Aux extra C2

P1

dada $f:\mathbb{R}^2 \to \mathbb{R}$ de clase C^2 homogénea de grado $m \geq 2$, es decir, para todo $\lambda \in \mathbb{R}$ verifica

$$f(\lambda x, \lambda y) = \lambda^m f(x,y) \quad \forall x, y \in \mathbb{R}$$

Demuestre que f verifica la igualdad

$$x^2 \frac{\partial^2 f}{\partial x^2}(x,y) + 2xy \frac{\partial^2 f(x,y)}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2}(x,y) = m(m-1) f(x,y)$$

Sol

sea $\phi(\lambda) = f(\lambda x, \lambda y)$

así $\phi'(\lambda) = \frac{\partial f}{\partial x}(\lambda x, \lambda y) \cdot x + \frac{\partial f}{\partial y}(\lambda x, \lambda y) \cdot y$

entonces

$$\phi''(\lambda) = x \cdot \left[\frac{\partial^2 f}{\partial x^2}(\lambda x, \lambda y) \cdot x + \frac{\partial^2 f}{\partial y \partial x}(\lambda x, \lambda y) \cdot y \right] + y \cdot \left[\frac{\partial^2 f}{\partial x \partial y}(\lambda x, \lambda y) \cdot x + \frac{\partial^2 f}{\partial y^2}(\lambda x, \lambda y) \cdot y \right]$$

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derivadas cruzadas

$$\phi''(\lambda) = x^2 \frac{\partial^2 f}{\partial x^2}(\lambda x, \lambda y) + 2xy \frac{\partial^2 f}{\partial x \partial y}(\lambda x, \lambda y)$$

$$+ y^2 \frac{\partial^2 f}{\partial y^2}(\lambda x, \lambda y)$$

$$\Rightarrow \phi''(1) = x^2 \frac{\partial^2 f}{\partial x^2}(x,y) + 2xy \frac{\partial^2 f}{\partial x \partial y}(x,y) + y^2 \frac{\partial^2 f}{\partial y^2}(x,y)$$

Por otro lado $\phi(\lambda) = f(\lambda x, \lambda y) = \lambda^m f(x,y)$

$$\Rightarrow \phi''(\lambda) = m(m-1)\lambda^{m-2} f(x,y)$$

$$\therefore \phi''(1) = m(m-1) f(x,y)$$

así se tiene

$$x^2 \frac{\partial^2 f}{\partial x^2}(x,y) + 2xy \frac{\partial^2 f}{\partial x \partial y}(x,y) + y^2 \frac{\partial^2 f}{\partial y^2}(x,y) = m(m-1) f(x,y)$$

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P2

(1) Considere la función $f: \mathbb{R}^2 \to \mathbb{R}$ definida por

$$f(x,y) = \ln(1+x^2+y^2)$$

a) Calcule $\nabla f(x,y)$ y $Hf(x,y)$ en cada punto $(x,y) \in \mathbb{R}^2$

Sol

$$\frac{\partial f}{\partial x} = \frac{1}{1+x^2+y^2} \cdot 2x$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+x^2+y^2} \cdot 2y$$

así $$\nabla f(x,y) = \begin{pmatrix} \dfrac{2x}{1+x^2+y^2} \\ \dfrac{2y}{1+x^2+y^2} \end{pmatrix}$$

Por otro lado

$$\frac{\partial^2 f}{\partial x^2} = \frac{2y^2-2x^2+2}{(1+x^2+y^2)^2}, \quad \frac{\partial^2 f}{\partial y \partial x} = \frac{-4xy}{(1+x^2+y^2)^2}$$

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$$\frac{\partial^2 f}{\partial y^2} = \frac{2x^2-2y^2+2}{(x^2+y^2+1)^2}$$

así

$$Hf(x,y) = \frac{2}{(x^2+y^2+1)^2}\begin{pmatrix} 1+y^2-x^2 & -2xy \\ -2xy & 1+x^2-y^2 \end{pmatrix}$$

b) Desarrollo limitado de segundo orden en torno a $(0,0)$

$$\to P_2 f(x,y) = f(0,0) + \nabla f(0,0)\cdot\begin{pmatrix} x-0 \\ y-0 \end{pmatrix} + \frac{1}{2}(x-0, y-0)\, Hf(0,0)\begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$$

$$= \ln(1+0^2+0^2) + \begin{pmatrix} \frac{2\cdot 0}{1+0^2+0^2} \\ \frac{2\cdot 0}{1+0^2+0^2} \end{pmatrix}\cdot\begin{pmatrix} x \\ y \end{pmatrix}$$

$$+\frac{1}{2}(x,y)\cdot\left[\frac{2}{(0^2+0^2+1)^2}\right]\begin{pmatrix} 1+0^2-0^2 & -2\cdot 0\cdot 0 \\ -2\cdot 0\cdot 0 & 1+0^2-0^2 \end{pmatrix}$$

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$\therefore\ P_2 f(x,y) = \frac{1}{2}(x,y) \cdot 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$= (x,y) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$= (x,y) \begin{pmatrix} x \\ y \end{pmatrix}$

$= x^2 + y^2$ //

c) calcular puntos criticos y decir su naturaleza

Sol//

$\Rightarrow \nabla f(x,y) = 0$

$\Rightarrow \frac{2x}{(1+x^2+y^2)^2} = \frac{2y}{(1+x^2+y^2)^2} = 0$

$\Rightarrow x = y = 0$

así el único punto critico es $(0,0)$

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Y

$$HF(0,0) = 2\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

que es semi-def Positiva

$\therefore$ (0,0) es minimo local //

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(2) considere la función $f: \mathbb{R}^2 \to \mathbb{R}$ definida por

$$f(x,y) = xy e^{-4x^2}$$

a) encuentre los puntos críticos y clasifíquelos

Sol

$$\frac{\partial f}{\partial x} = y e^{-4x^2} + xy \cdot e^{-4x^2} \cdot -8x$$

$$\frac{\partial f}{\partial y} = x e^{-4x^2}$$

$$\Rightarrow \quad \nabla f = 0$$

$$e^{-4x^2}[y - 8x^2 y] = 0$$

$$x e^{-4x^2} = 0$$

$$\Rightarrow \quad x = 0 \Rightarrow y = 0$$

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$\therefore$ el único punto crítico es $(0,0)$

vemos su Hessiano

$$\frac{\partial f}{\partial x} = y e^{-4x^2}[1-8x^2]$$

$$\frac{\partial^2 f}{\partial x^2} = y\, 8 e^{-4x^2} x(8x^2-3)$$

$$\frac{\partial^2 f}{\partial y \partial x} = e^{-4x^2}[1-8x^2]$$

$$\frac{\partial f}{\partial y} = x e^{-4x^2}$$

$$\frac{\partial^2 f}{\partial y} = 0$$

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así $Hf(0,0) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Calculamos sus valores propios

$\rightarrow P(\lambda) = \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix}$

$= +\lambda^2 - 1$

así $\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$\therefore$ es un punto silla

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b) Por lo anterior sabemos que no tiene, otra forma de verlo es ver que se va a infinito en alguna dirección

Sea $y = e^{4x^2}$

$\Rightarrow (x,y) \to +\infty$

$\therefore \lim_{x \to +\infty} f(x, e^{4x^2})$

$= \lim_{x \to \infty} x e^{4x^2} \cdot e^{-4x^2}$

$= \lim_{x \to \infty} x$

$= +\infty$ así no tiene máximo.

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P3

a) $A \subseteq \mathbb{R}^n$ convexo compacto, $f: A \to \mathbb{R}$ clase C^1. Demuestre que f es uniformemente continua. Es decir

$\forall \varepsilon > 0, \exists \delta > 0$ $\forall x, y \in A$, $\|x-y\| \leq \delta \Rightarrow \|f(x)-f(y)\| \leq \varepsilon$

Sol)

el TVM nos dice que si f es diferenciable $\exists c \in [a,b]$

$\Rightarrow f(b) - f(a) = \langle \nabla f(c) ; b-a \rangle$

asi por C-S

$\|f(b)-f(a)\| \leq \|\nabla f(c)\| \cdot \|b-a\|$

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ahora como A es convexo

$\Rightarrow \forall x, y \in A, \exists c \in [x, y] \subseteq A$

$\|f(x) - f(y)\| \leq \|\nabla f(c)\| \cdot \|x - y\|$

ahora consideremos

la función

$h: A \longrightarrow \mathbb{R}$

$x \longmapsto h(x) = \|\nabla f(x)\|$

Como $\nabla f(x)$ es continuo ($f \in C^1$)

y $\|\cdot\|$ es continua

$\Rightarrow$ h es continua en un

compacto $\therefore$ alcanza su

maximo y suminimo

$\Rightarrow \exists \bar{x} \in A$ t.q $\|\nabla f(x)\| \leq \|\nabla f(\bar{x})\|$

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así como $c \in [x,y] \subseteq A$

(1)

$\Rightarrow \|f(x)-f(y)\| \leq \|\nabla f(\bar{x})\| \, \|x-y\| \quad \forall x,y \in A$

donde $\bar{x}$ no depende de x,y

así;

Sea $\varepsilon > 0$, p.d.q

$\exists \delta > 0$, $\|x-y\| \leq \delta \Rightarrow \|f(x)-f(y)\| \leq \varepsilon$

Por (1)

$$\|f(x)-f(y)\| \leq \|\nabla f(\bar{x})\| \cdot \delta$$

así si $\|\nabla f(\bar{x})\| \neq 0$

$$\Rightarrow \delta = \frac{\varepsilon}{\|\nabla f(\bar{x})\|}$$

$$\Rightarrow \|f(x)-f(y)\| \leq \varepsilon \quad \forall x,y$$

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Si $\|\nabla f(\bar{x})\| = 0$

$\Rightarrow f(x) = f(y) \ \forall x, y$

$\therefore \forall \varepsilon > 0 \ \|f(x) - f(y)\| = 0 \leq \varepsilon$

y se va a cumplir

b) $f: [0,1] \to [0,1]$ $f(x) = \frac{1}{4} e^{x}$

Pdq f posee un único punto fijo

Sol)

Por el teo. del Punto Fijo nos basta probar que f es contractante

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ahora por TVM

$\exists\, c \in [0,1]$ t.q

$$f(x) - f(y) = f'(c) \cdot (x - y)$$

$$\rightarrow |f(x) - f(y)| = |f'(c)| \cdot |x - y|$$

$$f'(x) = \frac{1}{4} e^{x}$$

$$\Rightarrow |f'(c)| \leq \frac{1}{4} e^{1} < 1 \quad \xrightarrow{\ } e \leq 3$$

$\therefore\ |f(x) - f(y)| < |x - y| \quad \forall x, y$

así f contractante

y $\therefore$ tiene un único punto fijo.

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c) Demuestre que el sistema no lineal de ecuaciones

$$2 - x + \sin(y) = 0$$
$$6 + e^{-x^2} - 2y = 0$$

Posee una única solución en $\mathbb{R}^2$. Para ello, considere la función $f: \mathbb{R}^2 \to \mathbb{R}^2$ dada por

$$f(x,y) = \left(1 + \frac{x}{2} + \frac{1}{2}\sin(y), 3 + \frac{1}{2}e^{-x^2}\right)$$

Use apropiadamente el TVM para mostrar que f es contractante y concluya

Indicación: puede utilizar

$$x^2 e^{-2x^2} \leq \frac{1}{4} \quad \forall x \in \mathbb{R}$$

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sol)

notemos que

$f(x,y)=(x,y)$

$\Longleftrightarrow \quad 1+\frac{x}{2}+\frac{1}{2}\sin(y)=x$

$\quad\quad 3+\frac{1}{2}e^{-x^2}=y$

$\Longleftrightarrow \quad 2+x+\sin(y)=2x$

$\quad\quad 6+e^{-x^2}=2y$

$\Longleftrightarrow \quad 2-x+\sin(y)=0$

$\quad\quad 6+e^{-x^2}-2y=0$

$\therefore$ Si existe un unico $(\bar{x},\bar{y})$ T.q

$f(\bar{x},\bar{y})=(\bar{x},\bar{y})$ el sistema

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Tiene solución única.

Para esto sea

$$f_1(x,y) = 1 + \frac{x}{2} + \frac{1}{2}\sin(y)$$

$$f_2(x,y) = 3 + \frac{1}{2}e^{-x^2}$$

$\exists\, c \in \mathbb{R}^2$

$$\Rightarrow \quad f_1(x,y) - f_1(u,v) = \left\langle \nabla f_1(c), \begin{pmatrix} x-u \\ y-v \end{pmatrix} \right\rangle$$

$$f_2(x,y) - f_2(u,v) = \left\langle \nabla f_2(w), \begin{pmatrix} x-u \\ y-v \end{pmatrix} \right\rangle$$

$\exists\, w \in \mathbb{R}^2$

así

$$\| f(x,y) - f(u,v) \|^2$$

$$= (f_1(x,y) - f_1(u,v))^2 + (f_2(x,y) - f_2(u,v))^2$$

$$= \left(\left\langle \nabla f_1(c), \begin{pmatrix} x-u \\ y-v \end{pmatrix} \right\rangle \right)^2 + \left(\left\langle \nabla f_2(w), \begin{pmatrix} x-u \\ y-v \end{pmatrix} \right\rangle \right)^2$$

$$\leq \|\nabla f_1(c)\|^2 \cdot \left\| \begin{pmatrix} x-u \\ y-v \end{pmatrix} \right\|^2 + \|\nabla f_2(w)\|^2 \left\| \begin{pmatrix} x-u \\ y-v \end{pmatrix} \right\|^2$$

C-S

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asi

$$\|f(x,y)-f(u,v)\| \leq \underbrace{\sqrt{\|\nabla f_1(c)\|^2+\|\nabla f_2(c)\|^2}}_{\text{Si es menor que 1 se tiene que}} \cdot \left\|\begin{pmatrix} x-u \\ y-v \end{pmatrix}\right\|$$

f es contractante y se concluye.

asi

$$\nabla f_1(x,y) = \begin{pmatrix} 1/2 \\ \frac{\cos(y)}{2} \end{pmatrix}$$

$$\|\nabla f_1(x,y)\|^2 = \frac{1}{4} + \frac{\cos^2(y)}{4}$$

$$\leq \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

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Para otro lado

$$\nabla f_2(x,y) = \begin{pmatrix} -x e^{-x^2} \\ 0 \end{pmatrix}$$

$$\Rightarrow \|\nabla f_2(x,y)\|^2 = \left(-x e^{-x^2}\right)^2$$

$$= x^2 e^{-2x^2}$$

$$\leq 1/4 \rightarrow \text{enunciado}$$

así

$$\|\nabla f_1(x,y)\|^2 + \|\nabla f_2(x,y)\|^2 \leq \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\Rightarrow \sqrt{\|\nabla f_1(x,y)\|^2 + \|\nabla f_2(x,y)\|^2} \leq \sqrt{\frac{3}{4}} < 1$$

$\forall (x,y) \in \mathbb{R}^2$ así f

es contractante y $\therefore$

existe un único punto fijo //

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P4)

a) sea $f: \mathbb{R} \to \mathbb{R}$ una función de clase $\mathcal{C}^1$. Sea

$$C = \{ (x,y,z) \in \mathbb{R}^3 \mid z = x f\left(\frac{x}{y}\right), y \neq 0 \}$$

pdq todos los planos tangentes a la superficie C pasan por el origen.

Sol)

lo que nos piden demostrar es equivalente a que

para $g(x,y) = x f\left(\frac{x}{y}\right)$ su plano tangente pase por el 0.

asi

$$z = g(x_0, y_0) + \nabla g(x_0, y_0) \cdot \begin{pmatrix} x - y_0 \\ y - y_0 \end{pmatrix}$$

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$$\nabla g(x,y) = \begin{pmatrix} f\left(\frac{x}{y}\right) + x f'\left(\frac{x}{y}\right)\cdot\frac{1}{y} \\ + x f'\left(\frac{x}{y}\right)\frac{-x}{y^2} \end{pmatrix}$$

$\rightarrow$

$$Z = x_0 f\left(\frac{x_0}{y_0}\right) + \begin{pmatrix} f\left(\frac{x_0}{y_0}\right) + \frac{x_0}{y_0} f'\left(\frac{x_0}{y_0}\right) \\ -\frac{x_0^2}{y_0^2} f'\left(\frac{x_0}{y_0}\right) \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$$

veamos

$$Z(0,0) = x_0 f\left(\frac{x_0}{y_0}\right) + \begin{pmatrix} f\left(\frac{x_0}{y_0}\right) + \frac{x_0}{y_0} f'\left(\frac{x_0}{y_0}\right) \\ -\frac{x_0^2}{y_0^2} f'\left(\frac{x_0}{y_0}\right) \end{pmatrix} \begin{pmatrix} -x_0 \\ -y_0 \end{pmatrix}$$

$$= x_0 f\left(\frac{x_0}{y_0}\right) - x_0 f\left(\frac{x_0}{y_0}\right) - \frac{x_0^2}{y_0} f'\left(\frac{x_0}{x_0}\right)$$

$$+ \frac{x_0^2}{y_0} f'\left(\frac{x_0}{y_0}\right)$$

$$= 0$$

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$\therefore$ el punto $(0,0,0)$ esta en el grafo del plano tangente

$\Rightarrow$ Toda superficie tangente a C pasa por $(0,0,0)$

b) Encuentre el plano tangente al grafo de la función

$$f(x,y) = e^{x^2+y^2-x-2y}$$

en $(0,0)$

Sol

$$Z = f(0,0) + \nabla f(0,0)\begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$$

$$= 1 + \nabla f(0,0)\begin{pmatrix} x \\ y \end{pmatrix}$$

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$$\nabla f = \begin{pmatrix} e^{x^2+y^2-x-2y}(2x-1) \\ e^{x^2+y^2-x-2y}(2y-2) \end{pmatrix}$$

$$\Rightarrow \nabla f(0,0) = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\Rightarrow z = 1 + \begin{pmatrix} -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 - x - 2y$$

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P5| Considere un sistema de coordenadas Σ (en reposo), de modo que un punto del espacio P en un instante t. Se representa por (x, y, z, t).

Considere otro sistema Σ^* que se mueve en dirección x con respecto a Σ con velocidad constante $0 < k < c$ → velocidad de la luz

y cuando $t = 0$ coincide con Σ.

Sea $\phi(x, y, z, t)$ una función de clase $\mathcal{C}^2$ tal que:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$$

notación,

$$\phi_x = \frac{\partial \phi}{\partial x}, \quad \phi_y = \frac{\partial \phi}{\partial y}, \quad \phi_z = \frac{\partial \phi}{\partial z}$$

$$\phi_{xx} = \frac{\partial^2 \phi}{\partial x^2}, \quad \phi_{yx} = \frac{\partial^2 \phi}{\partial x \partial y} \quad \ldots$$

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así

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = \frac{1}{c^2}\phi_{\lambda\lambda}$$

a) Para representar un punto P de coordenadas (x, y, z, λ) en Σ en el sistema Σ^* se plantea el siguiente cambio de variables que corresponden al enfoque clásico (Galilea)

$$u = x - k\lambda, \quad v = y, \quad w = z, \quad \tau = \lambda$$

Deduzca el cambio de variable.

sea $\phi(x, y, z, \lambda) = \phi_1(u, v, w, \tau)$

demuestre que

$$\phi_{1uu} + \phi_{1vv} + \phi_{1ww} = \frac{1}{c^2}\left[\phi_{1\tau\tau} + k^2\phi_{1uu} - 2k\,\phi_{1u\tau}\right]$$

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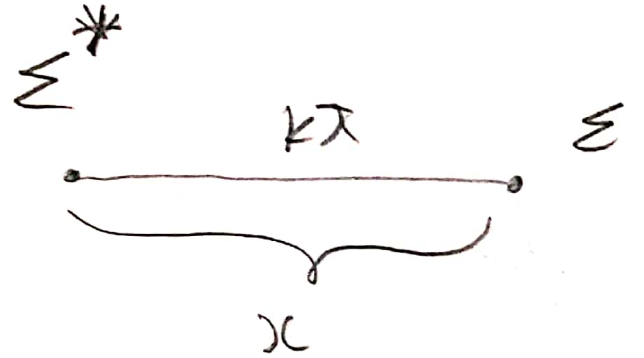
Sol,

como Σ^* se mueve en 'x' se tiene

$v = y$, $w = z$; a su vez el tiempo

no cambia $\lambda = \tau$ en x



$\Rightarrow \quad u = x - k\tau$

ahora

$$\phi(x, y, z, \tau) = \phi_1(x - k\tau, y, z, \tau)$$

así

$$\phi_x = \phi_{1u} \ , \ \phi_y = \phi_{1v} \ , \ \phi_z = \phi_{1w}$$

$\Rightarrow$ así $\phi_{1uu} + \phi_{1vv} + \phi_{1ww} = \phi_{xx} + \phi_{yy} + \phi_{zz}$

$$= \frac{1}{c^2} \phi_{\tau\tau}$$

$\therefore$ Falta calcular $\phi_{\lambda\lambda}$

$$\phi_{\lambda} = \phi_{1u}(x-k\lambda, y, z, \lambda) \cdot -k$$
$$+ \phi_{1\tau}(x-k\lambda, y, z, \lambda)$$

$$\phi_{\lambda\lambda} = -k\left[\phi_{1uu}(x-k\lambda, y, z, \lambda) \cdot -k + \phi_{1u\tau}(x-k\lambda, y, z, \lambda)\right]$$
$$+ \phi_{1\tau u}(x-k\lambda, y, z, \lambda) \cdot -k$$
$$+ \phi_{1\tau\tau}(x-k\lambda, y, z, \lambda)$$

$$= k^2 \phi_{1uu} - 2k\phi_{1u\tau} + \phi_{1\tau\tau}$$

así

$$\phi_{1uu} + \phi_{1vv} + \phi_{1ww} = \frac{1}{c^2}\left[\phi_{1\tau\tau} + k^2\phi_{1uu} - 2k\phi_{1u\tau}\right]$$

b) Tambiem se Plamtea el siguiente cambio de variables que corresponde al enfoque relavista moderno (Lorentz)

$$u = \gamma_0 (x - k\lambda), \quad v = y, \quad w = z$$

$$\tau = \gamma_0 \left(\lambda - \frac{k}{c^2} x\right)$$

donde $\gamma_0 \in \mathbb{R}$ es una cte de valor

$$\gamma_0 = \left(1 - \left(\frac{k}{c}\right)^2\right)^{-1/2}$$

sea $\phi_2(u, v, w, \tau) = \phi(x, y, z, \lambda)$

1) pdq

$$\frac{\partial \phi}{\partial x} = \gamma_0 \left[\frac{\partial \phi_2}{\partial u} - \frac{k}{c^2} \frac{\partial \phi_2}{\partial \tau} \right]$$

$$\frac{\partial \phi}{\partial \lambda} = \gamma_0 \left[\frac{\partial \phi_2}{\partial \tau} - k \frac{\partial \phi_2}{\partial u} \right]$$

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¿ ademas argumente que

$$\frac{\partial \phi}{\partial y} = \frac{\partial \phi_2}{\partial v}, \quad \frac{\partial \phi}{\partial z} = \frac{\partial \phi_2}{\partial w}$$

Sol] las ultimas dos ecuaciones se deben a que no hay cambio en esas variables.

Por otro lado

$$\phi(x,y,z,\lambda) = \phi_2\left(\gamma_0(x-k\lambda), y, z, \gamma_0\left(\lambda - \frac{k}{c^2}x\right)\right)$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi_2}{\partial u}\cdot\gamma_0 - \frac{k\gamma_0}{c^2}\frac{\partial \phi_2}{\partial \lambda}$$

$$= \gamma_0\left[\frac{\partial \phi_2}{\partial u} - \frac{k}{c^2}\frac{\partial \phi_2}{\partial \lambda}\right]$$

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$$\frac{\partial \phi}{\partial \lambda} = \frac{\partial \phi_2}{\partial u} \cdot -k\gamma_0 + \gamma_0 \frac{\partial \phi_2}{\partial \lambda}$$

$$= \gamma_0 \left[\frac{\partial \phi_2}{\partial \lambda} - k \frac{\partial \phi_2}{\partial u} \right]$$

2] Finalmente, usando lo anterior demuestre que

$$\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi_2}{\partial v^2} + \frac{\partial^2 \phi_2}{\partial w^2} = \frac{1}{c^2} \frac{\partial^2 \phi_2}{\partial \tau^2}$$

Sol] calculemos

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial \phi}{\partial x}\right]$$

$$= \frac{\partial}{\partial x}\left[\gamma_0 \left[\frac{\partial \phi_2}{\partial u}\left(\gamma_0(x-k\lambda), y, z, \gamma_0\left(\lambda - \frac{k}{c^2}x\right)\right) - \frac{k}{c^2} \frac{\partial \phi_2}{\partial \lambda}\left(\gamma_0(x-k\lambda), y, z, \gamma_0\left(\lambda - \frac{k}{c^2}x\right)\right)\right]\right]$$

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$$= \gamma_0 \left[\frac{\partial^2 \phi_2}{\partial u^2} \cdot \gamma_0 + \frac{\partial^2 \phi_2}{\partial \lambda \partial u} \cdot - \frac{\gamma_0 K}{c^2} \right.$$

$$\left. - \frac{K}{c^2} \left[\frac{\partial^2 \phi_2}{\partial u \partial \lambda} \gamma_0 + \frac{\partial^2 \phi_2}{\partial \lambda^2} - \frac{\gamma_0 K}{c^2} \right] \right]$$

$$= \gamma_0^2 \left[\frac{\partial^2 \phi_2}{\partial u^2} - \frac{K}{c^2} \frac{\partial \phi_2}{\partial \lambda \partial u} - \frac{K}{c^2} \frac{\partial^2 \phi_2}{\partial u \partial \lambda} + \frac{K^2}{c^4} \frac{\partial^2 \phi_2}{\partial \lambda^2} \right.,$$

$$= \gamma_0^2 \left[\frac{\partial^2 \phi_2}{\partial u^2} - 2 \frac{K}{c^2} \frac{\partial \phi_2}{\partial u \partial \lambda} + \frac{K}{c^4} \frac{\partial^2 \phi_2}{\partial \lambda^2} \right]$$

por otro lado

$$\frac{\partial \phi}{\partial \lambda} = \gamma_0 \left[\frac{\partial \phi_2}{\partial \tau} \left(\gamma_0 (x - K\lambda), y, z, \gamma_0 \left(\lambda - \frac{K}{c^2} x \right) \right) \right.$$

$$\left. - K \frac{\gamma_0 \phi_2}{\partial u} \left(\gamma_0 (x - K\lambda), y, z, \gamma_0 \left(\lambda - \frac{K}{c^2} x \right) \right) \right]$$

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$$\frac{\partial^2 \phi}{\partial x^2} = \gamma_0 \Big[\frac{\partial^2 \phi_2}{\partial u \partial \tau} \cdot -\gamma_0 K + \frac{\partial^2 \phi_2}{\partial \tau^2} \cdot \gamma_0 - K \Big[-\gamma_0 K \cdot \frac{\partial^2 \phi_2}{\partial u^2} + \gamma_0 \frac{\partial^2 \phi_2}{\partial u \partial \lambda} \Big] \Big]$$

$$= \gamma_0^2 \Big[-K \frac{\partial^2 \phi_2}{\partial u \partial \lambda} + \frac{\partial^2 \phi_2}{\partial \tau^2} + K^2 \frac{\partial^2 \phi_2}{\partial u^2} - K \frac{\partial^2 \phi}{\partial u \partial \lambda} \Big]$$

$$= \gamma_0^2 \Big[K^2 \frac{\partial^2 \phi_2}{\partial u^2} - 2K \frac{\partial^2 \phi_2}{\partial u \partial \lambda} + \frac{\partial^2 \phi_2}{\partial \tau^2} \Big]$$

asi obtenemos, que como

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial \lambda^2}$$

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$$\gamma_0^2\left(\frac{\partial^2\phi}{\partial u^2} - \frac{2K}{c^2}\frac{\partial^2\phi}{\partial u\partial\tau} + \frac{K^2}{c^4}\frac{\partial^2\phi_2}{\partial\lambda^2}\right)$$

$$+\frac{\partial^2\phi_2}{\partial v^2} + \frac{\partial^2\phi_2}{\partial w^2} = \frac{\gamma_0^2}{c^2}\left(K^2\frac{\partial^2\phi_2}{\partial u^2} - 2K\frac{\partial^2\phi_2}{\partial u\partial\tau} + \frac{\partial^2\phi_2}{\partial\tau^2}\right)$$

$\rightarrow$

$$\gamma_0^2\frac{\partial^2\phi}{\partial u^2} - \frac{2K}{c^2}\cancel{\gamma_0^2}\frac{\partial^2\phi}{\partial u\partial\tau} + \frac{\gamma_0^2K^2}{c^4}\frac{\partial^2\phi_2}{\partial\tau^2}$$

$$+\frac{\partial^2\phi_2}{\partial v^2} + \frac{\partial^2\phi_2}{\partial w^2}$$

$$= \frac{K^2\gamma_0^2}{c^2}\frac{\partial^2\phi_2}{\partial u^2} - \frac{2K\gamma_0^2}{c^2}\frac{\partial^2\phi}{\partial u\partial\lambda}$$

$$+\frac{\gamma_0^2}{c^2}\frac{\partial^2\phi_2}{\partial\lambda^2}$$

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$$\partial_0^2 \left(1 - \frac{K^2}{C^2}\right) \frac{\partial^2 \phi}{\partial u^2}$$

$$+ \frac{\partial^2 \phi_2}{\partial v^2} + \frac{\partial^2 \phi_2}{\partial w^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial \lambda^2} \partial_0^2 \left[1 - \frac{K^3}{C^2}\right]$$

$$\cancel{\partial_0^2} \frac{1}{\cancel{\partial_0^2}} \frac{\partial^2 \phi_2}{\partial u^2} + \frac{\partial^2 \phi_2}{\partial v^2} + \frac{\partial^2 \phi_2}{\partial w^2}$$

$$= \frac{1}{C^2} \frac{\partial^2 \phi_2}{\partial \lambda^2} \cancel{\partial_0^2} \cdot \frac{1}{\cancel{\partial_0^2}}$$

$$\rightarrow \frac{\partial^2 \phi_2}{\partial u^2} + \frac{\partial^2 \phi_2}{\partial v^2} + \frac{\partial^2 \phi_2}{\partial w^2} = \frac{1}{c^2} \frac{\partial^2 \phi_2}{\partial \lambda^2}$$

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