\Sigma M_A: R_2 \cdot L - P \cdot \bar{x} = 0$$

$$\Rightarrow \boxed{R_2 = \frac{P\bar{x}}{L}} \quad (2)$$

* $\bar{x}$ se obtiene de imponer $\int_0^{\bar{x}} w(x)\,dx = \int_{\bar{x}}^{L} w(x)\,dx \rightarrow$ depende de $w(x)$

De (1) y (2) $\Rightarrow \boxed{R_1 = P\left(1 - \frac{\bar{x}}{L}\right)}$

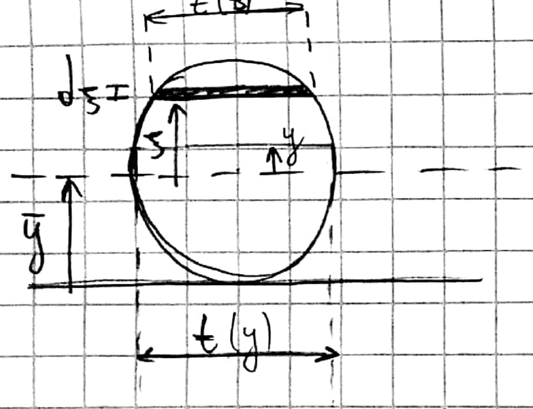


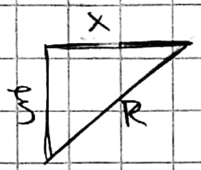
$$R_1 + V(x) - \int_0^x w(\xi)\,d\xi = 0$$

$$\Rightarrow \boxed{V(x) = \int_0^x w(\xi)\,d\xi - P\left(1 - \frac{\bar{x}}{L}\right)}$$

-> Ahora vamos con el área transversal

$$I_z = \frac{\pi D^4}{64}$$





$\xi^2 + x^2 = R^2$

$\Rightarrow x = \sqrt{R^2 - \xi^2}$

$\Rightarrow \boxed{z(\xi) = 2x = 2\sqrt{R^2 - \xi^2}}$

Lo mismo para

$t(y) = 2\sqrt{R^2 - y^2}$

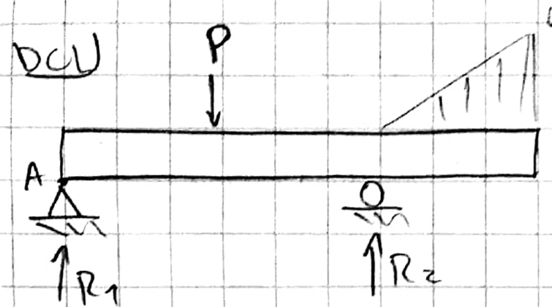
$dA = z(\xi)\,d\xi \Rightarrow \int_y^c \xi\,dA = \int_y^R \xi \cdot 2\sqrt{R^2 - \xi^2}\,d\xi = \frac{2}{3}\left(R^2 - y^2\right)^{3/2}$

$\Rightarrow \tau_{xy} = \frac{V(x)}{I_z \cdot t(y)} \cdot \int_y^c \xi\,dA =$

$= \left[\int_0^x w(\xi)\,d\xi - P\left(1 - \frac{\bar{x}}{L}\right)\right] \frac{64}{\pi D^4} \cdot \frac{\cancel{2}/3\,(R^2 - y^2)^{\cancel{3/2}}}{\cancel{2\sqrt{R^2 - y^2}}}$

$$\boxed{\Rightarrow \tau_{xy} = \left[\int_0^x w(\xi)\,d\xi - P\left(1 - \frac{\bar{x}}{L}\right)\right] \frac{64}{3\pi D^4}\left(\frac{D^2}{4} - y^2\right)}$$

P2 DCL



$\Sigma M_A: -P\cdot b + R_2\cdot 2b - \frac{W_o\cdot a}{2}\cdot\left(2b+\frac{2a}{3}\right)=0$

$\Rightarrow \boxed{R_2 = \frac{P}{2} + \frac{W_o\, a}{2} + \frac{W_o\, a^2}{6b}}$

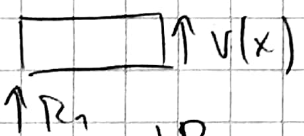
$R_1 = 25$ [N]
$R_2 = 1525$ [N]

$\Sigma F_y: R_1 - P + R_2 - \frac{W_o\cdot a}{2} = 0$

$\Rightarrow \boxed{R_1 = \frac{P}{2} - \frac{W_o\, a^2}{6b}}$

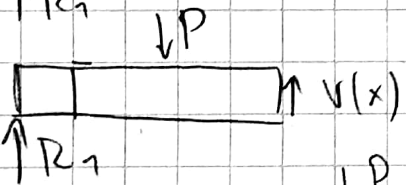
Corte por secciones

$0 < x < b$



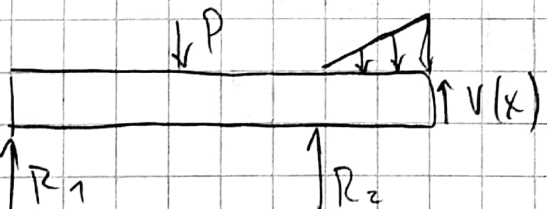
$\Rightarrow \boxed{V(x) = -25}$

$b < x < 2b$



$\Rightarrow \boxed{V(x) = P - R_1 = 775}$

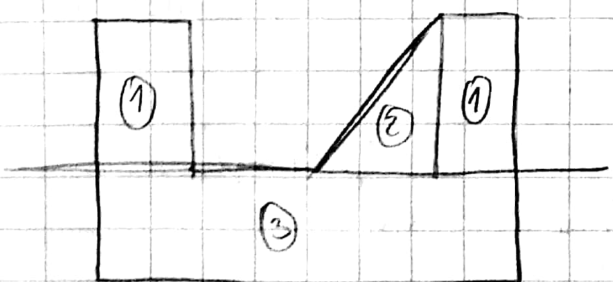
$2b < x < (2b+a)$



$V(x) = P - R_1 - R_2 + \frac{W_o}{2a}(x-2b)^2 = -750 + \frac{1000}{3}(x-2)^2$

$\Rightarrow$ máximo corte para $b < x < 2b \Rightarrow \boxed{V(x) = 775}$

Ahora con la sección transversal:



$y_1 = h_1 + \frac{h_2}{2} = 0{,}035$ [m] $\quad A_1 = t_1\cdot h_2 = 0{,}0003$

$y_2 = h_1 + \frac{h_2}{3} = 0{,}03 \quad A_2 = t_1\cdot h_2\cdot\frac{1}{2} = 0{,}00015$

$y_3 = \frac{h_1}{2} = 0{,}01 \quad A_3 = h_1(3t_1 + t_2) = 0{,}001$

$$\bar{y} = \frac{2y_1 \cdot A_1 + y_2 A_2 + y_3 A_3}{2A_1 + A_2 + A_3} = 0{,}0202857 \; [m]$$

$$I_{z1} = t_1 \cdot h_2^3/12 = 2 \cdot 10^{-8} \qquad \bar{I}_{z1} = 2 \cdot 10^{-8} + A_1(\bar{y} - y_1)^2 = 8 \cdot 10^{-8}$$

$$I_{z2} = t_1 \cdot h_2^3/36 = 7{,}5 \cdot 10^{-9} \qquad \bar{I}_{z2} = 7{,}5 \cdot 10^{-9} + A_2(\bar{y} - y_2)^2 = 2 \cdot 10^{-8}$$

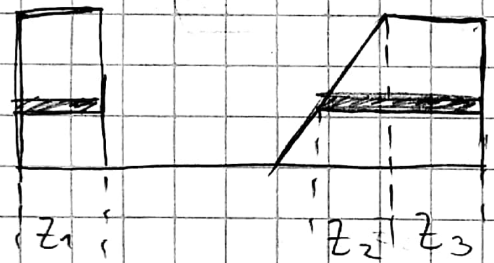
$$I_{z3} = (3t_1 + t_2) \cdot h_1^3/12 = 3 \cdot 10^{-8} \qquad \bar{I}_{z3} = 3 \cdot 10^{-8} + A_3(\bar{y} - y_3)^2 = 1{,}4 \cdot 10^{-7}$$

$$\boxed{\bar{I}_{TOT} = 2\bar{I}_{z1} + \bar{I}_{z2} + \bar{I}_{z3} = 3{,}2 \cdot 10^{-7}} = I_z$$

→ 2 casos para el espesor

- $-\bar{y} < y < -\bar{y} + h_1$
- $-\bar{y} + h_1 < y < h_1 + h_2 - \bar{y}$

① caso $-\bar{y} + h_1 < y < h_1 + h_2 - \bar{y}$



$$z_1 = t_1 = z_3$$

$$z_2 = -\frac{t_1}{h_2}\xi + t_1\left(\frac{h_1}{h_2} - \frac{\bar{y}}{h_2} + 1\right)$$

$$\Rightarrow z(\xi) = z_1 + z_2 + z_3 = -\frac{t_1}{h_2}\xi + t_1\left(\frac{h_1}{h_2} - \frac{\bar{y}}{h_2} + 3\right)$$

$$\Rightarrow \boxed{z(\xi) \simeq -0{,}\bar{3} \cdot \xi + 0{,}0299} \rightarrow \text{igual a } t(y)$$

$$\int_y^c \xi \, dA = \int_y^{h_1 + h_2 - \bar{y}} (-0{,}\bar{3}\xi^2 + 0{,}0299\xi)\, d\xi \simeq 0{,}\bar{1} y^3 - 0{,}01495 y^2 + 1{,}029 \cdot 10^{-5}$$

$$\Rightarrow \tau_{xy} = \frac{V(x)}{I_z \, t(y)} \int_y^c \xi \, dA = \frac{775}{3{,}2 \cdot 10^{-7}} \cdot \frac{0{,}\bar{1} y^3 - 0{,}01495 y^2 + 1{,}029 \cdot 10^{-5}}{-0{,}\bar{3} y + 0{,}0299}$$

caso $-\bar{y} < y < -\bar{y} + h_1 \Rightarrow \boxed{t(y) = 3t_1 + t_2} = 0{,}05$

$$z(\xi) \begin{cases} 3t_1 + t_2 & \text{si } y < \xi < -\bar{y} + h_1 \\ -0{,}\bar{3}\xi + 0{,}0299 & \text{si } -\bar{y} + h_1 < \xi < h_1 + h_2 - \bar{y} \end{cases}$$

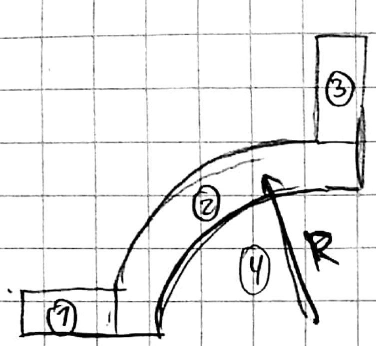
$$\Rightarrow \int_y^c \xi \, dA = \underbrace{\int_y^{-\bar{y}+h_1} \xi (3t_1 + t_2) d\xi}_{-0{,}025(y^2 - 8\cdot 10^{-8})} + \underbrace{\int_{-\bar{y}+h_1}^{h_1+h_2-\bar{y}} (-0{,}\bar{3}\xi^2 + 0{,}0299\xi) d\xi}_{1{,}029\cdot 10^{-5}}$$

$$\Rightarrow \int_y^c \xi \, dA \approx -0{,}025 y^2 + 1{,}029 \cdot 10^{-5}$$

$$\Rightarrow \tau_{xy} = \frac{V(x)}{I_z \, t(y)} \int_y^c \xi \, dA = \frac{775}{3{,}2\cdot 10^{-7} \cdot 0{,}05} \cdot \left(-0{,}025 y^2 + 1{,}029 \cdot 10^{-5}\right)$$

Scanned by CamScanner

P3

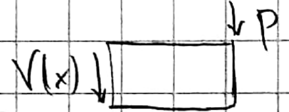


→ igual que siempre, se obtiene:

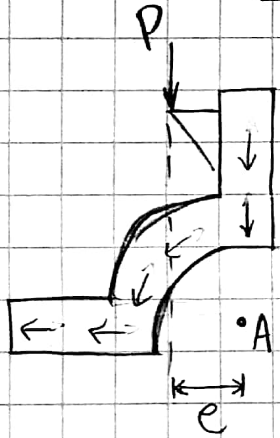
$\bar{Y} = 0{,}0351$ [m]

$\bar{I}_z = 8{,}2660 \cdot 10^{-7}$ [m^4]

(1) En el extremo con la carga P:

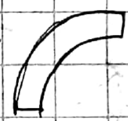


$\boxed{V(x) = -P}$ → ΣF_y.



$$\boxed{\Sigma M_A : Pe + \int r \cdot \tau_{xs} \, ds = 0} \quad (*)$$

→ se escoge la sumatoria de momento en torno al punto A porque se cancela la influencia del corte que existe en las zonas (1) y (3).

=> la única área de interés es: 

- $t(s) = t$
- $dA = t\left(R - \frac{t}{2}\right) d\theta \approx t \cdot R \, d\theta = ds$
- $\xi = \left(R - \frac{t}{2}\right)\cos(\theta) - \bar{Y} \approx R\cos(\theta) - \bar{Y}$

$$\Rightarrow \tau_{xs} = \frac{-P}{I_z \cancel{t}} \int_{\phi}^{\pi/2} (R\cos(\theta) - \bar{Y}) \cdot \cancel{t} R \, d\theta$$

$$\Rightarrow \tau_{xs} = \frac{-PR}{I_z \cdot 2} \left[2R(\sin\phi - 1) - \bar{Y}(2\phi - \pi) \right]$$

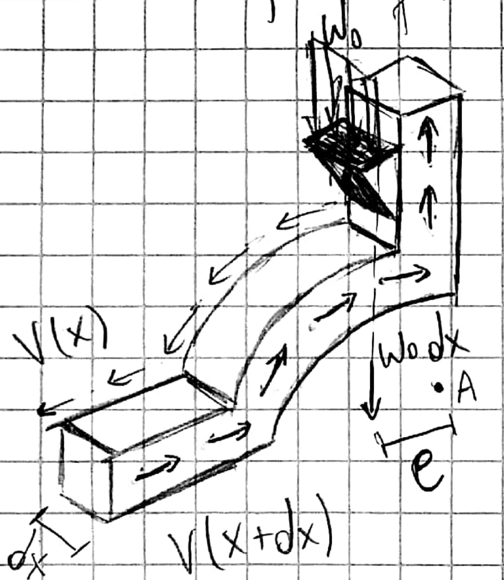
↳ en (*)

$$\cancel{P}e = +\int_s +\frac{\cancel{P}R^2}{I_z \cdot 2}\left[2R(\sin\phi - 1) - \bar{y}(2\phi - \pi)\right] ds \quad \text{con } ds = tR\,d\phi$$

$$\Rightarrow e = \frac{tR^3}{2I_z}\int_0^{\pi/2}\left[2R(\sin\phi - 1) - \bar{y}(2\phi - \pi)\right] d\phi$$

$$\Rightarrow \boxed{e \approx 0{,}0034\ [m]}$$

② En la zona con la carga distribuida, se toma una lonja diferencial.



→ como el corte $V(x)$ será distinto al corte $V(x+dx)$ y la carga es distribuida, hay que considerar la contribución del corte a ambos lados de la lonja.

$$\left.\begin{aligned} \tau_{xs}(x) &= \frac{V(x)}{I_z}\cdot\frac{R}{2}\cdot\left[2R(\sin\phi - 1) - \bar{y}(2\phi - \pi)\right] \\ \tau_{xs}(x+dx) &= \frac{V(x+dx)}{I_z}\frac{R}{2}\cdot\left[2R(\sin\phi - 1) - \bar{y}(2\phi - \pi)\right] \end{aligned}\right]$$

Ojo que este corte es sólo en la curva

$$\Sigma M_A: W_0 \cdot dx \cdot e + \int R\,\tau_{xs}(\phi)\,ds - \int R\,\tau_{x+dx,s}(\phi)\,ds = 0$$

$$\Leftrightarrow W_0 \cdot dx \cdot e + R\left[\frac{R}{2I_z}\left(V(x) - V(x+dx)\right)\right] \cdot \int_0^{\pi/2}\left[2R(\sin\phi - 1) - \bar{y}(2\phi - \pi)\right] \cdot tR\,d\phi = 0$$

como $V(x+dx) = V(x) + \frac{dV}{dx}dx$

$$\Leftrightarrow \cancel{W_0} \cdot \cancel{dx} \cdot e + \frac{R^3 t}{2I_z}\left(\cancel{V(x)} - \cancel{V(x)} - \underbrace{\frac{dV}{dx}}_{\cancel{W_0}}\cancel{dx}\right) \cdot \int_0^{\pi/2}\left[2R(\sin\phi - 1) - \bar{y}(2\phi - \pi)\right] d\phi = 0$$

$$\Leftrightarrow e = \frac{R^3 t}{2I_z} \cdot \int_0^{\pi/2}\left[2R(\sin\phi - 1) - \bar{y}(2\phi - \pi)\right] d\phi$$

↳ exactamente lo mismo que en el caso de P puntual.

$$\Rightarrow \boxed{e \approx 0{,}0034\ [m]}$$