10/03/2012

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

$$b_m^{n+1} = G\, b_m^n \quad ; \quad G \text{ es compleja}$$

$$G = |G|\, e^{i\phi} \quad ; \quad \phi = \frac{\mathrm{Im}(G)}{\mathrm{Re}(G)}$$

$$b_m^{n+1} e^{ik_m x_i} = |G_m|\, e^{i\phi_m}\, b_m^n\, e^{ik_m x_i}$$

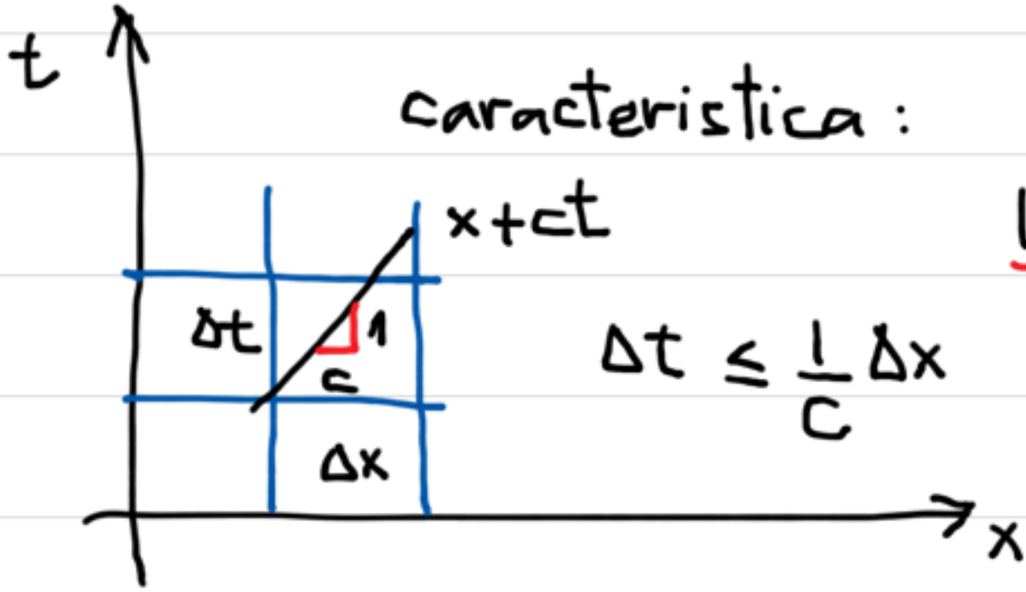
$$= |G_m|\, b_m^n\, e^{i(k_m x_i + \phi_m)}$$

$$\sum_m b^{n+1} e^{ik_m x_i} = \sum_m |G_m|\, b_m^n\, e^{i(k_m x_i + \phi_m)}$$

$|G_m|$: error de Difusión numérica

ϕ_m : error de fase

caracteristica :



Lax ; Upwind

$$\Delta t \le \frac{1}{c}\Delta x \Rightarrow \nu = \frac{c\,\Delta t}{\Delta x} \le 1$$

Implícito : (Euler - Implícito)

$$\frac{u_i^{n+1} - u_i^{n}}{\Delta t} + c\left(\frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x}\right) = 0$$

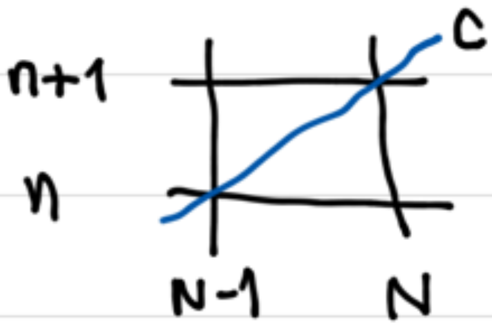
$$\begin{pmatrix} 1 & \nu/2 & 0 & 0 & & & \\ -\nu/2 & 1 & \nu/2 & 0 & & 0 & \\ 0 & -\nu/2 & 1 & \nu/2 & & & \\ & & & \ddots & & & \\ & 0 & & & -\nu/2 & 1 & \nu/2 \\ & & & & & -\nu/2 & 1 \end{pmatrix} \begin{pmatrix} u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_{N-2} \\ u_{N-1} \end{pmatrix}^{n+1} = \begin{pmatrix} u_2^n + \frac{\nu}{2}u_1^{n+1} \\ u_3^n \\ u_4^n \\ \vdots \\ \\ u_{N-1}^n - \frac{\nu}{2}u_N^{n+1} \end{pmatrix}$$

- Condición de borde de Aguas Arriba:
  (o aguas abajo)

$$u_1^{n+1} : \text{conocida}$$

- Condición de borde de Aguas Abajo:
  (o aguas arriba)

$$u_N^{n+1} : \text{desconocida}$$



Se recurre al metodo de las caracteristicas

$$\Rightarrow u_N^{n+1} = u_{N-1}^{n}$$

$$\frac{\partial u}{\partial t} = u_t \ ; \ \frac{\partial u}{\partial x} = u_x \Rightarrow u_t + c\,u_x = 0$$

Para Euler-Implícito:

$$u_t + c u_x = \left(\frac{c^2}{2}\Delta t\right)\underbrace{u_{xx}}_{\text{dif. numérica}} - \left(\frac{c}{6}(\Delta x)^2 + \frac{c^3}{3}(\Delta x)^2\right)\underbrace{u_{xxx}}_{\text{error fase}} + \cdots$$

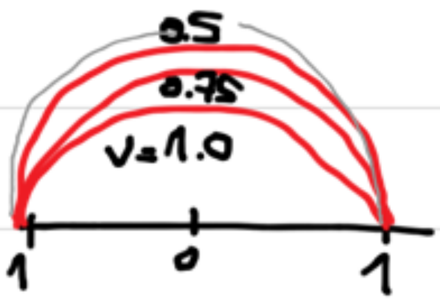
$$G_m = \frac{1}{1 - i\nu \operatorname{sen}\beta} = \frac{1 - i\nu \operatorname{sen}\beta}{1 + \nu^2 \operatorname{sen}^2\beta}$$

$$|G_m| = \frac{1}{1 + \nu^2 \operatorname{sen}^2\beta} \quad ; \quad \phi = \tan^{-1}(-\nu \operatorname{sen}\beta)$$

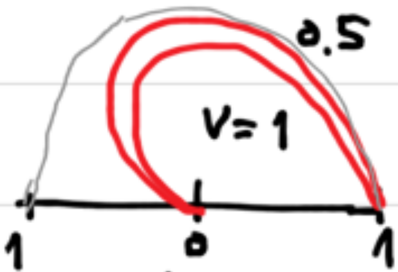
$$G_{\text{Teórico}} = \cos(\nu\beta) - i \operatorname{sen}(\nu\beta)$$

$$|G_{\text{TEO}}| = 1 \quad ; \quad \tan\phi_{\text{TEO}} = \tan(\nu\beta) \Rightarrow \phi_{\text{TEO}} = \nu\beta$$

$$\frac{|G_m|}{|G_{\text{TEO}}|} = \frac{1}{1 + \nu^2 \operatorname{sen}^2\beta} \quad ; \quad \frac{\phi}{\phi_{\text{TEO}}} = \frac{\tan^{-1}(-\nu \operatorname{sen}\beta)}{\nu\beta}$$



$\frac{|G|}{|G_{\text{TEO}}|}$



ϕ/ϕ_{TEO}