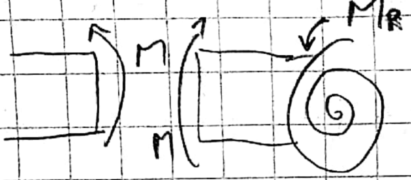
Aux: Deflexión

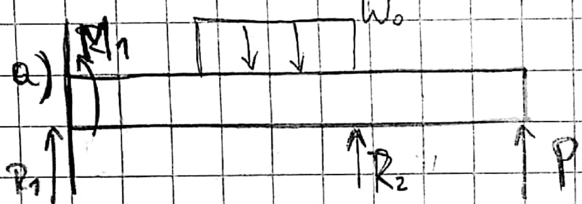


$$M - M_R = 0 \Rightarrow M = M_R$$

M_R es lineal con el ángulo: $\boxed{M_R = C \cdot \frac{dy}{dx}}$

$$\Rightarrow \boxed{M = EI_z \cdot \frac{d^2y}{dx^2} = C \cdot \frac{dy}{dx}}$$

P1 a)



→ estáticamente indeterminado

↳ se usa como una fuerza

D.C.L

$$\Sigma F_y: R_1 - W_0 \cdot a + R_2 + P = 0$$

$$\Rightarrow \boxed{R_1 = W_0 \cdot a - R_2 - P}$$

$$\Sigma M_z: M_1 - \frac{W_0 \cdot a^2 \cdot 3}{2} + R_2 \cdot 2a + P \cdot 3a = 0$$

$$\Rightarrow \boxed{M_1 + 2 \cdot R_2 \cdot a = \frac{3}{2} W_0 a^2 - 3P \cdot a}$$

$$\boxed{W(x) = W_0 \cdot r(x-a) - W_0 \cdot r(x-2a) - R_2 \cdot \delta(x-2a)}$$

$$\Rightarrow \frac{d^4y(x)}{dx^4} = \frac{1}{EI_z} \cdot \left[-W_0 \, r(x-a) + W_0 \, r(x-2a) + R_2 \cdot \delta(x-2a)\right]$$

$$\Rightarrow \frac{d^3y(x)}{dx^3} = \frac{1}{EI_z} \left[-W_0 (x-a) \cdot r(x-a) + W_0 (x-2a) r(x-2a) + R_2 \, r(x-2a)\right] + A$$

$$\frac{d^2y(x)}{dx^2} = \frac{1}{EI_z}\left[-w_0\frac{(x-a)^2}{2}\cdot r(x-a) + w_0\frac{(x-2a)^2}{2}\cdot r(x-2a)\right.$$

$$\left. + R_2(x-2a)\cdot r(x-2a)\right] + Ax + B$$

$$\frac{dy(x)}{dx} = \frac{1}{EI_z}\left[-w_0\frac{(x-a)^3}{6}\cdot r(x-a) + w_0\frac{(x-2a)^3}{6}\cdot r(x-2a)\right.$$

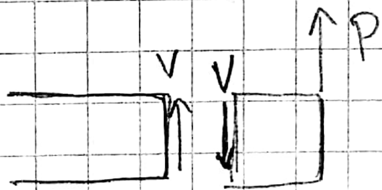
$$\left. + R_2\frac{(x-2a)^2}{2}\cdot r(x-2a)\right] + \frac{Ax^2}{2} + Bx + C$$

$$\boxed{y(x) = \frac{1}{EI_z}\left[-w_0\frac{(x-a)^4}{24}\cdot r(x-a) + w_0\frac{(x-2a)^4}{24}\cdot r(x-2a) + R_2\frac{(x-2a)^3}{6}\, r(x-2a)\right] + \frac{Ax^3}{6} + \frac{Bx^2}{2} + Cx + D}$$

C.B.) ① $y(0) = 0$
② $y'(0) = 0$ } Lado izquierdo

③ $y''(3a) = 0$

④ $y'''(3a) = \frac{-P}{EI_z}$



$-V + P = 0$
$\Rightarrow V(3a) = P$

$$\boxed{① \Rightarrow D = 0 \quad ; \quad ② \Rightarrow C = 0}$$

$$③ \Leftrightarrow \frac{1}{EI_z}\left[-\frac{w_0}{2}(2a)^2 + \frac{w_0}{2}\cdot(a)^2 + R_2\cdot a\right] + A\cdot 3a + B = 0$$

$$\Rightarrow \boxed{B = \left[\frac{3w_0a^2}{2} - R_2\cdot a\right]\cdot\frac{1}{EI_z} - 3A\cdot a}$$

$$③ \Rightarrow \frac{1}{EI_z}\left[-W_0 \cdot 2a + W_0 \cdot a + R_2\right] + A = \frac{-P}{EI_z}$$

$$\Rightarrow \boxed{A = \left[W_0 \cdot a - R_2\right] \cdot \frac{1}{EI_z} - \frac{P}{EI_z} = \left[W_0 \cdot a - R_2 - P\right] \cdot \frac{1}{EI_z}}$$

$$\Rightarrow B = \left[\frac{3W_0 a^2}{2} - R_2 \cdot a\right]\frac{1}{EI_z} - \left[3W_0 a^2 - 3R_2 \cdot a - 3Pa\right] \cdot \frac{1}{EI_z}$$

$$\Rightarrow \boxed{B = \left[-\frac{3}{2} W_0 a^2 + 2R_2 \cdot a + 3P \cdot a\right] \cdot \frac{1}{EI_z}}$$

→ tenemos todas las constantes de $y(x)$
pero aún no conocemos R_2

→ Se impone $y(2a) = 0$

$$y(2a) = \frac{1}{EI_z}\left[-\frac{W_0 a^4}{24}\right] + \frac{1}{EI_z}\left[\frac{W_0 a^4 8}{6} - \frac{8R_2 a^3}{6} - \frac{8Pa^3}{6}\right]$$

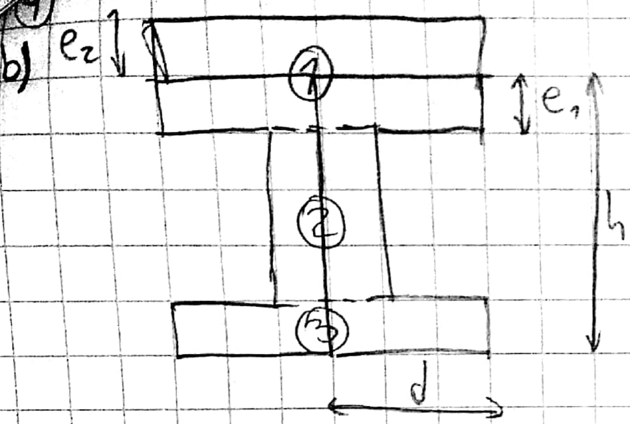
$$+ \frac{1}{EI_z}\left[-3W_0 a^4 + 4R_2 a^3 + 6 \cdot P \cdot a^3\right] \stackrel{!}{=} 0$$

↳ Se despeja R_2 con la T.I.

$$R_2 = \frac{41 \cdot a \cdot W_0 - 112 \cdot P}{64} \Rightarrow \boxed{R_2 = -50\,962{,}5\,[N]}$$

$$\Rightarrow \boxed{R_1 = 23.362{,}5\,[N]}$$

4

b) 

$\bar{y}_1 = \frac{e_1 + e_2}{2} + h - e_1 = 0{,}305$ [m]

$A_1 = (e_1 + e_2) \cdot 2d = 0{,}012$ [m²]

$\bar{y}_2 = e_1 + \frac{(h - 2e_1)}{2} = 0{,}15$ [m]

$A_2 = 2e_1 \cdot (h - 2e_1) = 0{,}0056$ [m²]

$\bar{y}_3 = \frac{e_1}{2} = 0{,}005$

$A_3 = e_1 \cdot 2d = 0{,}004$ [m²]

$I_1 = 2d \cdot (e_1 + e_2)^3 / 12 = 9 \cdot 10^{-7}$

$I_2 = 2e_1 \cdot (h - 2e_1)^3 / 12 = 3{,}659 \cdot 10^{-5}$

$I_3 = 2d \cdot e_1^3 / 12 = 3 \cdot 10^{-8}$

$\Rightarrow \boxed{\bar{y}_T = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3} = 0{,}2093 \text{ [m]}}$

$\bar{I}_1 = I_1 + A_1 \cdot (\bar{y}_1 - \bar{y}_T)^2 = 1{,}108 \cdot 10^{-4}$

$\bar{I}_2 = I_2 + A_2 (\bar{y}_2 - \bar{y}_T)^2 = 5{,}6 \cdot 10^{-5}$

$\bar{I}_3 = I_3 + A_3 (\bar{y}_3 - \bar{y}_T)^2 = 1{,}67 \cdot 10^{-4}$

$\Rightarrow \boxed{\bar{I}_T = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 3{,}3407 \cdot 10^{-4}}$

c) $y(a) = \frac{A \cdot a^3}{6} + \frac{B \cdot a^2}{2} = \frac{1}{EI_z}\left[\frac{W_0 a^4}{6} - \frac{R_2 a^3}{6} - \frac{P \cdot a^3}{6} - \frac{3}{4} \cdot W_0 a^4 + R_2 a^3 + \frac{3}{2} \cdot P a^3\right]$

$\Rightarrow \boxed{y(a) = -0{,}00156339 \text{ [m]}}$

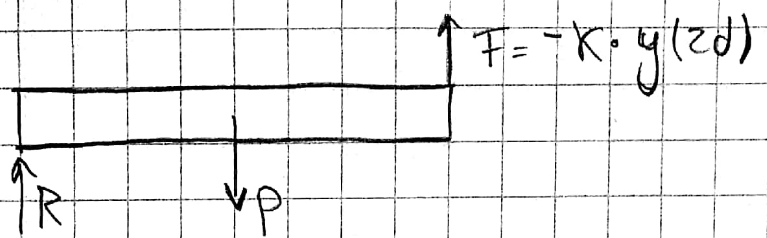
$y(3a) = \frac{1}{EI_z}\left[-\frac{W_0 (2a)^4}{24} + \frac{W_0 \cdot a^4}{24} + \frac{P_2 a^3}{6}\right] + \frac{1}{EI_z}\left[\frac{27 W_0 a^4}{6} - \frac{27 R_2 a^3}{6} - \frac{27 P a^3}{6}\right]$

$+ \frac{1}{EI_z}\left[-\frac{3 \cdot 9}{4} W_0 a^4 + 9 R_2 a^3 + \frac{3 \cdot 9}{2} P a^3\right]$

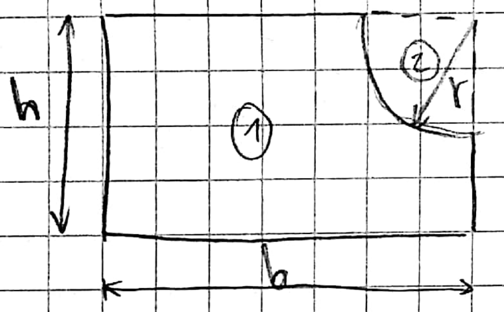
5

$y(3a) = 0{,}0102138\ [cm]$

P2



a)



$\bar{y}_1 = \frac{h}{2} = 0{,}05\ [m]$

$A_1 = h \cdot b = 0{,}25\ [m^2]$

$y_2 = (h - r) + \left(r - \frac{4r}{3\pi}\right) = 0{,}078779\ [m]$

$A_2 = \frac{\pi r^2}{4} = 0{,}0019635\ [m^2]$

$$\boxed{\bar{y}_T = \frac{\bar{y}_1 A_1 - \bar{y}_2 A_2}{A_1 - A_2} = 0{,}0497722\ [m]}$$

$I_1 = \frac{b \cdot h^3}{12} = 1{,}25 \cdot 10^{-5}$

$I_2 = 0{,}1098\, r^4 = 6{,}9 \cdot 10^{-7}$

$\Rightarrow \bar{I}_1 = I_1 + A_1 (\bar{y}_1 - \bar{y}_T)^2 = 1{,}251 \cdot 10^{-5}$

$\bar{I}_2 = I_2 + A_2 (\bar{y}_2 - \bar{y}_T)^2 = 2{,}34 \cdot 10^{-6}$

$\Rightarrow \boxed{\bar{I}_T = \bar{I}_1 - \bar{I}_2 = 1{,}017 \cdot 10^{-5}}$

b) $W(x) = P \cdot \delta(x - d)$

$$\frac{d^4 y}{dx^4} = \frac{1}{EI_z}\left(-P \cdot \delta(x - d)\right)$$

$$\frac{d^3 y}{dx^3} = \frac{1}{EI_z}\left[-P \cdot \gamma(x - d)\right] + A$$

$$\frac{d^2 y}{dx^2} = \frac{1}{EI_z}\left[-P(x - d) \cdot \gamma(x - d)\right] + Ax + B$$

④ $$\frac{dy}{dx} = \frac{1}{EI_z}\left[-P\frac{(x-d)^2}{2}r(x-d)\right] + \frac{Ax^2}{2} + Bx + C$$

$$y = \frac{1}{EI_z}\left[-P\frac{(x-d)^3}{6}r(x-d)\right] + \frac{Ax^3}{6} + \frac{Bx^2}{2} + Cx + D$$

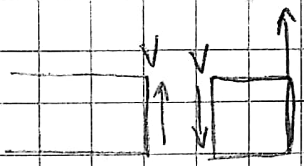
C.B.)

① $y(0) = 0 \Rightarrow D = 0$ pasador

② $y''(0) = 0 \Rightarrow B = 0$

③ $y'''(2d) = \frac{Ky(2d)}{EI_z}$

④ $y''(2d) = 0$



$-V + F = 0$

$\Rightarrow \boxed{V = -Ky(2d)}$

④ $$\frac{1}{EI_z}\left[-P\cdot d\right] + A\cdot 2d = 0$$

$$\Rightarrow \boxed{A = \frac{P}{2EI_z}}$$

③ $$\frac{1}{\cancel{EI_z}}\left[-P\right] + \frac{P}{2\cancel{EI_z}} = \frac{K}{\cancel{EI_z}}\cdot\left(\frac{1}{EI_z}\left[-P\frac{d^3}{6}\right] + \frac{P}{2EI_z}\frac{8d^3}{6} + C\cdot 2d\right)$$

$$\frac{-P}{2} = \frac{1}{EI_z}\left(-\frac{P\cdot Kd^3}{6} + \frac{2}{3}PKd^3 + EI_z\cdot KC\cdot 2d\right)$$

$$\Rightarrow C = \frac{1}{EI_z K\cdot 2d}\left[\frac{-P\cdot EI_z}{2} + \frac{PKd^3}{6} - \frac{2}{3}PKd^3\right]$$

$$\Rightarrow \boxed{C = \frac{-P}{EI_z K 2d}\left[\frac{EI_z}{2} + \frac{Kd^3}{2}\right]}$$

③

$$y(d) = \frac{A d^3}{6} + C \cdot d$$

$$= \frac{P d^3}{12 E I_z} - \frac{P}{E I_z K \cdot 2}\left[\frac{E I_z}{2} + \frac{K d^3}{2}\right]$$

$$\Rightarrow \boxed{y(d) = -0{,}0158\ [m]}$$

c) $M(x) = E I_z \cdot \frac{d^2 y}{dx^2}$

$$\Rightarrow M(x) = -P(x-d)\cdot r(x-d) + \frac{P}{2}\cdot x$$

$\Rightarrow$ $\underline{0 < x < d}$

$$M(x) = \frac{P}{2}\cdot x \rightarrow \text{creciente}$$

$\underline{d < x < 2d}$

$$M(x) = -Px + Pd + \frac{P}{2}x$$

$$\Rightarrow M(x) = P\left(d - \frac{x}{2}\right) \rightarrow \text{decreciente}$$

$\Rightarrow$ máximo en $\boxed{x = d}$ $\Rightarrow$ $\boxed{M(d) = \frac{Pd}{2}}$

$\sigma_x = -\frac{M}{I_z}\cdot y$ como $\bar{y}_T < \frac{h}{2}$ se usará $y = h - y_T$

$$\Rightarrow \sigma_x = -\frac{Pd}{2 I_z}\cdot(h - y_T) \Rightarrow \boxed{\sigma_x = -37{,}04\ [MPa]}$$

$\rightarrow$ Compresión porque es negativo