* Pauta Aux 1

"Cálculo Avanzado y Aplicaciones"

P1 Cuando nos piden las curvas de nivel de una función $f(x,y)$, nos están pidiendo encontrar la ecuación del lugar geométrico $f(x,y)=\alpha$, con α una constante (en principio $\alpha \in \mathbb{R}$, pero puede tener cotas dependiendo de $f(x,y)$).

(a) $f(x,y)=\sqrt{64-x^2-y^2}$

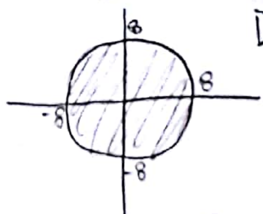
Aplicando directamente $f(x,y)=\alpha$:

$$\Rightarrow \sqrt{64-x^2-y^2}=\alpha \Rightarrow 64-x^2-y^2=\alpha^2 \Rightarrow x^2+y^2=64-\alpha^2$$

$$\Rightarrow \boxed{x^2+y^2=\left(\sqrt{64-\alpha^2}\right)^2} \longrightarrow$$ Circunferencias con centro en el origen y radio $r=\sqrt{64-\alpha^2}$

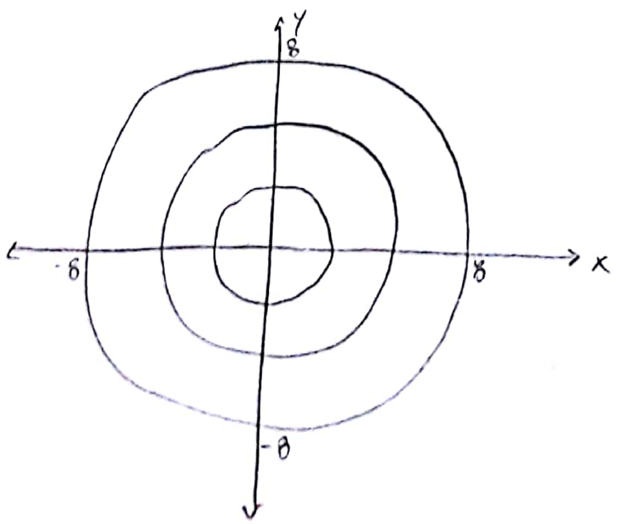
Sin embargo, hay que notar que el dominio está restringido:

$$64-x^2-y^2 \geq 0 \Rightarrow x^2+y^2 \leq 64 \Rightarrow x^2+y^2 \leq 8^2$$



Dominio

Por lo tanto, las curvas de nivel pueden dibujarse hasta $\alpha=8$, así:



Curvas de nivel de $f(x,y)$

b) $g(x,y) = x^2 - y^2$

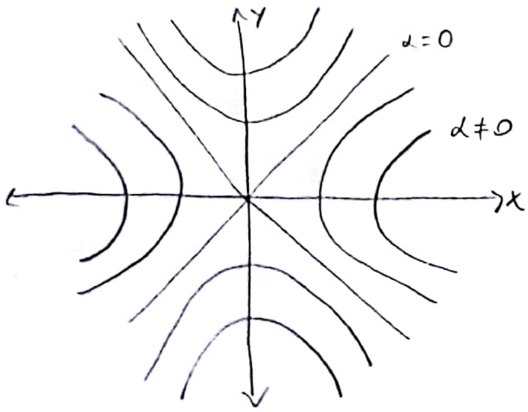
Curvas de nivel:

$$g(x,y) = \alpha \Rightarrow x^2 - y^2 = \alpha \Rightarrow \boxed{\frac{x^2}{\alpha} - \frac{y^2}{\alpha} = 1} \rightarrow$$ Hipérbolas con centro en el origen!!!

El caso especial es para $\alpha = 0$, en ese caso:

$$x^2 - y^2 = 0 \Rightarrow y^2 = x^2 \Rightarrow y = \pm\sqrt{x^2} \Rightarrow \boxed{y = \pm|x|} \rightarrow$$ Rectas!

Entonces:

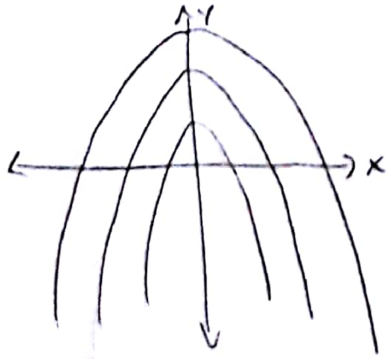


De la ecuación obtenida $\frac{x^2}{\alpha} - \frac{y^2}{\alpha} = 1$ se desprende que las hipérbolas son horizontales para $\alpha > 0$, y verticales para $\alpha < 0$

c) $h(x,y) = x^2 + y - 2$

Curvas de nivel:

$$h(x,y) = \alpha \Rightarrow x^2 + y - 2 = \alpha \Rightarrow \boxed{y = -x^2 + 2 + \alpha} \rightarrow$$ Parábolas!!!



P2 b).

En este ejercicio se debe considerar que, para un campo escalar f, $\Delta f = \nabla^2 = \nabla\cdot(\nabla f)$ (el laplaciano es la divergencia del gradiente del campo escalar, en campos vectoriales <u>ES DISTINTO</u>). Entonces:

$$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

$$\frac{\partial V}{\partial x} = -GM\cdot\left[\frac{-\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{x^2+y^2+z^2}}\cdot\cancel{2}x}{(x^2+y^2+z^2)}\right] \Rightarrow \boxed{\frac{\partial V}{\partial x} = \frac{GM\cdot x}{(x^2+y^2+z^2)^{3/2}}}$$

El caso de $\frac{\partial V}{\partial y}$ y $\frac{\partial V}{\partial z}$ es análogo, así:

$$\nabla V = \frac{GM}{(x^2+y^2+z^2)^{3/2}}\cdot(x\hat{x} + y\hat{y} + z\hat{z})$$

Entonces:

$$\nabla\cdot(\nabla V) = \frac{\partial(\nabla V)_x}{\partial x} + \frac{\partial(\nabla V)_y}{\partial y} + \frac{\partial(\nabla V)_z}{\partial z}$$

Al ver ∇V nos damos cuenta de que podemos calcular $\frac{\partial(\nabla V)_x}{\partial x}$, y las otras dos derivadas son análogas, así:

$$\frac{\partial(\nabla V)_x}{\partial x} = \frac{\partial}{\partial x}\left(\frac{GMx}{(x^2+y^2+z^2)^{3/2}}\right) = GM\cdot\left[\frac{(x^2+y^2+z^2)^{3/2} - x\cdot\frac{3}{\cancel{2}}\sqrt{x^2+y^2+z^2}\cdot\cancel{2}x}{(x^2+y^2+z^2)^3}\right]$$

$$\Rightarrow \frac{\partial(\nabla V)_x}{\partial x} = GM\cdot\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3x^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$\Rightarrow \frac{\partial(\nabla V)_y}{\partial y} = GM\cdot\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3y^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$\Rightarrow \frac{\partial(\nabla V)_z}{\partial z} = GM\cdot\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3z^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$\Rightarrow \nabla\cdot(\nabla V) = \nabla^2 V = \Delta V = GM\cdot\left[\frac{3}{(x^2+y^2+z^2)^{3/2}} - \frac{3\cdot(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$\Rightarrow \Delta V = GM\cdot\left[\cancel{\frac{3}{(x^2+y^2+z^2)^{3/2}}} - \cancel{\frac{3}{(x^2+y^2+z^2)^{3/2}}}\right]$$

$$\Rightarrow \boxed{\Delta V = 0} \;//$$

P3) (a) Aplicando directamente la definición:

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$\Rightarrow \nabla\times(\nabla f) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix} = \left(\frac{\partial^2 f}{\partial y\partial z} - \frac{\partial^2 f}{\partial z\partial y}\right)\hat{x} - \left(\frac{\partial^2 f}{\partial x\partial z} - \frac{\partial^2 f}{\partial z\partial x}\right)\hat{y} + \left(\frac{\partial^2 f}{\partial x\partial y} - \frac{\partial^2 f}{\partial y\partial x}\right)\hat{z}$$

Y, por el teo. de Schwarz, se tiene que las derivadas cruzadas son iguales, por lo tanto se cancelan y se obtiene que

$$\nabla\times(\nabla f) = \vec{0} \;//$$

(b) Aplicando directamente:

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$$

$$\Rightarrow \nabla\cdot(\nabla\times\vec{F}) = \frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)+\frac{\partial}{\partial y}\left(-\left(\frac{\partial F_z}{\partial x}-\frac{\partial F_x}{\partial z}\right)\right)+\frac{\partial}{\partial z}\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)$$

$$\Rightarrow \nabla\cdot(\nabla\times\vec{F}) = \frac{\partial^2 F_z}{\partial x\partial y}-\frac{\partial^2 F_y}{\partial x\partial z}-\frac{\partial^2 F_z}{\partial y\partial x}+\frac{\partial^2 F_x}{\partial y\partial z}+\frac{\partial^2 F_y}{\partial z\partial x}-\frac{\partial^2 F_x}{\partial z\partial y}$$

Y nuevamente por Teo. de Schwarz, se cancelan términos y se llega a que $\nabla\cdot(\nabla\times\vec{F})=0$ //

(c) Sea $\vec{F}=F_x\hat{x}+F_y\hat{y}+F_z\hat{z}$ y $\vec{G}=G_x\hat{x}+G_y\hat{y}+G_z\hat{z}$. Haciendo el producto cruz:

$$\vec{F}\times\vec{G}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ F_x & F_y & F_z\\ G_x & G_y & G_z\end{vmatrix} = \underbrace{(F_2G_3-F_3G_2)}_{\text{I}}\hat{x}-\underbrace{(F_1G_3-F_3G_1)}_{\text{II}}\hat{y}+\underbrace{(F_1G_2-F_2G_1)}_{\text{III}}\hat{z}$$

$$\Rightarrow \nabla\cdot(\vec{F}\times\vec{G}) = \frac{\partial \text{I}}{\partial x}+\frac{\partial \text{II}}{\partial y}+\frac{\partial \text{III}}{\partial z}$$

$$= \frac{\partial}{\partial x}(F_2G_3-F_3G_2)-\frac{\partial}{\partial y}(F_1G_3-F_3G_1)+\frac{\partial}{\partial z}(F_1G_2-F_2G_1)$$

$$= G_3\frac{\partial F_2}{\partial x}+F_2\cdot\frac{\partial G_3}{\partial x}-G_2\frac{\partial F_3}{\partial x}-F_3\frac{\partial G_2}{\partial x}-G_3\frac{\partial F_1}{\partial y}-F_1\frac{\partial G_3}{\partial y}$$

$$+G_1\frac{\partial F_3}{\partial y}+F_3\frac{\partial G_1}{\partial y}+G_2\frac{\partial F_1}{\partial z}+F_1\frac{\partial G_2}{\partial z}-G_1\frac{\partial F_2}{\partial z}-F_2\frac{\partial G_1}{\partial z}$$

Agrupando:

$$\Rightarrow \nabla\cdot(\vec{F}\times\vec{G}) = G_1\cdot\left(\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z}\right)+G_2\left(\frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x}\right)+G_3\left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right)$$

$$+F_1\left(\frac{\partial G_2}{\partial z}-\frac{\partial G_3}{\partial y}\right)+F_2\left(\frac{\partial G_3}{\partial x}-\frac{\partial G_1}{\partial z}\right)+F_3\left(\frac{\partial G_1}{\partial y}-\frac{\partial G_2}{\partial x}\right)$$

Recordando la definición de rotor:

$$\nabla \times \vec{H} = \left(\frac{\partial H_3}{\partial y} - \frac{\partial H_2}{\partial z}\right)\hat{x} - \left(\frac{\partial H_3}{\partial x} - \frac{\partial H_1}{\partial z}\right)\hat{y} + \left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\hat{z}$$

Reordenando convenientemente:

$$\Rightarrow \nabla \times (\vec{F} \times \vec{G}) = G_1 \cdot \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - G_2\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + G_3\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

$$- \left[F_1\left(\frac{\partial G_3}{\partial y} - \frac{\partial G_2}{\partial z}\right) - F_2\left(\frac{\partial G_3}{\partial x} - \frac{\partial G_1}{\partial z}\right) + F_3\left(\frac{\partial G_2}{\partial x} - \frac{\partial G_1}{\partial y}\right)\right]$$

$$\Rightarrow \nabla \times (\vec{F} \times \vec{G}) = \vec{G} \cdot (\nabla \times \vec{F}) - \vec{F} \cdot (\nabla \times \vec{G}) \quad //$$

(d) Aplicando lo demostrado en (c):

$$\nabla \cdot (\nabla f \times \nabla g) = \nabla g \cdot (\nabla \times (\nabla f)) - \nabla f \cdot (\nabla \times (\nabla g))$$

y, aplicando lo demostrado en (e), $\nabla \times (\nabla f) = \nabla \times (\nabla g) = \vec{0}$, así:

$$\Rightarrow \nabla \cdot (\nabla f \times \nabla g) = 0 \quad //$$

P4 Para hacer este ejercicio se necesitan coordenadas esféricas y expresiones para el gradiente y la divergencia en estas coordenadas. Entonces, considere el sistema de coordenadas dado por:

$$\vec{r}(r, \theta, \phi) = (r \cdot sen(\phi)\cos(\theta), r \cdot sen(\phi) sen(\theta), r \cdot \cos(\phi))$$

Entonces:

(i) $\frac{\partial \vec{r}}{\partial r} = (sen(\phi)\cos(\theta), sen(\phi)sen(\theta), \cos(\phi)$

$$\Rightarrow h_r = \left\|\frac{\partial \vec{r}}{\partial r}\right\| = \sqrt{sen^2(\phi)\cos^2(\theta) + sen^2(\phi)sen^2(\theta) + \cos^2(\phi)}$$

$$= \sqrt{sen^2(\phi)\left(\cos^2(\theta) + sen^2(\theta)\right) + \cos^2(\phi)} = \sqrt{sen^2(\phi) + \cos^2(\phi)} = 1$$

$$\Rightarrow \hat{r} = (sen(\phi)\cos(\theta),\ sen(\phi)sen(\theta),\ \cos(\phi))$$

(ii) $$\frac{\partial \vec{r}}{\partial \theta} = (-r\cdot sen(\phi)sen(\theta),\ r\cdot sen(\phi)\cos(\theta),\ 0)$$

$$\Rightarrow h_\theta = \left\|\frac{\partial \vec{r}}{\partial \theta}\right\| = \sqrt{r^2\cdot sen^2(\phi)\cdot sen^2(\theta) + r^2\cdot sen^2(\phi)\cdot\cos^2(\theta)}$$

$$= \sqrt{r^2\cdot sen^2(\phi)\left(sen^2(\theta) + \cos^2(\theta)\right)} = \sqrt{r^2\cdot sen^2(\phi)} = r\cdot sen(\phi)$$

$$\Rightarrow \hat{\theta} = \frac{1}{\cancel{r\cdot sen(\phi)}}\cdot\left(-\cancel{r\cdot sen(\phi)}sen(\theta),\ \cancel{r\cdot sen(\phi)}\cos(\theta),\ 0\right) \Rightarrow \hat{\theta} = (-sen(\theta),\ \cos(\theta),\ 0)$$

(iii) $$\frac{\partial \vec{r}}{\partial \phi} = (r\cdot\cos(\phi)\cos(\theta),\ r\cdot\cos(\phi)sen(\theta),\ -r\cdot\sin(\phi))$$

$$\Rightarrow h_\phi = \left\|\frac{\partial \vec{r}}{\partial \phi}\right\| = \sqrt{r^2\cdot\cos^2(\phi)\cos^2(\theta) + r^2\cdot\cos^2(\phi)sen^2(\theta) + r^2\cdot\sin^2(\phi)}$$

$$= \sqrt{r^2\cdot\cos^2(\phi)\left(\cos^2(\theta)+\sin^2(\theta)\right) + r^2\cdot\sin^2(\phi)} = r$$

$$\Rightarrow \hat{\phi} = \frac{1}{\cancel{r}}\cdot\left(\cancel{r}\cdot\cos(\phi)\cos(\theta),\ \cancel{r}\cdot\cos(\phi)sen(\theta),\ -\cancel{r}\cdot\sin(\phi)\right)$$

$$\Rightarrow \hat{\phi} = (\cos(\phi)\cos(\theta),\ \cos(\phi)\sin(\theta),\ -\sin(\phi))$$

Entonces, se tiene el gradiente:

$$\boxed{\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r\cdot sen(\phi)}\cdot\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r}\cdot\frac{\partial f}{\partial \phi}\hat{\phi}}$$

Y la divergencia:

$$\nabla\cdot\vec{F} = \frac{1}{r^2\cdot sen(\phi)}\cdot\left[\frac{\partial}{\partial r}\left(F_r\cdot r^2\cdot sen(\phi)\right) + \frac{\partial}{\partial \theta}\left(F_\theta\cdot r\right) + \frac{\partial}{\partial \phi}\left(F_\phi\cdot r\cdot sen(\phi)\right)\right]$$

$$\Rightarrow \nabla\cdot\vec{F} = \frac{1}{r^2\cdot \cancel{sen(\phi)}}\cdot\cancel{sen(\phi)}\cdot\frac{\partial}{\partial r}\left(F_r\cdot r^2\right) + \frac{1}{r^{\cancel{2}}\cdot sen(\phi)}\cdot\cancel{r}\cdot\frac{\partial}{\partial \theta}\left(F_\theta\right) + \frac{1}{r^{\cancel{2}}\cdot sen(\phi)}\cdot\cancel{r}\cdot\frac{\partial}{\partial \phi}\left(F_\phi\cdot sen(\phi)\right)$$

$$\Rightarrow \boxed{\nabla\cdot\vec{F} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\cdot F_r\right) + \frac{1}{r\cdot sen(\phi)}\cdot\frac{\partial F_\theta}{\partial \theta} + \frac{1}{r\cdot sen(\phi)}\cdot\frac{\partial}{\partial \phi}\left(sen(\phi)\cdot F_\phi\right)}$$

Ahora, se tiene el potencial de Yukawa:

$$V(r) = \frac{-g^2\cdot e^{-Kmr}}{r}$$

Podemos notar que sólo depende de r. Para obtener el campo vectorial asociado se usa el gradiente, así:

$$\vec{F} = \nabla V = \frac{\partial V}{\partial r}\hat{r} \Rightarrow \vec{F} = -g^2\cdot\frac{\partial}{\partial r}\left(\frac{e^{-Kmr}}{r}\right)\hat{r}$$

$$\Rightarrow \vec{F} = -g^2\cdot\left(\frac{-Kmr\cdot e^{-Kmr} - e^{-Kmr}}{r^2}\right)\hat{r}$$

$$\Rightarrow \boxed{\vec{F} = g^2\cdot e^{-Kmr}\cdot\left(\frac{Kmr+1}{r^2}\right)\hat{r}} \rightarrow \text{Campo vectorial asociado...}$$

Ahora, para la divergencia:

$$\nabla\cdot\vec{F} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(\cancel{r^2}\cdot g^2\cdot e^{-kmr}\cdot\left(\frac{kmr+1}{\cancel{r^2}}\right)\right)$$

$$\Rightarrow \nabla\cdot\vec{F} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(g^2\cdot e^{-kmr}\cdot(kmr+1)\right)$$

$$\Rightarrow \nabla\cdot\vec{F} = \frac{1}{r^2}\cdot g^2\cdot\left(-km\cdot e^{-kmr}\cdot(kmr+1)+km\cdot e^{-kmr}\right)$$

$$\Rightarrow \nabla\cdot\vec{F} = \frac{1}{r^2}\cdot g^2 km\cdot e^{-kmr}\left(-kmr\cancel{-1}\cancel{+1}\right)$$

$$\Rightarrow \boxed{\nabla\cdot\vec{F} = \frac{-g^2k^2m^2\cdot e^{-kmr}}{r} \longrightarrow \text{Divergencia de } \vec{F}\ldots}$$

P5 (a) El procedimiento es análogo al ejercicio anterior, entonces, se tiene:

$$\vec{r}(x,\rho,\theta) = (x, \rho\cdot\cos(\theta), \rho\cdot\sin(\theta))$$

(i) $\frac{\partial\vec{r}}{\partial x} = (1,0,0) \Rightarrow h_x = \left\|\frac{\partial\vec{r}}{\partial x}\right\| = \sqrt{1} \Rightarrow h_x = 1$

(ii) $\frac{\partial\vec{r}}{\partial\rho} = (0, \cos(\theta), \sin(\theta)) \Rightarrow h_\rho = \left\|\frac{\partial\vec{r}}{\partial\rho}\right\| = \sqrt{\cos^2(\theta)+\sin^2(\theta)} \Rightarrow h_\rho = 1$

(iii) $\frac{\partial\vec{r}}{\partial\theta} = (0, -\rho\cdot\sin(\theta), \rho\cdot\cos(\theta)) \Rightarrow h_\theta = \left\|\frac{\partial\vec{r}}{\partial\rho}\right\| = \sqrt{\rho^2\cdot\sin^2(\theta)+\rho^2\cdot\cos^2(\theta)}$

$$\Rightarrow h_\theta = \rho$$

Entonces, de lo Anterior se deduce que:

$$\hat{x} = (1, 0, 0) \; ; \; \hat{\rho} = (0, \cos(\theta), \sin(\theta)) \; ; \; \hat{\theta} = (0, -\sin(\theta), \cos(\theta))$$

Estos vectores son ortogonales simplemente por el procedimiento como fueron obtenidos, sin embargo, se puede comprobar al hacer el producto punto entre ellos (notar que todos dan cero, es decir, son ortogonales).

Ahora:

$$\hat{\theta} \times \hat{x} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -\sin(\theta) & \cos(\theta) \\ 1 & 0 & 0 \end{vmatrix} = (0-0)\hat{x} - (0 - \cos(\theta))\hat{y} + (0 - (-\sin(\theta)))\hat{z}$$

$$\Rightarrow \hat{\theta} \times \hat{x} = (0, \cos(\theta), \sin(\theta)) \Rightarrow \boxed{\hat{\theta} \times \hat{x} = \hat{\rho}}$$

$$\hat{\theta} \times \hat{\rho} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -\sin(\theta) & \cos(\theta) \\ 0 & \cos(\theta) & \sin(\theta) \end{vmatrix} = (-\sin^2(\theta) - \cos^2(\theta))\hat{x} - (0-0)\hat{y} + (0-0)\hat{z}$$

$$\Rightarrow \hat{\theta} \times \hat{\rho} = (-1, 0, 0) \Rightarrow \boxed{\hat{\theta} \times \hat{\rho} = -\hat{x}}$$

(b) Usando las definiciones:

(i) Gradiente:

$$\boxed{\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho} \cdot \frac{\partial f}{\partial \theta}\hat{\theta}}$$

(ii) Divergencia:

$$\nabla\cdot\vec{F} = \frac{1}{\rho}\cdot\left[\frac{\partial}{\partial x}(F_x\,\rho) + \frac{\partial}{\partial \rho}\cdot(F_\rho\cdot\rho) + \frac{\partial}{\partial\theta}(F_\theta)\right]$$

$$\Rightarrow \boxed{\nabla\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}(\rho\cdot F_\rho) + \frac{1}{\rho}\cdot\frac{\partial F_\theta}{\partial\theta}}$$

(iii) Laplaciano:

Recordando que $\nabla^2 f = \nabla\cdot(\nabla f)$, se tiene que:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\cdot\left(\rho\cdot\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho}\cdot\frac{\partial}{\partial\theta}\left(\frac{1}{\rho}\cdot\frac{\partial f}{\partial\theta}\right)$$

$$\Rightarrow \boxed{\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\left(\rho\cdot\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\cdot\frac{\partial^2 f}{\partial\theta^2}}$$

(iv) Rotor:

$$\nabla\times\vec{F} = \frac{1}{\rho}\cdot\begin{vmatrix} \hat{x} & \hat{\rho} & \rho\cdot\hat{\theta} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\theta} \\ F_x & F_\rho & \rho\cdot F_\theta \end{vmatrix}$$

$$\Rightarrow \nabla\times\vec{F} = \frac{1}{\rho}\cdot\left[\left(\frac{\partial}{\partial\rho}(\rho\cdot F_\theta) - \frac{\partial F_\rho}{\partial\theta}\right)\hat{x} - \left(\frac{\partial}{\partial x}(\rho\cdot F_\theta) - \frac{\partial F_x}{\partial\rho}\right)\hat{\rho} + \left(\frac{\partial F_\rho}{\partial x} - \frac{\partial F_x}{\partial\rho}\right)\cdot\rho\,\hat{\theta}\right]$$

$$\Rightarrow \boxed{\nabla\times\vec{F} = \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho\cdot F_\theta) - \frac{\partial F_\rho}{\partial\theta}\right)\hat{x} - \left(\frac{\partial F_\theta}{\partial x} - \frac{1}{\rho}\cdot\frac{\partial F_x}{\partial\rho}\right)\hat{\rho} + \left(\frac{\partial F_\rho}{\partial x} - \frac{\partial F_x}{\partial\rho}\right)\hat{\theta}}$$

(c) Por definición se tiene que:

$$\vec{r} = (x, y, z) = (x, \rho \cdot \cos(\theta), \rho \cdot \sin(\theta))$$

$$\Rightarrow y = \rho \cdot \cos(\theta) \quad ; \quad z = \rho \cdot \sin(\theta)$$

$$\Rightarrow y^2 + z^2 = \rho^2 \cdot (\cos^2(\theta) + \sin^2(\theta)) \Rightarrow \boxed{y^2 + z^2 = \rho^2}$$

Sin embargo, se tiene que $y^2 + z^2 = f(x)^2$, entonces, $f(x)$ cumple el rol de ρ, Así:

$$\vec{r} = (x, f(x) \cdot \cos(\theta), f(x) \cdot \sin(\theta))$$

Notamos que $\hat{x} = (1, 0, 0)$ y $\hat{\rho} = (0, \cos(\theta), \sin(\theta))$, entonces:

$$\Rightarrow \boxed{\vec{r} = x\hat{x} + f(x)\hat{\rho}} \quad //$$