Aux 2

$P_1$ y $P_2$ soluciones en Aux1 ☺

P3) Considere el siguiente sistema lineal en los parámetros $\alpha, \beta \in \mathbb{R}$

$$
\begin{aligned}
2x_1 + 2x_3 + \alpha x_4 &= -1 \\
-x_2 + x_3 + (\alpha-\beta)x_4 &= 1 \\
-2x_1 - x_2 + (2\alpha-1)x_3 + (-\beta-1)x_4 &= 3-\beta \\
x_2 - x_3 + (\beta-1)x_4 &= 1 \\
x_1 + x_3 + \frac{\alpha}{2}x_4 &= 2\beta - \frac{3}{2}
\end{aligned}
$$

Determine los valores de $\alpha$ y $\beta$ para los cuales el sistema tiene

a) Una única solución

b) ninguna solución

c) Infinitas soluciones

el sistema lo escribimos de manera matricial

$$\begin{pmatrix} 2 & 0 & 2 & \alpha \\ 0 & -1 & 1 & \alpha-\beta \\ -2 & -1 & 2\alpha-1 & -\beta-1 \\ 0 & 1 & -1 & \beta-1 \\ 1 & 0 & 1 & \alpha/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 3-\beta \\ 1 \\ 2\beta-3/2 \end{pmatrix}$$

así hay que resolver

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left(\begin{array}{cccc|c} 2 & 0 & 2 & \alpha & -1 \\ 0 & -1 & 1 & \alpha-\beta & 1 \\ -2 & -1 & 2\alpha-1 & -\beta-1 & 3-\beta \\ 0 & 1 & -1 & \beta-1 & 1 \\ 1 & 0 & 1 & \alpha/2 & 2\beta-\frac{3}{2} \end{array}\right) \end{array}$$

$f_i$ = Fila i

$f_i'$ = nueva fila i

$$f_3' = f_1 + f_3 \;/\; E_{1,3}(1)A$$

$$\left(\begin{array}{cccc|c} 2 & 0 & 2 & \alpha & -1 \\ 0 & -1 & 1 & \alpha-\beta & 1 \\ 0 & -1 & 2\alpha+1 & \alpha-\beta-1 & 2-\beta \\ 0 & 1 & -1 & \beta-1 & 1 \\ 1 & 0 & 1 & \alpha/2 & 2\beta-3/2 \end{array}\right)$$

$$f_5' = f_5 - \frac{1}{2} f_1 \;/\; E_{1,5}(-1/2)A$$

$$\left(\begin{array}{cccc|c} 2 & 0 & 2 & \alpha & -1 \\ 0 & -1 & 1 & \alpha-\beta & 1 \\ 0 & -1 & 2\alpha+1 & \alpha-\beta-1 & 2-\beta \\ 0 & 1 & -1 & \beta-1 & 1 \\ 0 & 0 & 0 & 0 & 2\beta-1 \end{array}\right)$$

$$f_3' = f_3 - f_2 \quad / \; E_{23}(-1)$$

$$\left(\begin{array}{cccc|c} 2 & 0 & 2 & \alpha & -1 \\ 0 & -1 & 1 & \alpha-\beta & 1 \\ 0 & 0 & 2\alpha & -1 & 1-\beta \\ 0 & 1 & -1 & \beta-1 & 1 \\ 0 & 0 & 0 & 0 & 2\beta-1 \end{array}\right)$$

$$f_4' = f_4 + f_2 \quad / \; E_{2;4}(1)$$

$$\left(\begin{array}{cccc|c} 2 & 0 & 2 & \alpha & -1 \\ 0 & -1 & 1 & \alpha-\beta & 1 \\ 0 & 0 & 2\alpha & -1 & 1-\beta \\ 0 & 0 & 0 & \alpha-1 & 2 \\ 0 & 0 & 0 & 0 & 2\beta-1 \end{array}\right)$$

ahora observamos si

$2\beta - 1 \neq 0$ i.e $\beta \neq 1/2$

el sistema es incompatible

$\therefore$ <u>no</u> hay solución

si $\beta = 1/2$

entonces el sistema queda como

$$\left(\begin{array}{cccc|c} 2 & 0 & 2 & \alpha & -1 \\ 0 & -1 & 1 & \alpha - 1/2 & 1 \\ 0 & 0 & 2\alpha & -1 & 1/2 \\ 0 & 0 & 0 & \alpha - 1 & 2 \end{array}\right)$$

la ultima fila <u>no</u> entrega información, se puede ignorar asi notamos que

$(\alpha-1)x_4 = 2$

$\therefore$ si $\alpha = 1$ queda

$0 = 2 \rightarrow \in$

$\therefore$ si $\alpha=1$ no hay solución

si $\alpha \neq 1$

$\Rightarrow x_4 = \frac{2}{\alpha-1}$

de ahi

$2\alpha x_3 - x_4 = 1/2$

$2\alpha x_3 = \frac{1}{2} + \frac{2}{\alpha-1}$

si $\alpha = 0$

$0 = \frac{1}{2} - 2 = -3/2 \rightarrow \times$

$\therefore$ si $\alpha = 0$ no hay solución

$\therefore$ $\alpha \neq 0$

entonces

$$2\alpha x_3 = \frac{\alpha - 1 + 4}{2(\alpha-1)} = \frac{\alpha+3}{2(\alpha-1)}$$

$$\Rightarrow x_3 = \frac{\alpha+3}{4\alpha(\alpha-1)}$$

así

$$-x_2 + x_3 + (\alpha - 1/2)x_4 = 1$$

$\Rightarrow$

$$x_2 = x_3 + (\alpha - 1/2)x_4 - 1$$

$$= \frac{\alpha+3}{4\alpha(\alpha-1)} + \frac{(2\alpha-1)}{\alpha-1} - 1$$

$$= \frac{\alpha+3 + 4\alpha(2\alpha-1) - 4\alpha(\alpha-1)}{4\alpha(\alpha-1)}$$

$$= \frac{\alpha+3+8\alpha^2-\cancel{4\alpha}-4\alpha^2+\cancel{4\alpha}}{4\alpha(\alpha-1)}$$

$$= \frac{4\alpha^2+\alpha+3}{4\alpha(\alpha-1)}$$

Penultimo

$$2x_1+2x_3+\alpha x_4=-1$$

$$2x_1=-2x_3-\alpha x_4-1$$

$$= -2\cdot\frac{\alpha+3}{4\alpha(\alpha-1)} - \frac{\alpha\cdot 2}{\alpha-1} - 1$$

$$= -\frac{(\alpha+3)}{2\alpha(\alpha-1)} - \frac{2\alpha}{(\alpha-1)} - 1$$

$$\Rightarrow x_1 = -\frac{1}{2}\left[\frac{\alpha+3}{2\alpha(\alpha-1)} + \frac{2\alpha}{\alpha-1} + 1\right]$$

$$= -\frac{1}{2}\left[\frac{\alpha+3+4\alpha^2+2\alpha(\alpha-1)}{2\alpha(\alpha-1)}\right]$$

$$= -\frac{1}{4\alpha(\alpha-1)}\left[6\alpha^2-\alpha+3\right]$$ //

en resumen

$\beta \neq 1/2$ : <u>no</u> hay sol

$\boxed{\beta = 1/2}$

$\hookrightarrow \alpha = 1$ <u>no</u> hay sol

$\hookrightarrow \alpha \neq 1$
- $\alpha = 0 \rightarrow$ <u>no</u> hay sol
- $\alpha \neq 0 \rightarrow$ hay <u>sol</u> única

P4 Sea

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, encuentre

las condiciones necesarias

y suficientes para que

$A^{-1}$ exista y calcularla.

realizaremos el siguiente

sistema

$$\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right)$$

la intuición detras de

esto es resolver

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ y}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

asi intuitivamente

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Por eso resolvemos los dos al mismo tiempo

$$\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right)$$

ignorando las condiciones de indefinición calcularemos.

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$$f_2' = f_2 - \frac{c}{a} f_1$$

$$\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & d - \frac{bc}{a} & -c/a & 1 \end{array}\right)$$

Usualmente terminamos aqui pero seguimos con algo del estilo

$\hat{A}\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}$ Para lo que para solucionar eso vamos a despejar la identidad en el lado izquierdo

$$f_2' = a f_2$$

$$\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & ad-bc & -c & a \end{array}\right)$$

$$f_1' = \frac{1}{ad-bc} \cdot -b f_2 + f_1$$

$$\left(\begin{array}{cc|cc} a & 0 & 1+\frac{bc}{ad-bc} & \frac{-ba}{ad-bc} \\ 0 & ad-bc & -c & a \end{array}\right)$$

re ordenado

$$\left(\begin{array}{cc|cc} a & 0 & \frac{ad}{ad-bc} & -\frac{ba}{ad-bc} \\ 0 & ad-bc & -c & a \end{array}\right),$$

$$f_1' = \frac{1}{a} f_1$$

$$f_2' = \frac{1}{ad-bc} f_2$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right)$$

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este metodo nos garantiza encontrar la inversa mientras este todo definido así

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

es la inversa de una matriz de 2x2

notamos que

si $ad-bc \neq 0$

$\Rightarrow A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ esta bien definido y para el metodo utilizado, llamado de gauss

esto esta bien definido y es la invers.

y de todas maneras

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{ad-bc} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{ad-bc} \begin{pmatrix} ad-bc & a(-b)+ba \\ cd+d(-c) & c(-b)+da \end{pmatrix}$$

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$$= \frac{1}{ad-bc}\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

ahora probamos

$$ad-bc \neq 0 \Rightarrow A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

quepasa si $ad-bc=0$

notamos que buscamos
$x_1, x_2, y_1, y_2$ t.q

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

entonces

$ad = bc$ y

$$\begin{pmatrix} ax_1 + bx_2 & ay_1 + by_2 \\ cx_1 + dx_2 & cy_1 + dy_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\Rightarrow ad = bc$

$ax_1 + bx_2 = 1$

$ay_1 + by_2 = 0$

$cx_1 + dx_2 = 0$

$cy_1 + dy_2 = 1$

Si $c=0 \Rightarrow ad=0$

$$\left.\begin{array}{l} a x_1 + b x_2 = 1 \\ a y_1 + b y_2 = 0 \\ d \cdot x_2 = 0 \\ d y_2 = 1 \end{array}\right\} \quad d \neq 0 \Rightarrow a = 0$$

$$\Rightarrow \quad \begin{array}{ll} b x_2 = 1 & \\ b y_2 = 0 & (*) \\ d x_2 = 0 & \\ d y_2 = 1 \to y_2 \neq 0 & \therefore \text{ si} \end{array}$$

$+ (*) \Rightarrow b = 0$

$\Rightarrow 0 = 1 \to \Leftarrow$

$\therefore \quad c \neq 0$

$$\Rightarrow b = \frac{ad}{c}$$

asi

$$a x_1 + \frac{ad}{c} x_2 = 1$$

$$a y_1 + \frac{ad}{c} y_2 = 0$$

$$c x_1 + d x_2 = 0$$

$$c y_1 + d y_2 = 1$$

$acx_1 + ad\,x_2 = c \quad (\ast)$

$acy_1 + ad\,y_2 = 0$

$c\,x_1 + d\,x_2 = 0$

$cy_1 + d\,y_2 = 1$

$cx_1 + d\,x_2 = 0$

$\Rightarrow acx_1 + adx_2 = 0$

pero por $(\ast)$ $c = acx_1 + adx_2$

$\Rightarrow c = 0 \longrightarrow \Leftarrow$

así no hay solución

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a

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\therefore$ Si $ad = bc$ no hay inversa

$\therefore$ $A^{-1}$ existe si y solo si $ad - bc \neq 0$

y $A^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

P5] considere las matrices

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

a) probar que P es invertible y calcule $P^{-1}$

para eso usamos el metodo de gauss

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$F_1 \Leftrightarrow F_2$, Permutamos filas.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$F_3' = F_3 + F_1$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 1 \end{array}\right)$$

$$f_3' = f_3 + f_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right)$$

$$f_2' = f_2 - \frac{1}{3} f_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2/3 & -1/3 & -1/3 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right)$$

$f_1' = f_1 - \frac{1}{3} f_3$ y después $f_3' = \frac{1}{3} f_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 2/3 & -1/3 \\ 0 & 1 & 0 & 2/3 & -1/3 & -1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right)$$

así

$$P^{-1} = \frac{1}{3}\begin{pmatrix} -1 & 2 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

Y por el metodo sabemos que cumple ser la invers. de todas maneras verificamos

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix} \frac{1}{3}\begin{pmatrix} -1 & 2 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 2+1 & -1+1 & -1+1 \\ -1+1 & 2+1 & -1+1 \\ 1-2+1 & -2+1+1 & 1+1+1 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ahora probar que

$A = PDP^{-1}$ y encontrar

$A^{10}$

$$PD = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 1 & 1 & 2 \end{pmatrix}$$

$PDP^{-1}$

$$= \begin{pmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 1 & 1 & 2 \end{pmatrix} \frac{1}{3} \begin{pmatrix} -1 & 2 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} -2+2 & 1+2 & 1+2 \\ 1+2 & -2+2 & 1+2 \\ -1+2+2 & 2-1+2 & -1-1+2 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = A \text{ //}$$

notamos que

$$A = PDP^{-1}$$

$$A^2 = (PDP^{-1})(PDP^{-1})$$

$$= PDP^{-1}PDP^{-1}$$

$$= PD\underline{I}DP^{-1}$$

$$= PD^2P^{-1}$$

y darse cuenta que en general

$$A^n = PD^nP^{-1}$$

asi

$$A^{10} = P\,D^{10}\,P^{-1}$$

$$= \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^{10} & 0 & 0 \\ 0 & (-1)^{10} & 0 \\ 0 & 0 & 2^{10} \end{pmatrix} \frac{1}{3} \begin{pmatrix} -1 & 2 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^{10} \end{pmatrix} \begin{pmatrix} -1 & 2 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 0 & 1 & 2^{10} \\ 1 & 0 & 2^{10} \\ -1 & -1 & 2^{10} \end{pmatrix} \begin{pmatrix} -1 & 2 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 2+2^{10} & 2^{10}-1 & 2^{10}-1 \\ 2^{10}-1 & 2^{10}+2 & 2^{10}-1 \\ 2^{10}-1 & 2^{10}-1 & 2^{10}+2 \end{pmatrix} \text{//}$$

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P6)

$$A = \begin{pmatrix} -1 & 2 & 0 & 1 \\ -1 & 3 & -2 & 1 \\ 1 & -2 & -1 & 1 \\ -1 & 4 & -4 & 2 \end{pmatrix}$$

usamos gauss

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & 3 & -2 & 1 & 0 & 1 & 0 & 0 \\ 1 & -2 & -1 & 1 & 0 & 0 & 1 & 0 \\ -1 & 4 & -4 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$f_2' = f_2 - f_1$

$f_3' = f_3 + f_1$

$f_4' = f_4 - f_1$

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 2 & -4 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$$

$f_4' = f_4 - 2F_2$

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right)$$

$f_3' = f_3 - 2f_4$

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right)$$

$f_1' = f_1 - f_4$

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 0 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right)$$

$\downarrow \; f_2' = f_2 - 2f_3$

$$\left(\begin{array}{cccc|cccc} -1 & 2 & 0 & 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & -7 & -2 & 4 \\ 0 & 0 & -1 & 0 & -1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right)$$

$\downarrow \; f_1' = f_1 - 2f_2$

$$\left(\begin{array}{cccc|cccc} -1 & 0 & 0 & 1 & -2 & 16 & 4 & -9 \\ 0 & 1 & 0 & 0 & 1 & -7 & -2 & 4 \\ 0 & 0 & -1 & 0 & -1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right).$$

$\downarrow \; f_1' = -f_1 \; , \; f_3' = -f_3$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -16 & -4 & 9 \\ 0 & 1 & 0 & 0 & 1 & -7 & -2 & 4 \\ 0 & 0 & 1 & 0 & 1 & -4 & -1 & 2 \\ 0 & 0 & 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 2 & -16 & -4 & 9 \\ 1 & -7 & -2 & 4 \\ 1 & -4 & -1 & 2 \\ 1 & -2 & 0 & 1 \end{pmatrix} \quad //$$

P7) $A \in M_{mn}$ y $B \in M_{nm}$

pruebe que

a) $tr(AB) = tr(B^t A^t)$

recordamos traza es la suma de la diagonal de una matriz y la transpuesta es invertir filas con columnas

i.e. $E \in M_{nn}$

$$tr(E) = \sum_{i=1}^{n} E_{ii}$$

Si $C \in M_{nm}$

$(C^T)_{ij} = C_{ji}$ , $i \in \{1 \ldots m\}$
$j \in \{1 \ldots n\}$

$\therefore C^T \in M_{mn}$

ahana

$$(AB)_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

$\therefore Tr(AB)$

$$= \sum_{i=1}^{m} (AB)_{ii}$$

$$= \sum_{i=1}^{m} \sum_{k=1}^{m} a_{ik} b_{ki}$$

$$= \sum_{k=1}^{m} \sum_{i=1}^{m} b_{ki} a_{ik}$$

notamos que

$$(b^T)_{ij} = b_{ji}, \quad (a^T)_{ij} = a_{ji}$$

$$\therefore \; b_{ki} = (b^T)_{ik}, \quad a_{ik} = (a^T)_{ki}$$

así

$$Tr(AB) = \sum_{k=1}^{n} \sum_{i=1}^{m} (b^T)_{ik} (a^T)_{ki}$$

$$= \sum_{k=1}^{n} (B^T A^T)_{ii}$$

$$= Tr(B^T A^T) \; //$$

b) $(AB)^T = B^T A^T$

Si $E = AB$

$\Rightarrow (E)^T_{ij} = E_{ji}$

$$(E)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

asi

$$(E)^T_{ij} = \sum_{k=1}^{n} a_{jk} b_{ki}$$

$$= \sum_{k=1}^{n} b_{ki} a_{jk}$$

$$= \sum_{k=1}^{n} (b^T)_{ik} (a^T)_{kj}$$

$$= (B^T A^T)_{ij}$$

$$\Rightarrow (AB)^T_{ij} = (B^T A^T)_{ij}$$

$$\Rightarrow (AB)^T = B^T A^T$$

c) $(\lambda A + B)^T = \lambda A^T + B^T \quad \forall \lambda \in \mathbb{R}$

Sea $\lambda \in \mathbb{R}$

$$\Rightarrow E = \lambda A + B$$

$$E_{ij} = \lambda a_{ij} + b_{ij}$$

$$\Rightarrow (E^T)_{ij} = E_{ji}$$

$$= \lambda a_{ji} + b_{ji}$$

$$= \lambda (a^T)_{ij} + (b^T)_{ij} = \lambda A^T_{ij} + B^T_{ij}$$

$\Rightarrow (E^T)_{ij} = \lambda A^T_{ij} + B^T_{ij}$

$\Rightarrow (\lambda A + B)^T = \lambda A^T + B^T$ //

d) $(A^T)^{-1} = (A^{-1})^T$

PDQ

$A^T \cdot (A^{-1})^T = I$, Por unicidad

ya demostrado

$A^T \cdot (A^{-1})^T \stackrel{?}{=} (A^{-1} \cdot A)^T$

$= I^T$

Facil de ver $\hookleftarrow$ $= I$ //

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