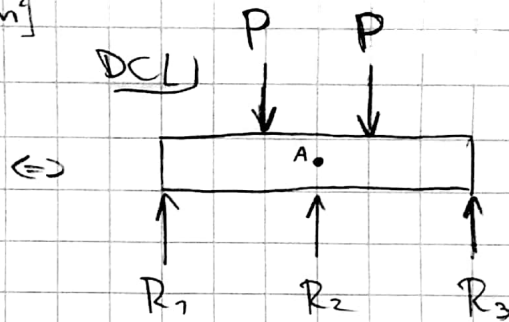
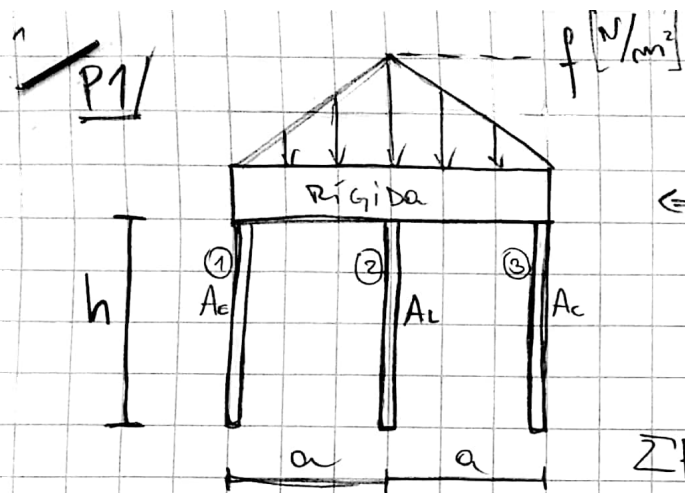
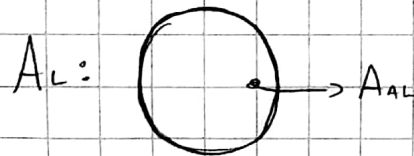
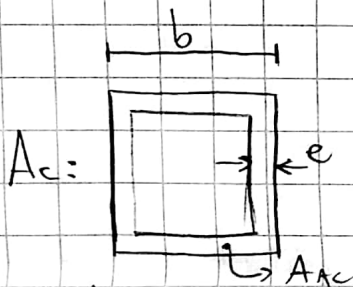




$$\sum F_y: R_1 + R_2 + R_3 = 2P \quad (1)$$

$$\sum M_A: P \cdot \frac{a}{3} + R_3 \cdot a - P \cdot \frac{a}{3} - R_1 \cdot a = 0$$

$$\Rightarrow R_1 = R_3 \quad (2)$$



* Plataforma rígida se mantiene horizontal.
 $\Rightarrow$ las deformaciones deben ser iguales.

* Tubos 1 y 3/

$$\Delta L_1 + \Delta L_{T1} = \Delta L_3 + \Delta L_{T3}$$

$$\Delta L = \frac{L_0 \cdot F}{A \cdot E}$$

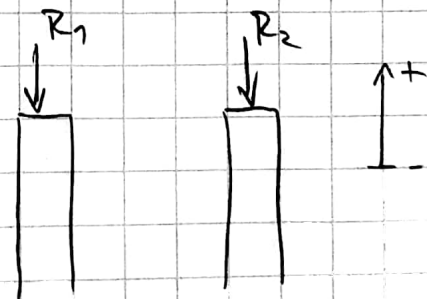
$$\frac{-h \cdot R_1}{A_{Ac} \cdot E_{Ac}} + \alpha_{Ac} \cdot \Delta T \cdot h = \frac{-h R_3}{A_{Ac} \cdot E_{Ac}} + \alpha_{Ac} \cdot \Delta T \cdot h$$

$$\Delta L_T = \alpha \cdot \Delta T \cdot L_0$$

$$\Rightarrow R_1 = R_3 \rightarrow \text{igual que (2)}$$

+ Tubos 1 y 2/

$$\Delta L_1 + \Delta L_{T1} = \Delta L_2 + \Delta L_{T2}$$



$$-\frac{R_1}{A_{AC} E_{AC}} + \alpha_{AC} \Delta T \cdot l = -\frac{R_2}{A_{AL} E_{AL}} + \alpha_{AL} \Delta T \cdot l$$

$$\Rightarrow R_2 = \left(\frac{R_1}{A_{AC} E_{AC}} + \Delta T (\alpha_{AL} - \alpha_{AC}) \right) A_{AL} E_{AL} \quad (3)$$

con (3) y (2) en (1)

$$\Rightarrow 2R_1 + A_{AL} E_{AL} \left[\frac{R_1}{A_{AC} E_{AC}} + \Delta T (\alpha_{AL} - \alpha_{AC}) \right] = 2P$$

$$\Leftrightarrow R_1 \left(2 + \frac{A_{AL} E_{AL}}{A_{AC} E_{AC}} \right) = 2P + A_{AL} E_{AL} \cdot \Delta T (\alpha_{AC} - \alpha_{AL})$$

$$\Rightarrow R_1 = \left[2P + A_{AL} E_{AL} \cdot \Delta T (\alpha_{AC} - \alpha_{AL}) \right] \frac{A_{AC} E_{AC}}{2A_{AC} E_{AC} + A_{AL} E_{AL}}$$

$$\Rightarrow R_2 = \left[\frac{2P + A_{AL} E_{AL} \Delta T (\alpha_{AC} - \alpha_{AL})}{2A_{AC} E_{AC} + A_{AL} E_{AL}} + \Delta T (\alpha_{AL} - \alpha_{AC}) \right] A_{AL} E_{AL}$$

($\Delta T = 0$)

$\Rightarrow$ Sin cambio de temperatura: $R_2 \approx 30 \text{ kN}$
 $\Rightarrow \sigma_2 \approx 95,5 \text{ MPa}$

Si el esfuerzo de fluencia del aluminio es de $\sigma_{oAL} = 120 \text{ MPa}$

$$\Rightarrow F.S. = \frac{\sigma_{oAL}}{\sigma_2} \approx 1,25$$

$$\hookrightarrow R_1 = R_3 = 110 \text{ kN}$$

$$\sigma_1 = \sigma_3 = 286 \text{ MPa}$$

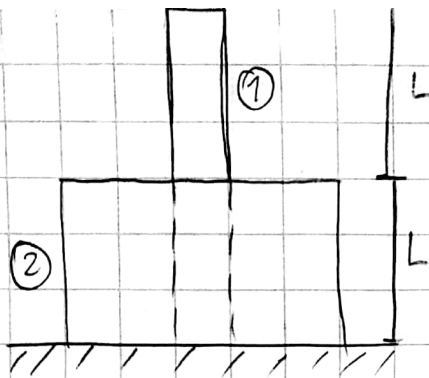
$\Rightarrow$ Con cambio de temperatura: $R_2 \approx 38,5 \text{ kN}$
 $\Rightarrow \sigma_2 \approx 122,6 \text{ MPa}$

$122,6 > \sigma_{oAL} \Rightarrow$ ~~Falla~~

$$\hookrightarrow R_1 = R_3 \approx 106 \text{ kN}$$

$$\sigma_1 = \sigma_3 \approx 275 \text{ MPa}$$

P2]



1° se supone que no toca la pared superior

zona 1

$$\Delta L_1 = \alpha_A \cdot \Delta T \cdot L$$

$$\Rightarrow \Delta L_1 = 0,001434 \text{ m}$$

zona 2

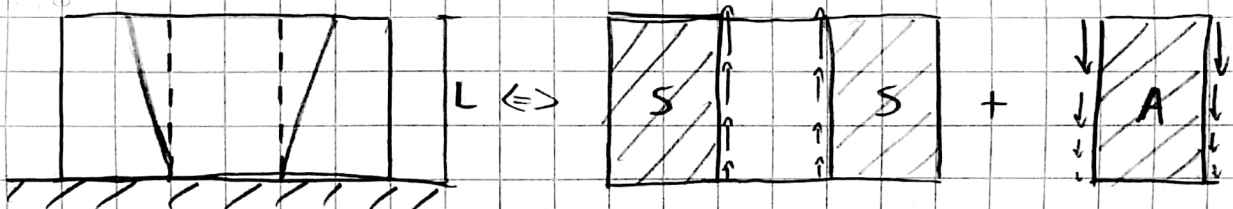
def acero = def. aluminio

$$\Delta L_{TS} + \Delta L_S = \Delta L_{TA} + \Delta L_A \quad (*)$$

$$\Delta L_{TS} = \alpha_S \cdot \Delta T \cdot L = 0,000648 \text{ m}$$

$$\Delta L_{TA} = 0,001434 \text{ m}$$

D.C.L



$$L \Rightarrow \left. \begin{array}{l} f(0) = 0 \\ f(L) = F \end{array} \right\} f(x) = \frac{F}{L} x$$

Aluminio

$$\frac{\Delta dx}{dx} = \frac{-f(x)}{A_A \cdot E_A} \quad \int_0^L \Delta dx = \frac{-1}{A_A E_A} \cdot \int_0^L f(x) dx$$

$$\Rightarrow \Delta L_A = \frac{-F}{2 A_A E_A} \cdot L \Rightarrow \Delta L_A = \frac{-F \cdot L}{2 A_A E_A}$$

¡cero! mismo resultado pero en sentido opuesto.

$$\Delta L_s = \frac{F \cdot L}{2 A_s E_s}$$

en (*) $0,000648 + \frac{F \cdot L}{2 A_s E_s} = 0,001434 - \frac{F \cdot L}{2 A_A E_A}$

$$\Rightarrow F = (0,001434 - 0,000648) \left(\frac{A_s E_s \cdot A_A E_A}{A_s E_s + A_A E_A} \right) \cdot \frac{2}{L}$$

con $A_s = 0,0027 \text{ [m}^2\text{]}$ $E_s = 205,8 \text{ [GPa]}$
 $A_A = 0,0035 \text{ [m}^2\text{]}$ $E_A = 68,6 \text{ [GPa]}$

$$\Rightarrow F = 878.517 \text{ [N]}$$

$$\Delta L_A = -0,00054884 \text{ [m]}$$

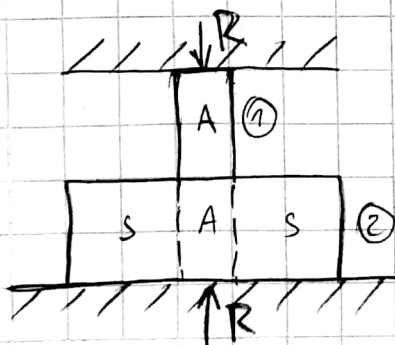
$$\Delta L_s = 0,00023716 \text{ [m]}$$

$$\Delta L_{TOT} = \Delta L_{TS} + \Delta L_s + \Delta L_1$$

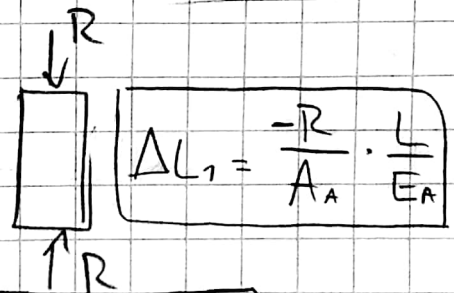
$$= 0,00231916$$

$\Rightarrow$ Si TOCA //

$$\Delta L_{TOT} - \Delta = 0,00131916 \text{ [m]}$$



Zona 1 D.C.L.



$$\Delta L_1 = \frac{-R}{A_A} \cdot \frac{L}{E_A}$$

$$\Delta L_{1TOT} = 0,001434 - \frac{R \cdot L}{A_A E_A}$$

Zona 2

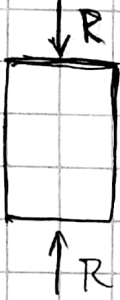
$\Delta L_{TS} = 0,000648$; $\Delta L_{TA} = 0,001434 \rightarrow$ igual que antes
 Ahora la eq. (*) queda:

$$\Delta L_{TS} + \Delta L_s + \Delta L_{RS} = \Delta L_{TA} + \Delta L_A + \Delta L_{RA} \quad \star\star$$

$\hookrightarrow$ se agrega la deformación por R

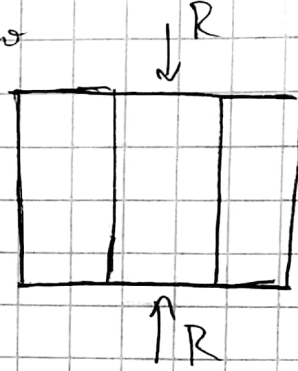
③

Aluminio



$$\Delta L_{RA} = -\frac{R \cdot L}{A_A E_A}$$

Acero



$$\Delta L_{RS} = \frac{-R \cdot L}{A_A E_A}$$

em (★★)

$$0,000648 + \frac{F \cdot L}{2 A_S E_S} - \frac{R \cdot L}{A_S E_S} = 0,001434 - \frac{F L}{2 A_A E_A} - \frac{R L}{A_A E_A}$$

$$\Rightarrow F = \frac{2(0,001434 - 0,000648) \left(\frac{A_A E_A \cdot A_S E_S}{A_A E_A + A_S E_S} \right)}{L} + 2R \left(\frac{A_A E_A - A_S E_S}{A_A E_A + A_S E_S} \right) = 878517,6 - R \cdot 0,7931$$

Por otro lado se debe cumplir que:

$$\Delta L_{1TOT} + \Delta L_{2TOT} = 0,001$$

como $\Delta L_{2TOT S} = \Delta L_{2TOT A}$
usaremos sólo el aluminio

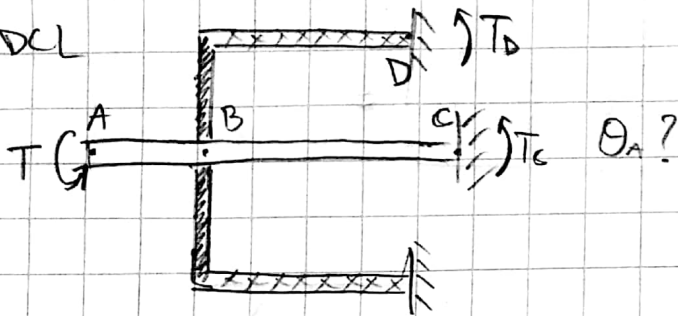
$$0,001434 - \frac{R L}{A_A E_A} + \Delta L_{TA} + \Delta L_A + \Delta L_{RA} = 0,001$$

$$\Leftrightarrow 0,001434 - \frac{R L}{A_A E_A} + 0,001434 - \frac{L}{2 A_A E_A} \cdot F - \frac{R L}{A_A E_A} = 0,001$$

$$\Rightarrow R = \frac{0,001165 (A_A E_A - L \cdot 2,3515 \cdot 10^8)}{L}$$

$$\Rightarrow R \approx 658.433 [N]$$

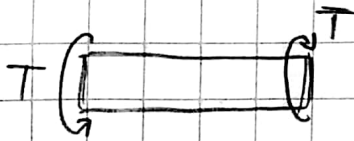
P3) DCL



$$\theta_A = \theta_{BA} + \theta_B \quad (1)$$

$$\sum M = T - T_c - T_d = 0 \Rightarrow T = T_c + T_d \quad (2)$$

θ_{BA}



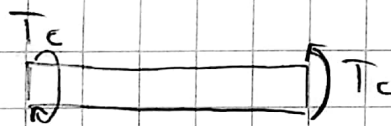
$$T = \frac{G \theta J}{L}$$

$$\Rightarrow \theta_{AB} = \frac{T \cdot L_1}{G_{AC} \cdot J_{AC}} \quad (3)$$

$$\text{con } J_{AC} = \frac{\pi D^4}{32}$$

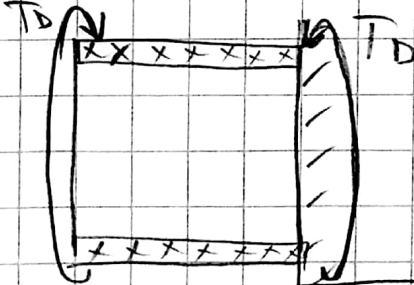
θ_B

barra



$$\Rightarrow \theta_B = \frac{T_c \cdot L_3}{G_{AC} \cdot J_{AC}} \quad (4)$$

tubo



$$\Rightarrow \theta_B = \frac{T_d \cdot L_2}{G_{br} \cdot J_{br}} \quad (5)$$

$$\text{con } J_{br} = \frac{\pi}{32} [D^4 - (D - 2t)^4]$$

$$(4) = (5) \Rightarrow T_c = \frac{L_2}{L_3} \cdot \frac{G_{AC} J_{AC}}{G_{br} J_{br}} \cdot T_d$$

en (2)

$$\Rightarrow T_d = T \left(\frac{1}{1 + \frac{L_2 G_{AC} J_{AC}}{L_3 G_{br} J_{br}}} \right)$$

em ⑤ $\Rightarrow \boxed{\Theta_B = T \left(\frac{L_2 L_3}{L_3 G_{br} J_{br} + L_2 G_{AC} J_{AC}} \right)} \quad \textcircled{6}$

⑥ y ③ em ①

$\Rightarrow \boxed{\Theta_A = \frac{T L_1}{G_{AC} J_{AC}} + T \left(\frac{L_2 L_3}{L_3 G_{br} J_{br} + L_2 G_{AC} J_{AC}} \right)}$